CARIBBEAN
EXAMINATIONS
COUNCIL

Caribbean Advanced Proficiency Examination®

SYLLABUS

APPLIED MATHEMATICS

CXC A9/U2/22

Effective for examinations from May–June 2023

CAPE®

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NOTE TO TEACHERS AND LEARNERS

This document CXC A9/U2/*22* replaces CXC A9/U2/07 issued in *2007*.

Please note that the syllabuses have been revised and amendments are indicated by italics.

First Issued 1999
Revised 2004
Revised 2007
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Please check the website www.cxc.org for updates on **CXC®**'s syllabuses.

Please access relevant curated resources to support teaching and learning of the syllabus at https://learninghub.cxc.org/

For access to short courses, training opportunities and teacher orientation webinars and workshops go to our Learning Institute at https://cxclearninginstitute.org/

PLEASE NOTE

This icon is used throughout the syllabus to represent key features which teachers and learners may find useful.

Introduction

The Caribbean Advanced Proficiency Examination (**CAPE®**) is designed to provide certification of the academic, vocational and technical achievement of students in the Caribbean who, having completed a minimum of five years of secondary education, wish to further their studies. The examinations address the skills and know ledge acquired by students under a flexible and articulated system where subjects are organised in 1-Unit or 2-Unit courses with each Unit containing three Modules. Subjects examined under **CAPE®** may be studied concurrently or singly.

The Caribbean Examinations Council offers three types of certification at the **CAPE®** level. The first is the award of a certificate showing each **CAPE®** Unit completed. The second is the **CAPE®** Diploma, awarded to candidates who have satisfactorily completed at least six Units, including Caribbean Studies. The third is the **CXC®** Associate Degree, awarded for the satisfactory completion of a prescribed cluster of *ten* **CAPE®** Units including Caribbean Studies, Communication Studies *and Integrated Mathematics. Integrated Mathematics is not a requirement for candidates pursuing the* ***CXC®*** *Associate Degree in Mathematics or pursuing Pure and Applied Mathematics Unit 1 OR 2.* The complete list of Associate Degrees may be found in the **CXC®** Associate Degree Handbook.

For the **CAPE®** Diploma and the **CXC®** Associate Degree, candidates must complete the cluster of required Units within a maximum period of five years. *To be eligible for a* ***CXC®*** *Associate Degree, the educational institution presenting the candidates for the award, must select the Associate Degree of choice at the time of registration at the sitting (year) the candidates are expected to qualify for the award.* Candidates will not be awarded an Associate Degree for which they were not registered.

Applied Mathematics Syllabus

♦ RATIONALE

The Caribbean society is an integral part of an ever-changing world. The impact of globalisation on most societies encourages this diverse Caribbean region to revisit the education and career opportunities of our current and future citizens. A common denominator is for Caribbean societies to create among its citizens a plethora of quality leadership with the acumen required to make meaningful projections and innovations for further development. Learning appropriate problem-solving techniques, inherent to the study of mathematics, is vital for such leaders. Mathematics promotes intellectual development, is utilitarian and applicable to all disciplines. Additionally, its aesthetics and epistemological approaches provide solutions fit for any purpose. Therefore, Mathematics is the essential tool to empower people with the knowledge, competencies and attitudes that are required for academia as well as quality leadership for sustainability in this dynamic world.

The main emphasis of the Applied Mathematics course of study is on developing the ability of the students to start with a problem in non-mathematical form and transform it into mathematical language. This will enable them to bring mathematical insights and skills in devising a solution, and then interpreting this solution in real-world terms. Students will accomplish *this through learning and assessment activities that require them to explore* problems using symbolic, graphical, numerical, physical and verbal techniques in the context of finite or discrete real-world situations. Furthermore, students will engage in mathematical thinking and modelling to examine and solve problems arising from a wide variety of disciplines including, but not limited to, economics, medicine, agriculture, marine science, law, transportation, engineering, banking, natural sciences, social sciences and computing.

This course of study incorporates the features of the Science, Technology, Engineering, and Mathematics (STEM) principles. On completion of this Syllabus, students will be able to make a smooth transition to further studies in Mathematics and other related subject areas or move on to career choices where a deeper knowledge of the general concepts of Mathematics is required. It will enable students to develop and enhance Twenty-first century skills including critical and creative thinking, problem solving, logical reasoning, modelling, collaboration, decision making, research and, information communication and technological competencies which are integral to everyday life and for life-long learning. This course thus provides insight into the exciting world of advanced mathematics, thereby equipping students with the tools necessary to approach any mathematical situation with confidence.

This Syllabus will contribute to the development of the Ideal Caribbean Person as articulated by the CARICOM Heads of Government. *This person will* demonstrate multiple literacies, independent and critical thinking; and *question the beliefs and practices of the past and present, bringing it to bear on the* innovative application of science and technology to problem solving. Such a person should also demonstrate a positive work *ethic* and values and display creative imagination and entrepreneurship. In keeping with the UNESCO Pillars of Learning, on completion of this course of study, students will *learn to know,* learn to do, learn to be, *learn to live together,* and learn to transform themselves and society.

♦ AIMS

The syllabus aims to enable students to:

1. *be* equip*ped* with *the skills needed* for data collection, organisation and analysis in order to make valid decisions and predictions;

2. use appropriate statistical language and form in written and oral presentations;

3. develop an awareness of the exciting applications of Mathematics;

4. develop a willingness to apply Mathematics to relevant problems that are encountered in daily activities;

5. understand certain mathematical concepts and structures, their development and their interrelationships;

6. use *relevant technology* to enhance mathematical investigations;

7. develop the skills of recognising essential aspects of real-world problems and translating these problems into mathematical forms;

8. develop the skills of defining the limitations of the model and the solution;

9. apply Mathematics across the subjects of the school curriculum;

10. acquire relevant skills and knowledge for access to advanced courses in Mathematics and/or its applications in other subject areas;

11. gain experiences that will act as a motivating tool for the use of technology;

12. *develop skills such as, critical and creative thinking, problem solving, logical reasoning, modelling, collaboration, decision making, research, information and communication and technological competencies which are integral to everyday life and for life-long learning; and,*

13. *integrate Information Communication and Technology (ICT) tools and skills in the learning process.*

♦ SKILLS AND ABILITIES TO BE ASSESSED

The assessment will test candidates' skills and abilities in relation to three cognitive levels.

1. **Conceptual knowledge** - *the ability to* ***recall****, and* ***understand*** *appropriate facts, concepts and principles in a variety of contexts.*

2. ***Algorithmic knowledge*** *- the ability to* ***manipulate*** *mathematical expressions and procedures using appropriate symbols and language, logical deduction and inferences.*

3. ***Reasoning*** *- the ability to* ***select*** *appropriate strategy or select,* ***use*** *and* ***evaluate*** *mathematical models and* ***interpret*** *the results of a mathematical solution in terms of a given real-world problem and engage in problem-solving.*

♦ PREREQUISITES OF THE SYLLABUS

Any person with a **good** grasp of the contents of the syllabus of the Caribbean Secondary Education Certificate (**CSEC®**) course in Mathematics, or equivalent, should be able to undertake the course. *However, persons with a **good** grasp of the contents of the syllabus of the Caribbean Secondary Education Certificate (**CSEC®**) course in Additional Mathematics would be better prepared to pursue this course of study.* Successful participation in the course will also depend on the possession of good verbal and written communication skills.

♦ STRUCTURE OF THE SYLLABUS

The *syllabus* is organised in two *(2)* Units. A Unit comprises three Modules each requiring 50 hours. The total time for each Unit is, therefore, expected to be 150 hours. Each Unit can independently offer students a comprehensive programme of study with appropriate balance between depth and coverage to provide a basis for further study in this field.

Unit 1: Statistical Analysis

Module 1	-	Collecting and Describing Data
Module 2	-	Managing Uncertainty
Module 3	-	Analysing and Interpreting Data

Unit 2: Mathematical Applications

Module 1	-	Discrete *Mathematics*
Module 2	-	Probability and Distributions
Module 3	-	Particle Mechanics

♦ APPROACHES TO TEACHING THE SYLLABUS

The Specific Objectives indicate the scope of the content and activities that should be covered. Teachers are encouraged to utilise a learner-centered approach to teaching and learning. They are also encouraged to model the process for completing, solving, and calculating mathematical problems. It is recommended that activities to develop these skills be incorporated in every lesson through the use of collaborative, integrative and practical teaching strategies. Note as well that additional notes and the formulae sheet are included in the syllabus.

♦ RECOMMENDED 2-UNIT OPTIONS FOR CAPE® MATHEMATICS

1. Pure Mathematics Unit 1 **AND** Pure Mathematics Unit 2.

2. Applied Mathematics Unit 1 **AND** Applied Mathematics Unit 2.

3. Pure Mathematics Unit 1 **AND** Applied Mathematics Unit 2.

♦ UNIT 1: STATISTICAL ANALYSIS
MODULE 1: COLLECTING AND DESCRIBING DATA

GENERAL OBJECTIVES

On completion of this Module, students should:

1. understand the concept of *sampling* and its role in data collection;
2. *develop appropriate skills for data collection and (describing) data analysis; and,*
3. appreciate that data can be represented both graphically and numerically.

SPECIFIC OBJECTIVES	**CONTENTS**
1. *Data Collection*	
Students should be able to:	
1.1 determine appropriate sources of data;	*Advantages and disadvantages of primary and secondary sources of data.*
1.2 distinguish *between types of data;*	*Types of data:* *(a)* primary and secondary; *(b) quantitative and qualitative; and,* *(c) discrete and continuous.*
1.3 distinguish *between statistical concepts;*	*Explanation of and distinction among statistical concepts:* (a) *population and sample;* (b) *census and sample survey; and,* (c) *parameter and statistic.* *Appropriate use of the concepts.*
1.4 determine appropriate sampling frames for given situations;	*Definition of sampling frame.* *Selection of appropriate* sampling frame *from the population*.
1.5 justify the need for sampling;	*Reasons for sampling.* *Ideal characteristics of a sample.*

SPECIFIC OBJECTIVES	CONTENTS
Data Collection (cont'd)	
Students should be able to:	
1.6 use sampling methods appropriately;	*Role of randomness in statistical work.* *Random sampling: simple random, stratified random, systematic random.* *Non-random sampling: cluster, quota, convenience and snowballing sampling.* *Advantages and disadvantages of sampling methods.*
1.7 use appropriate methods to obtain a simple random sample;	*Simple Random Methods to include:* *(a) random numbers (from a table or calculator); and,* *(b) lottery techniques.*
1.8 design *data collection instruments*;	*Appropriate use of the following instruments:* (a) questionnaires; (b) interviews; and, (c) observation schedules. *Structure and components of good data collection instruments.*
2. *Describing Data*	
Students should be able to:	
2.1 construct frequency distribution *tables* from raw data;	*Frequency distributions tables for grouped or ungrouped data.*
2.2 *interpret data from frequency distribution tables;*	*Class boundaries, class width, frequency density.*

SPECIFIC OBJECTIVES		CONTENT
Describing Data (cont'd)		
Students should be able to:		
2.3	*use appropriate statistical charts and diagrams to illustrate data;*	*Use the relative advantages and disadvantages to determine appropriate use:* *Pie charts, bar charts, stem-and-leaf diagrams and box-and-whisker plots, histograms, frequency polygons in data analysis.*
2.4	construct *statistical charts and diagrams based on given data;*	*Pie charts, bar charts, stem-and-leaf diagrams (including back-to-back diagrams), box-and-whisker plots, histograms, frequency polygons, cumulative frequency curves (ogives).*
2.5	*calculate measures of central tendency;*	*Ungrouped data:* *Mean, trimmed mean (given percentage from both ends), median, mode.* *Grouped data:* *Mean, mode and median.*
2.6	*use measures of central tendency appropriately;*	Advantages and disadvantages *of various measures of central tendency.* *Impact of outliers on measures of central tendency.*
2.7	calculate *measures of variability; and,*	*Calculations from ungrouped and grouped data:* *Range, quartiles, interquartile range, semi-interquartile range, percentiles, variance, standard deviation.* *From statistical charts, and diagrams and given data.*
2.8	*interpret results from statistical calculations.*	*Using measures of variability and measures of central tendency to determine:* *(a) shape of distribution (uniformity, symmetry and skewness); and,* *(b) impact of outliers.*

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Students should be encouraged to participate in small-group projects that require them* to collect data *using an appropriate data collection instrument.*

2. *Students should be guided in gathering secondary data on* the advantages and disadvantages of different sampling methods. *The findings should be presented, using PowerPoint or other technologies, and discussed in class. Students can also be encouraged to reflect on their experiences with the method (s) used to collect data in (a) above.*

3. *Students should be encouraged to, and be guided in, critiquing data collection instruments.*

4. *Students should be encouraged to construct questionnaires using an appropriate programme for example survey gizmo, to obtain information about the subjects, other than Mathematics done by students in the class.*

5. *Teachers are encouraged to incorporate the use of group work in which each group is encouraged to report on the results from the number of tosses of a fair die. Different groups can be asked to carry-out a set number of tosses and report on their results.*

6. *Students should be encouraged to work together to investigate the relationship between the height of a person and the distance with which the student throws a ball. Ask students to record their findings. The findings will be analysed when looking at Unit 1, Module 3.*

7. *Students should be encouraged to work in groups to investigate the number of siblings of students in the class. They should then report on their findings using either a table or graph.*

8. *Teachers are encouraged to demonstrate how* graphical representations such as histograms, pie-charts, box-and-whisker plots should be used for preliminary analysis of data. *Students should then be asked to represent data that has been collected in an appropriate form using the methods discussed.*

9. *Students should be encouraged to use back-to-back stem and leaf diagrams comparing money spent on lunch on different days of the week.*

10. *Teachers should encourage discussions on the relative advantages and usefulness of the mean, quartiles, standard deviation of grouped and ungrouped data and on the shape of frequency distributions. Students should also be guided in utilizing the functions of the calculator to calculate the mean and standard deviation of given values.*

11. *In groups of three or four, students should be required to measure the length of 10 leaves (from the base to the tip of the leaf) from a* ***single tree*** *in the school garden. Using tally charts, these measurements can then be put into groups, and an estimate given of the mean length of the leaves. Students should also calculate the standard deviation of the length.*

UNIT 1
MODULE 1: COLLECTING AND DESCRIBING DATA (cont'd)

RESOURCES

Anderson, D.R., Sweeney, D.J., Williams, T. A., Camm, J.D. and Cochran, J.J.	*Statistics for Business and economics (13th Ed.) Ohio: South-Western college Publisher, 2016.*
Crawshaw, J. and Chambers, J.	*A Concise Course in A-Level Statistics (4th Ed)*. Cheltenham: Stanley Thornes Limited, 2001.
Mahadeo, R.	*Statistical Analysis – The Caribbean Advanced Proficiency Examinations A Comprehensive Text*. Trinidad and Tobago: Caribbean Educational Publishers Limited, 2007.
Mann, P.S.	*Introductory Statistics (9th Ed). New Jersey: Wiley, 2016.*
Upton, G. and Cook, I.	*Introducing Statistics*. Oxford: Oxford University Press, 2001.

GENERAL OBJECTIVES

On completion of this Module, students should:

1. understand the concept of probability;

2. appreciate that probability models can be used to describe real world situations and to manage uncertainty; *and,*

3. *understand how to construct diagrams to facilitate probability problem solving.*

SPECIFIC OBJECTIVES	CONTENTS
1. Probability Theory	
Students should be able to:	
1.1 determine the outcomes of a given experiment;	*Elements of a possibility space.* *Elements of an event.*
1.2 calculate the probability of event A;	*Concept of probability.* $P(A)$, as the number of outcomes of A divided by the total number of possible outcomes $$P(A) = \frac{n(A)}{n(S)}$$
1.3 use the *basic rules of probability;*	*Basic rules:* (a) probability of an event A is a real number between *0 and 1 inclusive;* $(0 \leq P(A) \leq 1)$*;* (b) the sum of all the n probabilities of points in the sample space is 1; *and,* $$\sum_{i=1}^{n} p_i = 1$$

SPECIFIC OBJECTIVES		CONTENT
Probability Theory (cont'd)		
Students should be able to:		
		(c) $P(A') = 1 - P(A)$, where P(Á) is the probability that event *A* does not occur.
1.4	calculate *the probability of the union and the intersection of two sets;*	*Addition rule of probability* $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
1.5	*determine types of* events;	*Types of events:* (a) *Exhaustive events:* $P(A \cup B) = 1$ (b) *Mutually exclusive events:* $P(A \cap B) = 0$ *or* $P(A \cup B) = P(A) + P(B)$. (c) *Independent events:* $P(A \cap B) = P(A) \times P(B)$ *or* $P(A\|B) = P(A)$.
1.6	calculate *the conditional probability; and,*	*Conditional probability:* $P(A\|B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0.$ $P(A\|B) = P(A)$ *for independent events.*
1.7	*use possibility space diagrams to calculate probabilities.*	*Construction of possibility space diagrams.* *Possibility space diagrams (probability sample space), tree diagrams, Venn diagrams, contingency tables.*

SPECIFIC OBJECTIVES	CONTENT
2. Random Variables	
Students should be able to:	
2.1 construct probability distribution tables for discrete random variables;	Probability distribution *tables which* assign probabilities to values *of the discrete random variable x.*
2.2 use a given probability function of a discrete random variable;	*Probabilities:* $P(X = a), P(X > a)$, $P(X < b), P(X \geq b), P(X \leq a)$, *or any combination of these, where a and b are real numbers.*
2.3 *use the properties of the probability distribution* of a *discrete* random variable X;	*Properties of discrete random variables.* *0 ≤ P(X = x) ≤ 1 for all* X $\sum_{i=1}^{n} P_i = 1$ expected value $E(X) = \sum_{i=1}^{n} x_i p_i$ variance $\text{Var}(X) = E(X^2) - (E(X))^2$ and standard deviation $= \sqrt{Var(X)}$
2.4 use the laws of expectation and variance for one variable;	*Laws of expectation:* $E(aX \pm b)$ *Laws of variance:* $Var(aX \pm b)$
2.5 construct a cumulative distribution function table from a probability distribution table *for discrete random variables*;	*Cumulative distribution function table from a probability density function and vice versa.*
2.6 use cumulative distribution function tables-*for discrete random variables;*	*Calculation of probabilities.*

SPECIFIC OBJECTIVES | **CONTENT**

Random Variables (cont'd)

Students should be able to:

2.7 use the properties of a probability density function, $f(x)$ of a continuous random variable X; *and,*

2.8 use areas under the graph of a probability density function as measures of probabilities *for continuous random variables*.

Continuous random variables.

Probability density function.

$0 \leq f(x) \leq 1$ for all x

The total area under the graph is 1.

Measures of dispersion and Measures of central tendency

(integration will not be tested), $P(X = a) = 0$ for any continuous random variable X and real number a.

3. Binomial Distribution

Students should be able to:

3.1 state the assumptions made in modelling data by a binomial distribution;

3.2 use *the binomial distribution as a model of data, where appropriate;*

3.3 use the mean and variance of a binomial distribution; and,

3.4 calculate *binomial* probabilities.

Conditions for discrete data to be modelled as a binomial distribution.

Binomial distribution notation.

X ~ Bin (n, p), where n is the number of independent trials and p is the probability of a successful outcome in each trial.

Expected value E(X), and variance Var(X), of the binomial distribution.

Binomial probabilities

P(X = a), P(X > a),

$P(X < a), P(X \geq a), P(X \leq a)$ *or any combination of these, where X ~ Bin (n, p).*

$P(X = r) = {}^nC_r p^r(1-p)^{n-r}$

SPECIFIC OBJECTIVES	CONTENT
4. Normal Distribution	
Students should be able to:	
4.1 describe the main features of the normal distribution;	*Properties of the normal distribution.*
4.2 use the normal distribution as a model of data *representation*, as appropriate;	*Normal distribution notation,* $X \sim N\ (\mu, \sigma^2)$, *where* μ *is the population mean and* σ^2 *is the population variance.*
4.3 *standardize the normal distribution;*	*Standardization of normal distribution.* *Standard normal distribution* , $Z(0,1)$.
4.4 determine probabilities from tabulated values of the standard normal distribution, Z ~ N (0, 1);	The standard normal distribution and the use of standard normal distribution tables. $P(Z < a)$
4.5 *de-standardize the variable of the normal distribution;*	*Standard normal distribution* , $Z(0,1)$.
4.6 *solve problems involving probabilities* z-scores; and,	*Probabilities involving the normal distribution.*
4.7 use the normal distribution as an approximation to the binomial distribution.	*Continuity correction in the context of a normal distribution approximation to a binomial distribution.* *Normal approximation to the binomial distribution (np > 5 and nq > 5), applying an appropriate continuity correction* (± 0.5).

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Students should be encouraged to use the* probability calculations and the properties of probability based on data obtained from activities carried out under *Description of Data*, Module 1.

2. *Students should be asked to draw a tree diagram to show the total outcomes of tossing a coin three times. They could then be asked to determine the number of outcomes when a coin is tossed four or five times.*

3. *Students should be encouraged to work in groups. Each group should be given a frequency table(s), requiring them to use the results to calculate simple probabilities. Selected groups can share how they arrived at their answers.*

4. *Students should be engaged in discussions* on the concepts of mutually exclusive and independent events using real world concepts. A variety of ways to represent mutually exclusive and independent events should be utilised.

5. *Students should be encouraged to complete activities similar to the following: let the random variable X represent the times that horses take to complete a race on a given race day, and Y represent the times that cars take to complete a race on a race day.*

   (a) *Are X and Y independent events?*

   (b) *Are X and Y mutually exclusive events?*

6. *Students should be encouraged to respond to the scenario 'a die is loaded in such a way that each even number is twice as likely to occur as each odd number'. They should then be asked to calculate P(H), where H is the event that a number greater than 3 occurs in a single roll of the die.*

7. *Students should be encouraged to participate in t class discussions* and activities to clarify the concepts of discrete and continuous random variables.

8. *Students should be encouraged to use their knowledge of Binomial Distribution to do activities similar to the following:*

   (a) *The probability that a student will get exactly 8 correct answers in a multiple choice test which has 15 items and each item has 4 equally likely correct choices.*

   (b) *A shipment of 10 sewing machines includes 3 that are defective. What is the probability that of the 5 machines purchased by a school, exactly 2 are defective?*

9. *Teachers are encouraged to incorporate the use of video presentations explaining when to use normal distribution. Examples should be included which show that the distribution is symmetric (the mean, mode and median are approximately equal).*

10. *Students should be encouraged to sketch a normal curve and locate the mean of the distribution. Students should also be encouraged to shade the area under the curve showing the given or required probability. Allow students to convert from Normal distribution to standard normal distribution.*

    Examples should include:

    (a) *A manufacturer makes two sizes of chocolate bars. The smaller bar has a mean weight of 110 grams with a standard deviation of 2 grams. The weights are normally distributed. Calculate the proportion of bars that are likely to have a weight (a) less than 106 grams;(b) between 108 and 112 grams.*

    (b) *The bigger size bars have a mean weight of 115 grams with the same standard deviation. These weights are also normally distributed. What proportion of the larger bars will have a weight less than the mean of the smaller bars?*

    (c) *It has been determined that the probability that a bank will reject a loan application is 0.20. Calculate the probability that of the 225 loan applications made last week, the bank will reject at most 40.*

    (d) *Using the measurement of the leaves collected in Module one, calculate the median length and the modal length of the leaves collected by the entire class. Discuss the skewness of the distribution of the length of the leaves of the tree.*

11. *Students should be guided in using the data collected from the activity in Unit 1, Module 1 (relationship between height of a student and the distance with which they can throw the ball). Students should be encouraged to do the following:*

    (a) *Draw a scatter diagram for the data.*

    (b) *Determine the independent variable.*

    (c) *Estimate the distance that a person of a given height will throw a ball.*

UNIT 1
MODULE 2: MANAGING UNCERTAINTY (cont'd)

RESOURCES

Anderson, D.R., Sweeney, D.J., Williams, T.A., Camm, J.D. and Cochran, J.J.	*Statistics for Business and economics (13th Ed.). Ohio: South-Western college Publisher, 2016.*
Crawshaw, J. and Chambers, J.	*A Concise Course in A-Level Statistics (4th Ed.).* Cheltenham: Stanley Chambers, J. Thornes Limited, *2001*.
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Mann, P.	*Introductory Statistics (9th Ed). Wiley Global Education, 2016.*
Upton, G. and Cook, I.	*Introducing Statistics. Oxford: Oxford University Press, 2001.*

GENERAL OBJECTIVES

On completion of this Module, students should:

1. understand *the uses of* the sampling distribution and confidence intervals *in providing* information about a population;

2. understand the *relevance* of tests of hypotheses regarding statements about a population parameter;

3. *appreciate the use of statistical information to make inferences;*

4. appreciate that finding possible associations between variables and measuring their strengths are key ideas of statistical inference; and,

5. *understand the use of scatter diagrams to illustrate bivariate data.*

SPECIFIC OBJECTIVES	CONTENT
1. Sampling Distribution and Estimation	
Students should be able to:	
1.1 *formulate sampling distributions;*	*Distribution of sample mean,* $\bar{X}$ $E(\bar{X}) = \mu$ and Var $(\bar{X}) = \dfrac{\sigma^2}{n}$ where μ the population mean, σ^2 is the population variance and n is the sample size; *Distribution of sample proportion,* P $E(P) = p_s$ and $Var(P) = \dfrac{p_s(1-p_s)}{n}$ where p_s is the *sample proportion*, and n is the sample size;

SPECIFIC OBJECTIVES	CONTENT
Sampling Distribution and Estimation (cont'd)	
Students should be able to:	
1.2 *apply the Central Limit Theorem where appropriate;*	*Central Limit Theorem (no proof required).* *If the random variable X has a normal distribution, then the sample mean $\overline{X}$ will have a normal distribution.* *If the random variable X does not have a normal distribution, then the sample size n must be large in order to use the CLT.*
1.3 calculate *point* estimates;	*Unbiased estimators for the population mean, proportion and variance.*
1.4 calculate confidence intervals; *and,*	*Confidence intervals for a population mean or proportion using a large sample drawn from a population of known or unknown variance with the condition that $n \geq 30$.*
1.5 *interpret the confidence intervals.*	*Concept of confidence intervals for a population mean and proportion.* *Probability that the interval contains the mean.*
***2.* Hypothesis Testing**	
Students should be able to:	
2.1 formulate hypothesis *tests*;	Null hypothesis, H_0. *A*lternative hypotheses, H_1.
2.2 *determine* whether a one-tailed test or a two-tailed test is appropriate;	One-tailed (*upper tail or lower tail)* and two-tailed tests.
2.3 *describe the type of errors associated with hypothesis tests;*	*Type I and Type II errors.*

SPECIFIC OBJECTIVES	CONTENT
Hypothesis Testing (cont'd)	
Students should be able to:	
2.4 *determine* the critical values from tables for a given test;	Critical values *(given the level of significance).*
2.5 *identify* the critical *or rejection region* for a given test;	*Use of critical values to identify the critical region or the rejection region/s (given the level of significance).*
2.6 evaluate from sample data the test statistic for testing a population mean or proportion;	*Test statistic for the population mean, μ* $Z_{test} = \dfrac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$ *Test statistic for the population proportion p* $Z_{test} = \dfrac{p_s - p}{\sqrt{\frac{p(1-p)}{n}}}$
2.7 apply a z-test; *and,*	Hypothesis test for: (a) a population mean when a sample is drawn from a normal distribution of known variance*;* (b) *a population mean where a large sample* $(n \geq 30)$ *is drawn from a non-normal population using the Central Limit Theorem; and,* (c) *test for proportion when a large sample (n ≥ 30) is drawn from a binomial distribution,* with the appropriate *continuity correction.*
2.8 *interpret the results of the z-test.*	*Identification of the critical values and the rejection region for the test.* *Valid conclusion for the test (consider the level of significance).*

SPECIFIC OBJECTIVES	CONTENT
Hypothesis Testing (cont'd)	
3. t-test	
Students should be able to:	
3.1 determine if a t-test is appropriate for a given situation;	*t distribution.* $T \sim t(n-1)$ $n < 30$, *and the population variance is unknown.*
3.2 evaluate the t-test statistic;	t-test statistic $t_{test} = \frac{\overline{X}-\mu}{\frac{\hat{s}}{\sqrt{n-1}}}$
3.3 determine the appropriate number of degrees of freedom for a given data set;	Degrees of freedom *in the context of a t-test.*
3.4 *read* probabilities from *t*-distribution tables;	*Use of t-distribution tables.*
3.5 apply a *t-test* for a *population* mean*; and,*	Hypothesis test for a population mean using a small sample *(n < 30)* drawn from a normal population of unknown variance.
3.6 *interpret the results of the t-test.*	*Identification of the critical values and the rejection region for the test.* *Valid conclusion for the test.*

SPECIFIC OBJECTIVES	CONTENT
4. ***Chi-Squared*** **(χ^2-) Test**	
Students should be able to:	
4.1 *formulate a χ^2-test for independence;*	*Null and Alternative Hypotheses for chi-squared tests.* *Hypothesis test for independence. (2×2 contingency tables and cells with expected frequency of less than 5* ***not*** *included).*
4.2 *evaluate the χ^2-test critical value;*	*Degrees of freedom, in context of the Chi-squared test, for a contingency table.* *Reading and interpreting probabilities from χ^2-tables.*
4.3 *evaluate expected frequency for each cell;*	$E_{ij} = \left(\frac{R_i \times C_j}{G}\right)$, *where R_i is the total of the i^{th} row, C_j is the total of the j^{th} column and G is the grand total.*
4.4 *evaluate* the *χ^2-test* statistic; and,	*Chi-squared test for independence* $\chi^2{}_{test} = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$ *Where O_i is the observed frequency, E_i is the expected frequency and N is the total frequency.*
4.5 *interpret the results of the χ^2-test.*	*Identification of the critical values and the rejection region for the test.* *Valid conclusion for the test.*

SPECIFIC OBJECTIVES	CONTENT
5. Correlation and Linear Regression Bivariate Data	
Students should be able to:	
5.1 distinguish between dependent and independent variables;	*Bivariate data.* *Dependent and independent variables.*
5.2 draw scatter diagrams to represent bivariate data;	*Scatter diagrams.*
5.3 *interpret* scatter diagrams;	
5.4 interpret the value of r, as related to the data;	*Calculation of the value of (r), the Product moment correlation coefficient.* *Calculation of S_{xy}, S_{xx}, S_{yy}* *Strength of relationship based on the value of r ($-1 \leq r \leq 1$)* *Correlation Coefficient:* *0 = no correlation* $0 < \lvert r \rvert \leq 0.2$ *(very weak)* $0.2 < \lvert r \rvert \leq 0.4$ *(weak)* $0.4 < \lvert r \rvert \leq 0.6$ *(fair/moderate)* $0.6 < \lvert r \rvert \leq 0.8$ *(strong)* $0.8 < \lvert r \rvert \leq 1$ *(very strong)* *1 = Perfect correlation*
5.5 *draw* the regression line of y on x or x on y passing through $(\bar{x}, \bar{y})$ on a scatter diagram;	*Calculation of the regression coefficients for the line y on x or x on y using the values of* S_{xy}, S_{xx}, S_{yy} *Estimating the regression line in the form $\hat{y} = a + bx$ where a and b are regression coefficients.*
5.6 *interpret* the regression coefficients;	*Type of relationship.* *Using the regression coefficients in problem solving through practical examples.* *Interpretation of regression coefficients.*
5.7 make estimations using the appropriate *values in the* regression line; and,	*Estimation from regression lines.*

SPECIFIC OBJECTIVES	CONTENT
Correlation and Linear Regression Bivariate Data (cont'd)	
Students should be able to:	
5.8 outline the limitations of simple correlation and regression analyses.	*Limitations of simple correlation and regression analyses.*

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Students should be encouraged to work in groups to analyse data that has been collected. Students should then be required, using the data, to identify the population and the sample size. Teachers are also encouraged to guide students in discussing how the sample size was determined.*

2. *Students should be engaged in activities that require them to recognise* and use the sample mean, $\bar{X}$, as a random variable.

3. *Students should be encouraged* to obtain the means from samples of size 3 and construct a histogram of the sample means illustrating sampling distribution. *They should then be guided* in repeat*ing* the exercise with increasing sample sizes to illustrate the Central Limit Theorem.

4. *Students should be encouraged to work in groups of two or three, where they will be required to use the results of activities like the leaf collecting activity in Unit 1, Module 1, to d*etermine the sample mean and sample standard deviation from random samples of size 5 chosen from a population of size 50. For each sample, *they should be asked to* calculate a 95 *per cent* confidence interval and determine the number of confidence intervals which contain the population mean. *Groups can be selected to show how they arrived at their answer. Encourage students to use Excel or any other statistical programme or software.*

5. *Students should be guided in using regression and correlation to* test whether there is a relationship, *the type of relationship, and the strength of the relationship* between student performance in *English and student performance in* Mathematics.

RESOURCES

Anderson, D.R., Sweeney, D.J., Williams, T.A., Camm, J.D. and Cochran, J.J.	*Statistics for Business and economics (13th Ed.) Ohio: South-Western college Publisher, 2016.*
Crawshaw, J. and Chambers, J.	*A Concise Course in A-Level Statistics (4th ed.)*. Cheltenham: *Stanley Thornes Limited*, *2001*.
Mahadeo, R.	*Statistical Analysis – The Caribbean Advanced Proficiency Examinations A Comprehensive Text*. San Fernando: Caribbean Educational Publishers Limited, 2007.
Mann, P.	*Introductory Statistics (9th Ed). Wiley Global Education, 2016.*
Upton, G. and Cook, J.	*Introducing Statistics. Oxford: Oxford University Press, 2001.*

UNIT 2: MATHEMATICAL APPLICATIONS
MODULE 1: DISCRETE MATHEMATICS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. understand the concept of linear programming to formulate models in a real-world context;

2. understand graph theory;

3. understand basic network concepts;

4. understand basic concepts and applications of Boolean Algebra;

5. have the ability to *construct* truth tables to establish the validity of statements; and,

6. appreciate the application of discrete methods in efficiently addressing real-world situations.

SPECIFIC OBJECTIVES	**CONTENT**
1. Linear Programming	
Students should be able to:	
1.1 *design linear programming models from real-world data;*	*The relationship between variables, and constraints.* *Maximization and minimization of the objective functions.*
1.2 *graph linear inequalities in two variables;*	*Graphical representation of linear inequalities in two variables.* *Shading the side of the line that satisfies the inequality.*
1.3 *determine* the solution set that satisfies a set of linear inequalities in two variables; *and,*	*Identification of feasible (common) region of the inequalities.* *Solution set for linear inequalities in two variables.*
1.4 *determine* a unique optimal solution of a linear programming problem.	*Analysis of the vertices to determine the optimal solution.*

SPECIFIC OBJECTIVES		CONTENT
2.	**Assignment Models**	
Students should be able to:		
2.1	model a weighted assignment (or allocation) problem;	*Models of assignment problems using an m x n matrix where m is the number of rows and n is the number of columns.*
2.2	convert non-square matrix models to square matrix models;	*Dummy variables.*
2.3	convert a maximisation assignment problem into a minimisation; and,	*Maximum and minimum assignment problems (by changing the sign of each entry).*
2.4	solve a maximisation or a minimisation assignment problem *using* the Hungarian algorithm.	*Hungarian algorithm.* *Complexity of 5 x 5 or less.* *The convention of reducing rows before columns will be followed.*
3.	**Graph Theory, Critical Path Analysis and *Shortest Path***	
Students should be able to:		
3.1	determine *the components of a graph;*	*Graph theory terminology: vertex, edge, path, loop, degree (of a vertex).*
3.2	determine the degree of a vertex;	*Distinguishing among walk, trail and path.*
3.3	*construct activity networks;*	*Networks as models of real-world situations.* *The activity network algorithm in drawing a network diagram to model a real-world problem.* *(Activities will be represented by vertices and the duration of activities by edges.)*

SPECIFIC OBJECTIVES	CONTENT
Graph Theory, Critical Path Analysis and *Shortest Path (cont'd)*	
Students should be able to:	
3.4 *use* activity networks in decision making*; and,*	*Calculation of the earliest start time, latest start time, and float time.* *Identification of the critical path.*
3.5 *determine the shortest path in a network.*	*Dijkstra's algorithm.*
4. Logic and Boolean Algebra	
Students should be able to:	
4.1 formulate *propositions;*	*Propositions (in symbols or on words).* Simple propositions. The negation of simple propositions. Compound propositions that involve conjunctions, disjunctions and negations. Conditional and bi-conditional propositions.
4.2 *construct* truth *tables;*	*Truth tables.* The negation of simple propositions. Compound propositions that involve conjunctions, disjunctions and negations. Conditional and bi-conditional propositions.

SPECIFIC OBJECTIVES	CONTENT
Logic and Boolean Algebra (cont'd)	
Students should be able to:	
4.3 *analyse propositions using* truth tables;	*Truth tables for:* (a) tautology or a contradiction; (b) truth values of the converse, inverse and contrapositive of propositions; *and,* (c) equivalent propositions.
4.4 use the laws of Boolean algebra to simplify Boolean expressions;	*Application of algebra of propositions to mathematical logic.* *Idempotent, complement, identity, commutative, associative, distributive, absorption, de Morgan's Law.*
4.5 derive a Boolean expression from a given switching *circuit or logic gate*;	*Application of Boolean algebra to switching circuits and logic gates.*
4.6 represent a Boolean expression by a switching *circuit or logic gate*; and,	
4.7 use switching *circuits and logic gates* to model real-world situations.	

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Students should be encouraged to complete activities requiring them to design activity networks for local community projects (for example a small community center). They should then be required to establish the critical path to ensure that the building is completed as scheduled with a given budget.*

2. *Students should be guided in using the technique above (1) to map the shortest route from the airport to a local cricket field.*

3. *Teachers should incorporate activities which allows students to participate in classroom discussions. For example, discussions on* the route of a postman in a community with a crisscross of streets and houses situated on both sides of the street. *Students should then work in groups to plan* the route that best serves to save on time and avoid returning along the same street. A simple diagram may be used.

4. *Teachers are encouraged to present the following scenario students 'a furniture company produces dining tables and chairs. The production process for each is similar in that both require a certain number of hours for carpentry work and a certain number of labour hours in the finishing department. Each table takes 4 hours of carpentry and 2 hours in the finishing department. Each chair requires 3 hours of carpentry and 1 hour in the finishing department. The company has available 240 hours of carpentry time and 100 hours of finishing time'. Students should then be asked to use the Linear Method to do the following. Selected students can then be asked to share their responses.*

   (a) *Construct the necessary inequations given and a well labelled graph, clearly indicating the feasible region, to represent the information.*

   (b) *Using the fact that each table can be sold for a profit of $7 and each chair produced can be sold for a profit of $5, find the best combination of tables and chairs that the company should manufacture in order to reach the maximum profit.*

5. *Students should be encouraged to work in small groups where they will apply their knowledge of logic to guide a discussion on the scenario proposed that, 'Justin argues that it is bad to be depressed and watching the news makes him feel depressed. Therefore, it's good to avoid watching the news'. Each group will then be required to:*

   (a) *Write the statements in symbolic form.*

   (b) *Construct a truth table to show the argument.*

   (c) *Use Boolean algebra to prove his argument.*

UNIT 2:
MODULE 1: DISCRETE MATHEMATICS (cont'd)

RESOURCES

Bloomfield, I. and Stevens, J.	*Discrete & Decision.* Cheltenham: Nelson and Thornes, *2002.*
Bloomfield, I. and Stevens, J.	*Discrete & Decision.* Cheltenham: Teacher Resource File, *2002.*
Bryant, V.	*Advancing Mathematics for AQA Discrete Mathematics I. Oxford: Heinemann Educational Publishers, 2001.*
Peter, G. W. (Ed.)	*Discrete Mathematics.* Oxford: Heinemann Educational, 1992.
Ramirez, A. and Perriot, L.	*Applied Mathematics. Barbados: Caribbean Examinations Council, 2004.*

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *understand the use of* counting techniques and calculus in probability;

2. appreciate that probability models can be used to describe real-world situations;

3. *understand how to utilize* appropriate distributional approximations to data; and,

4. *appreciate* the appropriateness of distributions to data.

SPECIFIC OBJECTIVES		CONTENT
1.	**Probability**	
Students should be able to:		
1.1	*apply counting principles to probabilities;*	*Counting principles:* *Permutations* – number of ordered arrangements of *n* objects taken *r* at a time, with or without restrictions. *Combinations* – number of selections of *n* objects taken *r* at a time, with or without restrictions.
1.2	calculate probabilities of events (which may be combined by unions or intersections) using appropriate counting techniques;	*Probability – union and intersection of events.*
1.3	calculate *probabilities associated with conditional, independent or mutually exclusive events; and,*	*Probabilities associated with conditional, independent or mutually exclusive events.*
1.4	*use the results from probabilities to make decisions.*	*Data driven decision making from probability analysis.*

SPECIFIC OBJECTIVES	CONTENT
2. Discrete Random Variables	
Students should be able to:	
2.1 use a *given* probability function;	*f(x) = P(X = x) where f is a simple polynomial or rational function.*
2.2 use the laws of expectations and variance of a linear combination of independent random variables;	*Expectation and variance of a linear combination of independent random variables.* *Laws of expectation:* $E(aX \pm bY)$ $E(X_1 + X_2 + X_3 + \cdots + X_n)$ *Laws of variance:* $Var(aX \pm bY)$ $Var(X_1 + X_2 + X_3 + \cdots + X_n)$
2.3 model practical situations in which the discrete *distributions* are suitable;	*Discrete distributions: uniform, binomial, geometric and Poisson.*
2.4 *calculate probabilities for discrete random variables;*	*uniform:* X *is* $r(\frac{1}{b-a})$ $P(X = x) = \frac{1}{n}$, *where* $x = x_1, x_2, ..x_n$; *Binomial:* X *is* $Bin\ (n, p)$ *P (X = x) =* ${}^{n}C_{x}\, p^{x}(1-p)^{n-x}$ *x = 0, 1,2, …n;* *Geometric:* X *is* $Geo(p)$ $P(X = x) = q^{x-1}p$, *where x = 1, 2, 3.;* *Poisson:* X *is* $Po\ (\lambda)$ *P (X = x)=*$\frac{\lambda^x e^{-\lambda}}{x!}$*, x = 0, 1, 2, 3..;*

SPECIFIC OBJECTIVES | **CONTENT**

Discrete Random Variables (cont'd)

2.5 *use* the formulae for *E(X) and Var(X) of discrete;* and,

Expectation and variance where X follows:

Uniform

$\boldsymbol{E(X) = \frac{\sum x}{n}}$ *OR*

$\boldsymbol{E(X) = \sum x\, P(X = x)}$

$\boldsymbol{Var\ (X) = E\ (X^2) - (E(X))^2}$

OR

$\boldsymbol{E(X) = \frac{n+1}{2}}$

$\boldsymbol{Var\ (X) = \frac{n^2 - 1}{12}}$

Where x= 1,2,3,…n

Binomial

$\boldsymbol{E(X) = np}$

$\boldsymbol{Var(X) = n(1 - p)}$

Geometric

$\boldsymbol{E(X) = \frac{1}{p}}$

$\boldsymbol{Var\ (X) \frac{q}{p^2}}$

Poisson

$\boldsymbol{E(X) = \lambda}$

$\boldsymbol{Var(X) = \lambda}$

2.6 *use* the Poisson distribution as an approximation to the binomial distribution, where appropriate ($n > 50$ and $np < 5$).

Poisson approximation to binomial distribution.

SPECIFIC OBJECTIVES	CONTENT
3. Continuous Random Variables	
3.1 *calculate probabilities* of a continuous random variable X;	*Application of the properties of the probability density function.* $0 \leq f(x) \leq 1$ $\int_{-\infty}^{+\infty} f(x)\, dx = 1$ Where f is a probability density function (f will be restricted to simple polynomials)
3.2 *determine the* cumulative distribution function;	*Cumulative distribution function.* $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,\mathrm{d}x$
3.3 *calculate probabilities in a continuous distribution;*	*Probability.* $P(a \leq X \leq b) = \int_a^b f(x)dx = F(b) - F(a)$
3.4 *calculate measures of central tendency for a continuous distribution;*	*Expected value and medians using integration.*
3.5 *calculate measures of dispersion of a continuous distribution;*	*Variance, standard deviation, quartiles and percentiles using integration.*
3.6 *interpret various statistics related to continuous distributions; and,*	*Measures of central tendency.* *Measures of dispersion.*
3.7 use the normal distribution, *as an approximation to* the Poisson distribution.	*Normal approximation to the Poisson distribution* $(\lambda > 15)$ with a continuity correction as appropriate.

SPECIFIC OBJECTIVES	CONTENT
4. ***Chi-squared*** (χ^2) **test**	
Students should be able to:	
4.1 *formulate a χ^2 goodness-of-fit test;*	*Goodness–fit–test with appropriate number of degrees of freedom.* *Hypotheses test statistic.*
4.2 *evaluate expected frequency;*	*Expected values for situations modelled by a given ratio, discrete uniform, binomial, geometric, Poisson or normal distribution will be tested.* *Classes should be combined in cases where the expected frequency is less than 5.*
4.3 *evaluate the χ^2 critical value;*	*Degrees of freedom, critical values, and rejection region in context of the Chi-squared test.* *Reading and interpreting the χ^2-table.*
4.4 *evaluate the χ^2 test statistic; and,*	*Chi-squared test for goodness-fit* $$\chi^2{}_{test} = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$ *Where O_i is the observed frequency, E_i is the expected frequency and n is the number of groups.*
4.5 *use the results of the χ^2 test in problem solving.*	*Identification of the critical values and the rejection region for the test.* *Valid conclusion for the test.*

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Students should be encouraged to work in groups of three or four to complete the following:*

   Five friends, three girls and two boys, are waiting in a line to purchase lunch from a fast-food restaurant. However, the COVID-19 protocols restrict them from going to the cashier at the same time. As a result only one can go at a time.

   (a) Determine the number of different lines that they can make to go to the cashier.

   (b) If Mario wants to be third to order his lunch, in how many ways can they make their orders.

   (c) If Leroy insists of being first in line and Renee agrees to the last, how many lines can they make.

   (d) Calculate the probability that the three girls get their lunch first.

2. *Teacher should guide a class discussion on whether absenteeism from school is determined by the day of the week. Students should then be encouraged to work in groups to use a survey to gather data on the number of students absent by year groups on different days of the week. Using the data collected students should then determine whether the same number of students absent each day. Students should be guided to use their knowledge of Goodness-of-fit tests to:*

   (a) Clearly state their hypothesis.

   (b) Determine which test method they would use.

   (c) Carry out the testing procedures.

   (d) Make an appropriate conclusion of the test.

   Each group should be required to use a PowerPoint presentation or any other presentation method to present their findings to the class.

3. *Teachers are encouraged to model situations in which discrete distributions are appropriate. They can utilise PowerPoint Presentations and YouTube videos to concretize processes to solve problems. Activities like the following should be given to students and responses discussed. For example, students should be asked to name the distribution, giving the value(s) of its parameter(s) which may be used to model each of the following random variables.*

(a) *A is the number of persons in a group of 25 persons who are right-handed, assuming that 70% of all persons are right-handed.*

(b) *B is the number of flaws in a 15 metre length of sheet metal, given that there is an average of 2 flaws in every 10 metres of the sheet metal.*

(c) *C is the number of driving tests that an applicant takes before finally passing the test on the fourth try, if the probability of passing the test is 0.75.*

4. *Students should be encouraged to perform calculations using the Laws of Expectation and Variance.*

RESOURCES

Anderson, D.R., Sweeney, D.J., Williams, T.A., Camm, J.D. and Cochran, J.J.	*Statistics for Business and economics (13th Ed.). Ohio: South-Western college Publisher, 2016.*
Chambers, J., and Crawshaw, J.	A Concise Course in Advanced Level Statistics *(4th Ed.)*. Cheltenham: Stanley Thornes Limited, 2001.
Mann, P.S.	*Introductory Statistics (9th Ed.). New Jersey: Wiley, 2016.*

GENERAL OBJECTIVES

On completion of this Module, students should:

1. understand forces and their applications;

2. understand the concepts of work, energy and power; and,

3. appreciate the application of mathematical models to the motion of a particle.

SPECIFIC OBJECTIVES	CONTENT
1. Coplanar Forces and Equilibrium	
Students should be able to:	
1.1 *Illustrate how* forces act on a body in a given situation;	*Forces on a body to include but not limited to force normal to the plane, frictional, weight, gravitational, tractive.*
1.2 use vector notation to represent forces;	*Vectors.* *Forces as Vectors (including gravitational forces).*
1.3 *illustrate* the contact force between two surfaces;	*Normal and frictional component.*
1.4 *determine the components* of forces;	*Resolution of forces on particles, in mutually perpendicular directions (including those on inclined planes).*
1.5 calculate the resultant of two or more coplanar forces;	*Two or more coplanar forces.* *Resolving forces - particle in equilibrium* principle that *when a* particle *is* in equilibrium, the vector sum of its forces is zero, (or equivalently the sum of its components in any direction is zero.
1.6 *use Lami's Theorem with* concurrent forces;	*Lami's Theorem.* *Unknown forces and angles.* *Three forces acting at a point in equilibrium.*

SPECIFIC OBJECTIVES	CONTENT
Coplanar Forces and Equilibrium (cont'd)	
Students should be able to:	
1.7 *calculate friction; and,*	*Friction.* The appropriate relationship $F \leq \mu R$ for two *particles* in limiting equilibrium.
1.8 *interpret the results of mathematical solutions.*	*Valid conclusions from calculation (resultants, friction, system in equilibrium).*
2. Kinematics and Dynamics	
Students should be able to:	
2.1 distinguish between *Kinematics of motion in a straight*;	*Kinematics of motion:* *(a) distance and displacement; and,* *(b) speed and velocity.*
2.2 *construct graphs of motion;*	*Velocity-time and displacement-time graphs.*
2.3 *determine kinematic quantities;*	*Displacement, velocity and acceleration.* *Using graphs of motion.* *Using equations of motion.* *Assumptions – Constant acceleration.* *Motion in a straight line.*
2.4 apply Newton's laws of motion *to real world situations;*	*Newton's laws of motion* *Action of constant force.* *Rough or smooth planes.* a constant mass moving in a straight line under the action of a constant force. a particle moving vertically or on an inclined plane with constant acceleration. a system of two connected particles.

SPECIFIC OBJECTIVES	CONTENT
Kinematics and Dynamics (cont'd)	
Students should be able to:	
2.5 *calculate* rates of change *of motion;*	*Differential relationship between displacement (x), velocity (v) and acceleration.* $v = \frac{dx}{dt} = \dot{x}$ $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \ddot{x}$ *Where* $\dot{x}, \ddot{x}$ *represent velocity (v) and acceleration (a) respectively.*
2.6 solve first order differential equations of linear motion; *and,*	*Formulating and solving first-order differential equations as models of the linear motion of a particle when the applied force is proportional to its displacement.* *(Only differential equations where the variables are separable will be required).*
2.7 apply the principle of conservation of linear *momentum.*	*Linear momentum.* *Impulse.* *Direct impact of two inelastic particles moving in the same straight line* *Problems may involve two-dimensional vectors.*
3. Projectiles	
Students should be able to:	
3.1 model the projectile of a particle moving under constant gravitational force (neglecting air resistance);	*Modelling the projectile of a particle (including the use of vectors).*

SPECIFIC OBJECTIVES	CONTENT
Projectiles (cont'd)	
Students should be able to:	
3.2 formulate the equation of the trajectory of a projectile; *and,*	*Horizontal, inclined above and below the horizontal.* (Problems may involve velocity expressed in vector notation.)
3.3 use the equations of motion *for* projectiles*.*	*Properties of a projectile.*
	Motion of a projectile.
	Quadratic equations.
	Resolving velocities.
	Uniform accelerated motion.
	Time of flight $T = \frac{2V\sin\theta}{g}$
	Greatest height $H = \frac{V^2\sin^2\theta}{2g}$
	Horizontal range = $R = \frac{V^2\sin 2\theta}{g}$
	Equation of the trajectory
	$y = x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$
	$y = x\tan\theta - \frac{gx^2}{2V^2}\left(1+\tan^2\theta\right)$
4. Work, Energy and Power	
Students should be able to:	
4.1 calculate the work done;	*Work done by a constant force.*
	Work done by a variable force in one dimension.

SPECIFIC OBJECTIVES	CONTENT
Work, Energy and Power (Cont'd)	
Students should be able to:	
4.2 *calculate* kinetic energy and gravitational potential energy;	*Kinetic energy* $= \frac{1}{2}mv^2$ *Gravitational potential energy* $= mgh$ *Where* m *is mass,* v *is velocity,* g *is acceleration due to gravity and* h *is vertical height.*
4.3 *use the work-energy principle in real world situations; and,*	*Principle of conservation of energy.* *Application of the work-energy principle to calculate potential and kinetic energy* (KE) *(including change in kinetic energy).* $Work = \Delta KE$
4.4 *calculate* power.	*Definition of Power.* $P = Fv$ *Where* F *is force (driving or tractive) and* v *is velocity.* $P = \frac{work\ done}{time}$ *Measuring power.* $1\ watt\ (W) = 1\ joule\ (J)\ per\ second$ $1\ kilowatt\ (kW) = 1000\ watts\ (W)$

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Students should be encouraged to work in groups in which they will be required to research Newton's Laws of Motion and use examples to show how it should be applied when solving problems.*

Place students in groups and assign tasks outlined below:

2. Draw a diagram of a uniform ladder or beam resting on rough horizontal ground and leaning against a rough (smooth) vertical, with the figure of a man some way up the ladder. Show the forces acting on the ladder and on the man.

3. Draw a diagram of an inclined plane on which a body is placed and is about to be pulled up the plane by a force acting at an angle to the inclined plane. The plane may be smooth or rough. Show all the forces acting on the body. The system may also be considered as the body about to move down the plane.

4. Draw a diagram of a large smooth sphere of weight W resting inside a smooth cylinder and held in place by a small smooth sphere of weight, w. Show the forces acting on the large sphere and on the small sphere.

5. Draw diagrams showing forces acting on a block of wood which is:

   (a) sliding down a rough inclined plane at steady speed; and,

   (b) accelerating down a rough plane.

6. Draw a diagram showing the forces acting on a car which is driven up an incline at steady speed.

7. Draw a diagram of a car towing a caravan on level road and show the forces acting on the car and on the caravan.

8. Show the forces acting if air resistance is present when a stone is thrown through the air.

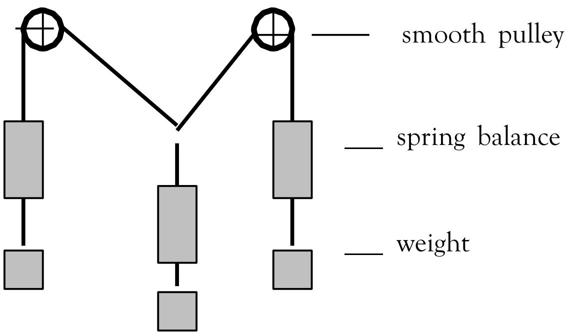
9. A man is standing alone in a moving lift. Draw a diagram to show the forces acting on:

   (a) the man;
   (b) the lift, when it is accelerating upwards;
   (c) the lift, when it is travelling at steady speed; and,
   (d) the lift, when it is accelerating downwards.

10. A railway engine is pulling a train up an incline against frictional resistances. If the combined engine and train are experiencing a retardation, draw diagrams to show the forces acting on the engine and forces acting on the train.

11. Students should be allowed to experiment with a system of three spring balances as illustrated in the figure below to investigate the resultant, resolution and equilibrium of forces.

Resource material: 2 pulleys, string, weights, a sheet of paper, 3 spring balances.



Students should consider:

(a) ***body*** *is an object to which a force can be applied; and,*

(b) ***particle*** *is a body whose dimensions, except mass, are negligible.*

RESOURCES

Bostock, L. and Chandler, S.	*Mechanics for A-Level,* Cheltenham. London: Stanley Thornes (Publishers) Limited, *1996*.
Graham, T.	*Mechanics (Collins Advanced Mathematics).* Oxford: *Harper* Collins Educational, *2011*.
Hebborn, J., Littlewood J. and Norton, F.	*Heinemann Modular Mathematics for London AS and A-Level Mechanics 1 and 2.* Oxford: Heinemann Educational Publishers, 1994.
Jefferson, B., and Beadsworth, T.	*Introducing Mechanics,* Oxford: University Press, *2000*.
Price, N. (Editor)	*AEB Mechanics for AS and A-Level*. Oxford: Heinemann Publishers, 1997.
Sadler, A.J., and Thorning, D.W.S.	*Understanding Mechanics.* Oxford: Oxford University Press, 1996.

♦ OUTLINE OF ASSESSMENT

A candidate's performance is reported as an overall grade and a grade on each Module. The assessment comprises two components, one external and one internal.

EXTERNAL ASSESSMENT		**(80 %)**
Paper 01 ***(1 hour 30 minutes)***	The Paper will consist of forty-five (45) multiple-choice items, fifteen (15) items on each Module. Each item is worth 1 mark *(weighted up to 2).*	30%
Paper 02 (2 hours 30 minutes)	The paper consists of six compulsory extended response questions, *two from each Module. Each question is worth 25 marks.*	50 %
SCHOOL-BASED ASSESSMENT		**(20 %)**
Paper 031	This paper is intended for candidates registered through schools or *other approved institutions.* **UNIT 1** The paper consists of a project designed and internally assessed by the teacher and externally moderated by **CXC®**. *The project is written work based on personal research or investigation involving collection, analysis, and evaluation of data.* ***UNIT 2*** *The paper consists of a portfolio designed and internally assessed by the teacher and externally moderated by **CXC®**. The portfolio is written work which shows how at least THREE mathematical concepts (ONE from EACH module) can be modules to solve real world problems.* See pages *54-70* for the details on the School Based Assessment (SBA)	
Paper 032 ***(2 hours)***	This paper is an alternative for Paper *031, the School-Based Assessment* and is intended for private candidates. The paper comprises three questions and *tests skills similar to those assessed in the School-Based Assessment.* The duration of the paper is 2 hours. *The case to be assessed in the papers will be given to candidates one week in advance of the examination dates.*	

MODERATION OF SCHOOL-BASED ASSSESMENT

All School-Based Assessment Record of marks must be submitted online using the SBA data capture module of the Online Registration System (ORS) *by stipulated deadlines*. Assignments will be requested by **CXC®** for moderation purposes. These assignments will be reassessed by **CXC®** Examiners who moderate the School-Based Assessment. Teachers' marks may be adjusted as a result of moderation. The Examiners' comments will be sent to schools.

Copies of the students' assignments must be retained by the school until three months after publication by **CXC®** of the examination results.

ASSESSMENT DETAILS

External Assessment by Written Papers (80% of Total Assessment)

Paper 01 (1 hour *30* minutes - 30% of Total Assessment)

1. Composition of papers

(a) This paper consists of forty-five multiple-choice items and is partitioned into three sections (Module 1, 2 and 3). Each section contains fifteen questions.

(b) All *items* are compulsory.

2. Syllabus Coverage

(a) Knowledge of the entire syllabus is required.

(b) The paper is designed to test candidates' knowledge across the breadth of the syllabus.

3. Question Type

Questions may be presented using words, symbols, tables, diagrams or a combination of these.

4. Mark Allocation

(a) Each item is allocated 1 mark, *which will be weighted to 2 marks.*

(b) Each Module is allocated 15 marks, *which will be weighted to 30 marks.*

(c) The total number of marks available for this paper is *45, which will be weighted to 90 marks.*

(d) This paper contributes *30* per cent towards the total assessment.

5. Award of Marks

Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

6. **Use of Calculators**

(a) Each candidate is required to have a silent non-programmable calculator and is responsible for its functioning.

(b) The use of calculators with graphical displays will not be permitted.

(c) Answers found by using a calculator, without relevant working shown, may not be awarded full marks.

(d) Calculators must not be shared during the examination.

Paper 02 (*2 hours* 30 minutes – *50 per cent* of Total Assessment)

1. **Composition of Paper**

(a) This paper consists of *six* questions, two questions from each Module.

(b) All questions are compulsory. A question may require knowledge of several topics in a Module. However, all topics in a Module may not be given equal emphasis.

2. **Syllabus Coverage**

(a) Each question may require knowledge from more than one topic in the Module from which the question is taken and will require sustained reasoning.

(b) Each question may address a single theme or unconnected themes.

3. **Question Type**

(a) Questions may require an extended response.

(b) Questions may be presented using words, symbols, diagrams, tables or combinations of these.

4. **Mark Allocation**

(a) Each question is worth *25* marks.

(b) The number of marks allocated to each sub-question will appear in brackets on the examination paper.

(c) Each M*odule* is allocated *50* marks.

(d) The total marks available for this paper is *150.*

(e) The paper contributes *50%* towards the final assessment.

5. **Award of Marks**

(a) Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

(b) Full marks are awarded for **correct** answers and the presence of **appropriate working**.

(c) It may be possible to earn partial credit for a correct method where the answer is incorrect.

(d) If an incorrect answer in an earlier question or part-question is used later in a section or a question, then marks may be awarded in the later part even though the original answer is incorrect. In this way, a candidate is not penalised twice for the same mistake.

(e) A correct answer given with no indication of the method used (in the form of written work) will receive no marks. Candidates are, therefore, advised to show all relevant working.

6. **Use of Calculators**

(a) Each candidate is required to have a silent non-programmable calculator and is responsible for its functioning.

(b) The use of calculators with graphical displays will not be permitted.

(c) Answers found by using a calculator, without relevant working shown, may not be awarded full marks.

(d) Calculators must not be shared during the examination.

7. Use of Mathematical Tables

A booklet of mathematical formulae and tables will be provided.

SCHOOL-BASED ASSESSMENT

School-Based Assessment is an integral part of student assessment in the course covered by this syllabus. It is intended to assist students in acquiring certain knowledge, skills, and attitudes that are associated with the subject. The activities for the School-Based Assessment are linked to the syllabus and should form part of the learning activities to enable the student to achieve the objectives of the syllabus. *Group work should be encouraged.*

During the course of study for the subject, students obtain marks for the competence they develop and demonstrate in undertaking their Sc*hool-Based Assessment* assignments. These marks contribute to the final marks and grades that are awarded to students for their performance in the examination.

The guidelines provided in this syllabus for selecting appropriate tasks are intended to assist teachers and students in selecting assignments that are valid for the purpose of *School-Based Assessment*.

The guidelines provided for the assessment of these assignments are intended to assist teachers in awarding marks that are reliable estimates of the achievement of students in the *School-Based Assessmen*t component of the course. In order to ensure that the scores awarded by teachers are not out of line with the **CXC®** standards, the Council undertakes the moderation of a sample of the *School-Based Assessment* assignments marked by each teacher.

School-Based Assessment provides an opportunity to individualise a part of the curriculum to meet the needs of students. It facilitates feedback to the student at various stages of the experience. This helps to build the self- confidence of students as they proceed with their studies. *School-Based Assessment* also facilitates the development of the critical skills and abilities emphasised by this **CAPE®** subject and enhance the validity of the examination on which candidate performance is reported. *School-Based Assessment*, therefore, makes a significant and unique contribution to both the development of relevant skills and the testing and rewarding of students for the development of those skills.

The Caribbean Examinations Council seeks to ensure that the *School-Based Assessment* scores are valid and reliable estimates of accomplishment. The guidelines provided in this syllabus are intended to assist in doing so.

CRITERIA FOR THE SCHOOL-BASED ASSESSMENT (SBA) (*Paper 031)*

Unit1

This paper is compulsory and consists of a project. *Candidates have the option to work in small groups (maximum of 5 members in a group) to complete their SBA's*

1. **The aims of the project are to:**

   (a) *develop candidates' ability to work collaboratively;*

   (b) develop candidates' insights into the nature of statistical analysis;

   (c) develop candidates' abilities to formulate their own questions about statistics;

   (d) encourage candidates to initiate and sustain a statistical investigation;

   (e) provide opportunities for all candidates to show, with confidence, that they have mastered the syllabus; and,

   (f) enable candidates to use the methods and procedures of statistical analysis to describe or explain real-life phenomena.

2. **Requirements**

   (a) The project is **written work** based on personal research or investigation involving **collection**, **analysis** and **evaluation** of data.

   (b) Each project should include:

      (i) a statement of the task;

      (ii) description of method of data collection;

      (iii) presentation of data;

      (iv) analysis of data or measures; and,

      (v) discussion of findings.

   (c) The project may *utilise* mathematical modelling, statistical applications or surveys.

   (d) Teachers are expected to guide candidates in choosing appropriate projects that relate to their interests and mathematical expertise.

   (e) Candidates should make use of mathematical and statistical skills from *ALL* Modules.

3. **Integration of Project into the Course**

(a) The activities related to project work should be integrated into the course so as to enable candidates to learn and practise the skills of undertaking a successful project.

(b) Class time should be allocated for general discussion of project work. For example, discussion of how data should be collected, how data should be analysed and how data should be presented.

(c) Class time should also be allocated for discussion between teacher and student, and student and student.

4. **Management of Project**

(a) **Planning**

An early start to planning project work is highly recommended and the schedule of the dates for submission should be developed by teachers and candidates.

(b) **Length**

The project must not exceed 1500 words. The word count does not include: Tables, References, Table of contents, Appendices and Figures. TEN percent of candidates' earned marks will be deducted for exceeding the word limit by 1000 words.

(c) **Guidance**

Each candidate should know the requirements of the project and its assessment process.

Although candidates may consult with resource persons besides the teacher the candidates' submission should be their own work.

Candidates are not expected to work on their own. The teacher is expected to give appropriate guidance at all stages of project work, for example, chapters to read, alternative procedures to follow and other sources of information.

(d) **Authenticity**

Teachers are required to ensure that all projects are the candidates' work.

The recommended procedures are to:

(i) engage candidates in discussion;

(ii) ask candidates to describe procedures used and summarise findings either orally or written; and,

(iii) ask candidates to explain specific aspects of the analysis.

ASSESSMENT CRITERIA FOR THE PROJECT

General

It is recommended that candidates be provided with assessment criteria before commencing the project.

1. For each component, the aim is to find the level of achievement reached by the candidates.

2. For each component, only whole numbers should be awarded.

3. It is recommended that the assessment criteria be available to candidates at all times.

ASSESSING THE PROJECT

The project will be marked out of a total of 60 marks. Marks for the Project will be allocated across Modules in the ratio 1:1:1. The marks earned by a student are assigned to each Module. For example, if a student earns 50 out of 60 for his School-Based Assessment, 50 marks will be assigned to Module 1, 50 marks to Module 2 and 50 marks to Module 3. The total score will be 50+50+50= 150 out of 180. The marks for the portfolio are allocated to each task as outlined below:

Project Descriptors

1.	***Project Title***	***2 marks***
	(a) *Title is clear and concise.* ***(1 mark)*** *(b)* *Title relates to the project.* ***(1 mark)***	
2.	***Purpose of Project***	***6 marks***
	(a) *Background describes the context of the problem.* ***(1 mark)*** *(b)* *Purpose Statement* - *The purpose of the project is relevant to the background.* ***(1 mark)*** - *Method of data analysis (hypothesis testing and/or correlation and regression analysis) is relevant to the purpose of the project.* ***(1 mark)*** - *Method of managing uncertainty is relevant to the purpose of the project.* ***(1 mark)*** *(c)* *Dependent variable(s) accurately identified.* ***(1 mark)*** *(d)* *Independent and/or random variable(s) accurately identified.* ***(1 mark)***	
3.	***Method of Data Collection***	***12 marks***
	(a) *Sampling (selection of a sample):* - *Population is clearly identified.* ***(1 mark)*** - *Clear justification of the use of a sample is presented.* ***(1 mark)*** - *A relevant sampling method is identified.* ***(1 mark)*** - *Justification of the selected sampling method is clearly stated.* ***(1 mark)*** - *Sampling Procedure* ***(4 marks)*** • *Award* ***1 mark*** *for a sequential procedure clearly describing how the sample method was used to select the sample.* • *Award* ***1 mark*** *for a procedure which aligns with the sampling method selected.* • *Award* ***1 mark*** *for stating the sample size.* • *Award* ***1 mark*** *for a sample size that is representative of the population.*	

	(b) *Instruments* ***(4 marks)*** - *Appropriate Data Collection Instruments identified.* ***(1 mark)*** - *Data Collection instruments described.* ***(2 marks)*** • *Award* ***1 mark*** *for description.* • *Award* ***1 mark*** *for evidence of the instrument used. (appendix)* - *Administration of the instrument is clearly described.* ***(1 mark)***	
4.	***Presentation of Data***	***12 marks***
	(a) ***Tables*** - *At least one table used.* ***(1 mark)*** - *An appropriate title is presented.* ***(1 mark)*** - *Table is clearly written (unambiguous and systematic).* ***(1 mark)*** - *Appropriate headers (columns and rows).* ***(1 mark)*** *(b)* ***Graph/Chart (as mentioned in the syllabus)*** - *At least one graph/chart used.* ***(1 mark)*** - *An appropriate title is presented.* ***(1 mark)*** - *Scale/Key.* ***(1 mark)*** - *Correct Labels (Axis/Sectors).* ***(1 mark)*** - *At least 4 correct values used.* ***(4 marks)*** • *Award* ***1 mark*** *for each correct value (candidate table).*	
5.	***Analysis of Data***	***18 marks***
	Probability Analysis ***Candidates must use the statistical technique stated in the purpose. No mark will be awarded for the calculations if the technique stated is not used.*** *(a)* *At least TWO probability calculations attempted.* ***(1 mark)*** *(b)* *Award* ***4 marks*** *for EACH probability calculation attempted as follows.* ***(8 marks)*** - *Accurate Formula/Equation shown.* ***(1 mark)***	

	- *Accurate utilization of the data presented in candidate's tables/charts/graphs.* ***(1 mark)*** - *Accurate steps are shown.* ***Candidates who do not show working will NOT be awarded marks for this section. (2 marks)*** • *Award* ***2 marks*** *if there are no errors.* • *Award* ***1 marks*** *if there are errors.* • *Award* ***zero*** *for completely inaccurate calculations.* ***Hypothesis (Z-test/t-test/Chi-squared) and/or Correlation and Regression Analysis.*** ***Candidates must use the statistical technique stated in the purpose. No mark will be awarded for the calculations if the technique stated is not used.*** *(a) A hypotheses OR correlation and regression analysis attempted.* ***(1 mark)*** *(b) Accurate Formula/Equation stated.* ***(1 mark)*** *(c) Accurate utilization of the data presented.* ***(1 mark)*** *(d) ALL steps of the analysis are shown.* ***(1 mark)*** *(e) Accurate steps are shown.* ***Candidates who do not show working will NOT be awarded marks for this section. (3 marks)*** - *Award* ***3 marks*** *if there are no errors.* - *Award* ***2 marks*** *if there are 1-2 errors.* - *Award* ***1 mark*** *if there are more than 2 errors.* - *Award* ***zero*** *for completely inaccurate calculations.* *(f) Candidate's Correct Answer.* ***(1 mark)***	
6.	***Discussions of Findings/Conclusion***	***7 marks***
	(a) Limitations - *Insights into the nature of the problem encountered in the tasks.* ***(1 mark)*** - *Insights into the resolution of problems encountered in the task.* ***(1 mark)*** *(b) Statement of most findings is clearly identified.* ***(1 mark)*** *(c) Statements of findings follow logically from the data gathered.* ***(1 mark)*** *(d) Conclusion is based on analysis of data.* ***(1 mark)*** *(e) Conclusion is related to the purpose of project.* ***(1 mark)*** *(f) At least ONE recommendation to improve the project is presented.* ***(1 mark)***	

7.	***Communication of Information***	***2 marks***
	- ***Award 2 marks*** *if information is communicated in a logical way using correct grammar, statistical jargon, and symbols most of the time.* - ***Award 1 mark*** *if there are more than two areas requiring improvement.*	
8.	***List of References***	***1 mark***
	References relevant, up to date, written using a consistent convention	
	TOTAL 60 MARKS	
For exceeding the word limit of 1,500 words by 1000 words, deduct 10 percent of the candidate's score.		

Unit 2

This paper is compulsory and consists of a *portfolio*. Candidates are encouraged to work in small groups *(maximum of 5 members in a group)* to complete their SBA's.

1. The aims of the assignment are to:

(a) *develop candidates' ability to work collaboratively;*

(b) enable the student to explore *research* possibilities *in Applied Mathematics*; and,

(c) develop mathematical ideas and communicate using mathematical tools, language and symbols.

2. Requirements

(a) *Candidates are required to create a portfolio which shows how at **least** THREE mathematical concepts (ONE from EACH module) can be utilised modules to solve real world problems;*

(b) At each stage of the task, the candidates must:

(i) describe and explain clearly their actions and thinking;

(ii) present all data (preferably in a table or chart or diagram or as a set of symbols, equations and inequalities);

(iii) process or analyse data using mathematical skills and available technology;

(iv) state assumptions and expected limitations of the selected process; and,

(v) discuss their expected findings.

(c) The *portfolio may utilise mathematical modelling, demonstrations, and investigations;*

(d) Teachers are expected to guide candidates in choosing appropriate topics that relate to candidates' interests and mathematical expertise. *During the identification-of-the-topic stage, candidates should be required to:*

(i) list <u>most</u> of the mathematics they expect to use in engaging the topic; and,

(ii) ensure that there is substantial mathematical content in their work.

(e) Candidates should make use of mathematical and statistical skills contained in **ALL THREE** Modules.

3. Integration of portfolio into the course

(a) The activities related to the *Portfolio* should be integrated into the course so as to enable students to learn and practise the skills of undertaking and completing a successful *Portfolio*.

(b) Class time should be allocated for general discussion of *Portfolio* work.

(c) Class time should also be allocated for discussion between teacher and students, and among students.

4. Management of *Portfolio*

(a) Planning

An early start to planning *Portfolio* work is highly recommended and the schedule of the dates for submission should be developed by teachers and candidates.

(b) Length

The Portfolio must not exceed 1500 words. The word count does not include: Tables, References, Table of contents, Appendices and Figures. TEN percent of the candidates' earned marks will be deducted for exceeding the word limit by 1000 words.

(c) *Guidance*

Each candidate should be provided with the requirements of the assignment and its assessment process.

Although candidates may consult with resource persons besides the teacher the candidates' submission should be their own work.

Candidates are not expected to work on their own. The teacher is expected to give appropriate guidance at all stages of project work, for example, chapters to read, alternative procedures to follow and other sources of information.

(d) Authenticity

Teachers are required to ensure that all assignments are the candidates' work. The recommended procedures are to:

(i) *engage candidates in discussion;*

(ii) *require candidates to describe procedures used and summarise findings either orally or written; and,*

(iii) *require candidates to replicate parts of the analysis.*

ASSESSMENT CRITERIA FOR THE *PORTFOLIO*

General

It is recommended that candidates be provided with an assessment criterion before commencing the Assignment.

1. For each component, the aim is to ascertain the level of achievement reached by the candidates.

2. For each component, fractional marks should not be awarded.

3. It is recommended that the assessment criteria be available to candidates at all times.

ASSESSING THE *PORTFOLIO*

The portfolio will be marked out of a total of 60 marks. Marks for the portfolio will be allocated across Modules in the ratio 1:1:1. The marks earned by a student are assigned to each Module. For example, if a student earns 50 out of 60 for his School-Based Assessment, 50 marks will be assigned to Module 1, 50 marks to Module 2 and 50 marks to Module 3. The total score will be 50+50+50= 150 out of 180. The marks for the portfolio are allocated to each task as outlined below:

1.	*Sections A – Module 1* *Section B – Module 2* *Section C – Module 3* ***Each section will be marked out of 19 using the outline below:***	***(57 marks)***
	Statement of Task (4 marks)	
	(a) Background detailing a real-world problem is presented. ***(1 mark)*** *(b) Purpose Statement.* ***(2 marks)*** - *Award* ***1 mark*** *if the purpose stated is relevant to the background.* - *Award* ***1 mark if*** *the purpose statement includes a relevant method of analysis.* *(c) The relevant variable/s to be used is/are identified.* ***(1 mark)***	
	Methodology (15 marks) *Description of the plan for carrying out task (and mathematics involved).*	
	(a) Method of ***solving details*** *how mathematical processes will be carried out correctly in analysing the data.* ***<u>Candidates may use provided data, data from conducting their own investigations or simulated data.</u>*** - *Justification of the appropriateness of the method of solving.* ***(2 marks)*** • *Award* ***1 mark*** *if* ***ALL*** *assumptions/conditions of the method of analysis are presented and accurate. (****1 mark)*** • *Award* ***1 mark*** *if the justification provided links* ***ALL*** *assumptions/conditions of the method of analysis to the problem described.* ***(1 mark)*** - *Data is presented using appropriate mathematical tools. (****4 marks)*** • *An appropriate title is presented.* ***(1 mark)*** • *The data presentation is clear and systematic.* ***(1 mark)*** • *The mathematical tool is appropriately labelled (headings/scale/keys/sectors). (****1 mark)*** • *The data presentation is without flaws.* ***(1 mark)*** - *Data Analysis* ***(6 marks)*** • *accurate Algorithm/Formula/Equation stated.* ***(1 mark)*** • *accurate utilization of the data presented.* ***(1 mark)*** • *ALL steps of the analysis are shown.* ***(1 mark)*** • *The steps shown are accurate.* ***Candidates who do not show working will NOT be awarded marks for this section. (3 marks)*** - *Award* ***3 marks*** *if there are no errors* - *Award* ***2 marks*** *if there are 1-2 errors* - *Award* ***1 mark*** *if there are more than 2 errors*	

	(b) Method of interpreting ***(3 marks)*** - *At least ONE relevant expected limitation/challenge is clearly stated.* ***(1 mark)*** - *At least ONE relevant conclusion based on candidates' analysis is stated.* ***(1 mark)*** - *At least ONE recommendation to improve the project is presented.* ***(1 mark)***	
2.	***Communication of Information (Overall Portfolio)***	***(2 marks)***
	Award 2 marks *if information is communicated in a logical way using correct grammar, statistical jargon, and symbols most of the time.* ***Award 1 mark*** *if there are more than two areas requiring improvement.*	
3.	***List of References (Overall Portfolio)***	***(1 mark)***
	At least ONE relevant reference for EACH section is provided and is written using a consistent convention.	*1*
	TOTAL 60 MARKS	
For exceeding the word limit of 1,500 words by 1000 words, deduct 10 percent of the candidate's score.		

GENERAL GUIDELINES FOR TEACHERS

1. *All School-Based Assessment Record of marks must be submitted online using the SBA data capture module of the Online Registration System (ORS).*

2. **CXC®** requires *that ALL* assignments *are submitted* for external moderation.

3. Teachers should note that the reliability of marks awarded is a significant factor in School-Based Assessment, and has far-reaching implications for the candidate's final grade.

4. Candidates who do not fulfill the requirements of the School-Based Assessment will be considered absent from the whole examination.

5. Teachers are asked to note the following:

    (a) the relationship between the marks for the assignments and those submitted to **CXC®** on the School-Based Assessment form should be clearly shown; *and,*

    (b) the standard of marking should be consistent.

♦ REGULATIONS FOR PRIVATE CANDIDATES

Candidates who are registered privately will be required to sit Paper 01, Paper 02 and Paper 032. Paper 032 will be 2 hours' duration and will contribute 20 per cent of the total assessment of a candidate's performance on that Unit.

The paper comprises THREE questions and tests skills similar to those assessed in the School-Based Assessment. ***This case to be assessed in the papers will be given to candidates ONE week in advance of the examination dates***

♦ REGULATIONS FOR RESIT CANDIDATES

CAPE® candidates may reuse any moderated SBA score within a two-year period. In order to assist candidates in making decisions about whether or not to reuse a moderated SBA score, the Council will continue to indicate on the preliminary results if a candidate's moderated SBA score is less than 50% in a particular Unit. Candidates reusing SBA scores should register as "Resit candidates" and must provide the previous candidate number when registering. Resit candidates must complete Papers 01 and 02 of the examination for the year in which they register.

♦ ASSESSMENT GRID

The Assessment Grid for this Unit contains marks assigned to papers and to Modules, and percentage contributions of each paper to total scores.

Papers	Module 1	Module 2	Module 3	Total	(%)
External Assessment **Paper 01** (1 hour 30 minutes) Multiple Choice	30 *(15 raw)*	*30* *(15 raw)*	*30* *(15 raw)*	*90* *(45 raw)*	(30)
Paper 02 (2 hours 30 minutes) Extended Response	*50*	*50*	*50*	*150*	(50)
Paper 03 ***Paper 031* - School Based Assessment (SBA)** **Papers 032** *(2 hours)*	*20* *(60 raw)*	*20* *(60 raw)*	*20* *(60 raw)*	*60* *(180 raw)*	(20)
Total (Weighted)	100	100	100	300	(100)

♦ APPLIED MATHEMATICS NOTATION

The following list summarises the notation used in Applied Mathematics papers of the Caribbean Advanced Proficiency Examinations.

Set Notation

$\in$	is an element of
$\notin$	is not an element of
$\{x: \ldots\}$	the set of all x such that …
n(A)	the number of elements in set A
$\emptyset$	the empty set
U	the universal set
A'	the complement of the set A
W	the set of whole numbers {0, 1, 2, 3, …}
N	the set of natural numbers {1, 2, 3, …}
Z	the set of integers
Q	the set of rational numbers
$\overline{Q}$	the set of irrational numbers
R	the set of real numbers
C	the set of complex numbers
$\subset$	is a subset of
$\not\subset$	is not a subset of
$\cup$	union
$\cap$	intersection
[a, b]	the closed interval $\{x \varepsilon \mathbf{R} : a \leq x \leq b\}$
(a, b)	the closed interval $\{x \varepsilon \mathbf{R} : a < x < b\}$
$[a, b)$	the interval $\{x \varepsilon \mathbf{R} : a \leq x < b\}$
$(a, b]$	the interval $\{x \varepsilon \mathbf{R} : a < x \leq b\}$

♦ MISCELLANEOUS SYMBOLS

$\equiv$	is identical to
$\approx$	is approximately equal to
$\propto$	is proportional to
∞	infinity

Operations

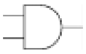
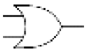
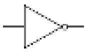
$\sum_{i=1}^{n} x_i$	$x_1 \; x_2 \; \ldots \; x_n$
$\angle \lvert x \angle \rvert$	the modulus of the real number x
$n!$	n factorial, $1\times 2\times 3\times \ldots\times n$, for $n \in \mathbf{N}$ $(0! = 1)$
${}^{n}C_{r}$	the binomial coefficient, $\frac{n!}{(n-r)!\,r!}$, for $n \in \mathbf{R}$, $0 \le r \le n$
${}^{n}P_{r}$	$\frac{n!}{(n-r)!}$

Functions

Δx, δx	an increment of x
$\frac{dy}{dx}$, y'	the derivative of y with respect to x
$\frac{d^n y}{dx^n}$, $y^{(n)}$	the nth derivative of y with respect to x
$f'(x), f''(x), \cdots, f^{(n)}(x)$	the first, second, …, nth derivatives of $f(x)$ with respect to x
$\dot{x}, \ddot{x}$	the first and second derivatives of x with respect to time
e	the exponential constant
$\ln x$	the natural logarithm of x (to base e)
$\lg x$	the logarithm of x to base 10

Logic

p, q, r	propositions
$\wedge$	conjunction
$\vee$	(inclusive) disjunction
$\sim$	negation
$\rightarrow$	conditionality
$\leftrightarrow$	bi-conditionality
•	implication
$\Leftrightarrow$	equivalence

	AND gate
	OR gate
	NOT gate
T, 1	true
F,0	false

Probability and Statistics

S	the possibility space
A, B, …	the events *A*, *B*, …
$P(A)$	the probability of the event A occurring
P(A′)	the probability of the event not occurring
P(A∠│ B)	the conditional probability of the event A given the event B
X, Y, R …	random variables
x, y, r …	values of the random variable X, Y, R …
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	the frequencies with which the observations $x_1, x_2, \ldots$ occur
f(x)	the probability density function of the random variable X
F(x)	the value of the cumulative distribution function of the random variable X
E(X)	the expectation of the random variable X
Var(X)	the variance of the random variable X
μ	the population mean
$\bar{x}$	the sample mean
σ^2	the population variance
s^2	the sample variance
$\hat{\sigma}^2$	an unbiased estimate of the population variance
r	the linear product-moment correlation coefficient
Bin(n, p)	the binomial distribution, parameters n and p
Po(λ)	the Poisson distribution, mean and variance λ
N(μ, σ^2)	the normal distribution, mean μ and variance σ2
N(μ, σ^2)	the normal distribution, mean μ and variance σ2 readσ^2
Z	standard normal random variable
Φ	cumulative distribution function of the standard normal distribution N(0, 1)
χ^2_v	the chi-squared distribution with ν degrees of
t_v	freedom the *t*-distribution with ν degrees of freedom

Vectors

$\underset{\sim}{a}$, **a**, $\overrightarrow{AB}$	vectors
$\hat{\mathbf{a}}$	a unit vector in the direction of **a**
$\angle\|\mathbf{a}\angle\|$	the magnitude of **a**
a . b	the scalar product of **a** and b
i, **j**, **k**	unit vectors in the direction of the cartesian coordinate axes
$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	$x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

Mechanics

x	displacement
v, $\dot{x}$	velocity
a, $\dot{v}$ $\ddot{x}$	acceleration
μ	the coefficient of
F	force
R	normal reaction
T	tension
m	mass
g	the gravitational
W	work
P	power
I	impulse

♦ GLOSSARY OF EXAMINATION TERMS

WORD	DEFINITION	NOTES
Analyse	examine in detail	
Annotate	add a brief note to a label	Simple phrase or a few words only.
Apply	use knowledge/principles to solve problems	Make inferences/conclusions.
Assess	present reasons for the importance of particular structures, relationships or processes	Compare the advantages and disadvantages or the merits and demerits of a particular structure, relationship or process.
Calculate	arrive at the solution to a numerical problem	Steps should be shown; units must be included.
Classify	divide into groups according to observable characteristics	
Comment	state opinion or view with supporting reasons	
Compare	state similarities and differences	An explanation of the significance of each similarity and difference stated may be required for comparisons which are other than structural.
Construct	use a specific format to make and/or draw a graph, histogram, pie chart or other representation using data or material provided or drawn from practical investigations, build (for example, a model), draw scale diagram	Such representations should normally bear a title, appropriate headings and legend.
Deduce	make a logical connection between two or more pieces of information; use data to arrive at a conclusion	
Define	state concisely the meaning of a word or term	This should include the defining equation/formula where relevant.
Demonstrate	show; direct attention to...	

WORD	DEFINITION	NOTES
Derive	to deduce, determine or extract from data by a set of logical steps some relationship, formula or result	This relationship may be general or specific.
Describe	provide detailed factual information of the appearance or arrangement of a specific structure or a sequence of a specific process	Description may be in words, drawings or diagrams or any appropriate combination. Drawings or diagrams should be annotated to show appropriate detail where necessary.
Determine	find the value of a physical quantity	
Design	plan and present with appropriate practical detail	Where hypotheses are stated or when tests are to be conducted, possible outcomes should be clearly stated and/or the way in which data will be analysed and presented.
Develop	expand or elaborate an idea or argument with supporting reasons	
Diagram	simplified representation showing the relationship between components	
Differentiate/Distinguish (between/among)	state or explain briefly those differences between or among items which can be used to define the items or place them into separate categories	
Discuss	present reasoned argument; consider points both for and against; explain the relative merits of a case	
Draw	make a line representation from specimens or apparatus which shows an accurate relation between the parts	In the case of drawings from specimens, the magnification must always be stated.
Estimate	make an approximate quantitative judgement	
Evaluate	weigh evidence and make judgements based on given criteria	The use of logical supporting reasons for a particular point of view is more important than the view held; usually both sides of an argument should be considered.

WORD	DEFINITION	NOTES
Explain	give reasons based on recall; account for	
Find	locate a feature or obtain as from a graph	
Formulate	devise a hypothesis	
Identify	name or point out specific components or features	
Illustrate	show clearly by using appropriate examples or diagrams, sketches	
Interpret	explain the meaning of	
Investigate	use simple systematic procedures to observe, record data and draw logical conclusions	
Justify	explain the correctness of	
Label	add names to identify structures or parts indicated by pointers	
List	itemise without detail	
Measure	take accurate quantitative readings using appropriate instruments	
Name	give only the name of	No additional information is required.
Note	write down observations	
Observe	pay attention to details which characterise a specimen, reaction or change taking place; to examine and note scientifically	Observations may involve all the senses and/or extensions of them but would normally exclude the sense of taste.
Outline	give basic steps only	
Plan	prepare to conduct an investigation	
Predict	use information provided to arrive at a likely conclusion or suggest a possible outcome	
Record	write an accurate description of the full range of observations made during a given procedure	This includes the values for any variable being investigated; where appropriate, recorded data may be depicted in graphs,

WORD	DEFINITION	NOTES
		histograms or tables.
Relate	show connections between; explain how one set of facts or data depend on others or are determined by them	
Sketch	make a simple freehand diagram showing relevant proportions and any important details	
State	provide factual information in concise terms outlining explanations	
Suggest	offer an explanation deduced from information provided or previous knowledge. (... a hypothesis; provide a generalisation which offers a likely explanation for a set of data or observations.)	No correct or incorrect solution is presumed but suggestions must be acceptable within the limits of scientific knowledge.
Use	apply knowledge/principles to solve problems	Make inferences/conclusions.

♦ GLOSSARY OF MATHEMATICAL TERMS

WORDS	MEANING
Absolute Value	The absolute value of a real number x, denoted by $\|x\|$, is defined by $\|x\| = x$ if $x > 0$ and $\|x\| = -x$ if $x < 0$. For example, $\|-4\| = 4$.
Algorithm	A process consisting of a specific sequence of operations to solve a certain types of problems. See **Heuristic**.
Argand Diagram	An Argand diagram is a rectangular coordinate system where the complex number $x + iy$ is represented by the point whose coordinates are x and y. The x-axis is called the real axis and the y-axis is called the imaginary axis.
Argument of a Complex Number	The angle, $\theta = tan^{-1}\left(\frac{y}{x}\right)$, is called the argument of a complex number $z = x + iy$.
Arithmetic Mean	The average of a set of values found by dividing the sum of the values by the amount of values.
Arithmetic Progression	An arithmetic progression is a sequence of elements, $a1, a2, a3$,….., such that there is a common difference of successive terms. For example, the sequence {2, 5, 8, 11, 14, …} has common difference, $d = 3$.
Asymptotes	A straight line is said to be an asymptote of a curve if the curve has the property of becoming and staying arbitrarily close to the line as the distance from the origin increases to infinity.
Augmented Matrix	If a system of linear equations is written in matrix form $Ax = b$, then the matrix $[A\|b]$ is called the augmented matrix.
Average	The average of a set of values is the number which represents the usual or typical value in that set. Average is synonymous with measures of central tendency. These include the mean, mode and median.
Axis of symmetry	A line that passes through a figure such that the portion of the figure on one side of the line is the mirror image of the portion on the other side of the line.
Bar Chart	A bar chart is a diagram which is used to represent the frequency of each category of a set of data in such a way that the height of each bar if proportionate to the frequency of the category it represents. Equal space should be left between consecutive bars to indicate it is not a histogram.

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Base	In the equation $y = log_a x$, the quantity a is called the base. The base of a polygon is one of its sides; for example, a side of a triangle. The base of a solid is one of its faces; for example, the flat face of a cylinder. The base of a number system is the number of digits it contains; for example, the base of the binary system is two.
Bias	Bias is systematically misestimating the characteristics of a population (parameters) with the corresponding characteristics of the sample (statistics).
Biased Sample	A biased sample is a sample produced by methods which ensures that the statistics is systematically different from the corresponding parameters.
Bijective	A function is bijective if it is both injective and surjective; that is, both one-to-one and unto.
Bimodal	Bimodal refers to a set of data with two equally common modes.
Binomial	An algebraic expression consisting of the sum or difference of two terms. For example, $(ax + b)$ is a binomial.
Binomial Coefficients	The coefficients of the expansion $(x + y)^n$ are called binomial coefficients. For example, the coefficients of $(x + y)^3$ are 1, 3, 3 and 1.
Box-and-whiskers Plot	A box-and-whiskers plot is a diagram which displays the distribution of a set of data using the five number summary. Lines perpendicular to the axis are used to represent the five number summary. Single lines parallel to the axis are used to connect the lowest and highest values to the first and third quartiles respectively and double lines parallel to the axis form a box with the inner three values.
Categorical Variable	A categorical variable is a variable measured in terms possession of quality and not in terms of quantity.
Class Intervals	Non-overlapping intervals, which together contain every piece of data in a survey.
Closed Interval	A closed interval is an interval that contains its end points; it is denoted with square brackets $[a, b]$. For example, the interval $[-1,2]$ contains -1 and 2. For contrast see **open interval**.

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Composite Function	A function consisting of two or more functions such that the output of one function is the input of the other function. For example, in the composite function $f\big(g(x)\big)$ the input of f is g.
Compound Interest	A system of calculating interest on the sum of the initial amount invested together with the interest previously awarded; if A is the initial sum invested in an account and r is the rate of interest per period invested, then the total after n periods is $A(1+r)^n$.
Combinations	The term combinations refers to the number of possible ways of selecting r objects chosen from a total sample of size n if you don't care about the order in which the objects are arranged. Combinations is calculated using the formula $nCr = \binom{n}{r} = C_r^n = \frac{n!}{r!(n-r)!}$. See **factorial**.
Complex Numbers	A complex number is formed by adding a pure imaginary number to a real number. The general form of a complex number is $z = x + iy$, where x and y are both real numbers and i is the imaginary unit: $i^2 = -1$. The number x is called the real part of the complex number, while the number y is called the imaginary part of the complex number.
Conditional Probability	The conditional probability is the probability of the occurrence of one event affecting another event. The conditional probability of event A occurring given that even B has occurred is denoted $P(A\|B)$ (read "probability of A given B"). The formula for conditional probability is $P(A\|B) = \frac{P(A \text{ and } B)}{P(B)}$.
Conjugate of a Complex Number	The conjugate of a complex number $z = x + iy$ is the complex number $\bar{z} = x - iy$, found by changing the sign of the imaginary part. For example, if $z = 3 - 4i$, then $\bar{z} = 3 + 4i$.
Continuous	The graph of $y = f(x)$ is continuous at a point a if: 1. $f(a)$ exists, 2. $\lim\limits_{x \to a} f(x)$ exists, and 3. $\lim\limits_{x \to a} f(x) = f(a)$. A function is said to be continuous in an interval if it is continuous at each point in the interval.
Continuous Random Variable	A continuous random variable is a random variable that can take on any real number value within a specified range. For contrast, see **Discrete Random Variable.**
Coterminal	Two angles are said to be coterminal if they have the same initial and terminal arms. For example, $\theta = 30^o$ is coterminal with $\alpha = 390^o$.

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Critical Point	A critical point of a function $f(x)$ is the point $P(x, y)$ where the first derivative, $f'(x)$ is zero. See also **stationary points.**
Data	Data (plural of datum) are the facts about something. For example, the height of a building.
Degree	1. The degree is a unit of measure for angles. One degree is $\frac{1}{360}$ of a complete rotation. See also **Radian.** 2. The degree of a polynomial is the highest power of the variable that appears in the polynomial. For example, the polynomial $p(x) = 2 + 3x - x^2 + 7x^3$ has degree 3.
Delta	The Greek capital letter delta, which has the shape of a triangle: Δ, is used to represent "change in". For example Δx represents "change in x".
Dependent Events	In Statistics, two events A and B are said to be dependent if the occurrence of one event affects the probability of the occurrence of the other event. For contrast, see **Independent Events.**
Derivative	The derivative of a function $y = f(x)$ is the rate of change of that function. The notations used for derivative include: $y' = f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$.
Descriptive Statistics	Descriptive statistics refers to a variety of techniques that allows for general description of the characteristics of the data collected. It also refers to the study of ways to describe data. For example, the mean, median, variance and standard deviation are descriptive statistics. For contrast, see **Inferential Statistics.**
Determinant	The determinant of a matrix is a number that is useful for describing the characteristics of the matrix. For example if the determinant is zero then the matrix has no inverse.
Differentiable	A continuous function is said to be differentiable over an interval if its derivative exists for every point in that interval. That means that the graph of the function is smooth with no kinks, cusps or breaks.
Differential Equation	A differential equation is an equation involving the derivatives of a function of one or more variables. For example, the equation $\frac{dy}{dx} - y = 0$ is a differential equation.
Differentiation	Differentiation is the process of finding the derivative.
Discrete	A set of values are said to be discrete if they are all distinct and separated from each other. For example the set of shoe sizes where the elements of this set can only take on a limited and distinct set of

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	values. See **Discrete Random Variables.**
Discrete Random Variable	A discrete random variable is a random variable that can only take on values from a discrete list. For contrast, see **Continuous Random Variables.**
Estimate	The best guess for an unknown quantity arrived at after considering all the information given in a problem.
Even Function	A function $y = f(x)$ is said to be even if it satisfies the property that $f(x) = f(-x)$. For example, $f(x) = \cos x$ and $g(x) = x^2$ are even functions. For contrast, see Odd Function.
Event	In probability, an event is a set of outcomes of an experiment. For example, the even A may be defined as obtaining two heads from tossing a coin twice.
Expected Value	The average amount that is predicted if an experiment is repeated many times. The expected value of a random variable X is denoted by $E[X]$. The expected value of a discrete random variable is found by taking the sum of the product of each outcome and its associated probability. In short, $E[X] = \sum_{i=1}^{n} x_i p(x_i)$.
Experimental Probability	Experimental probability is the chances of something happening, based on repeated testing and observing results. It is the ratio of the number of times an event occurred to the number of times tested. For example, to find the experimental probability of winning a game, one must play the game many times, then divide by the number of games won by the total number of games played.
Exponent	An exponent is a symbol or a number written above and to the right of another number. It indicates the operation of repeated multiplication.
Exponential Function	A function that has the form $y = a^x$, where a is any real number and is called the base.
Extrapolation	An extrapolation is a predicted value that is outside the range of previously observed values. For contrast, see **Interpolation**.
Factor	A factor is one of two or more expressions which are multiplied together. A prime factor is an indecomposable factor. For example, the factors of $(x^2 - 4)(x + 3)$ include $(x^2 - 4)$ and $(x + 3)$, where $(x + 3)$ is prime but $(x^2 - 4)$ is not prime as it can be further decomposed into $(x - 2)(x + 2)$.
Factorial	The factorial of a positive integer n is the product of all the integers

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	from 1 up to n and is denoted by $n!$, where $1! = 0! = 1$. For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.
Function	A correspondence in which each member of one set is mapped unto a member of another set.
Geometric Progression	A geometric progression is a sequence of terms obtained by multiplying the previous term by a fixed number which is called the common ratio. A geometric progression is of the form $a, ar, ar^2, ar^3, \ldots$.
Graph	A visual representation of data that displays the relationship among variables, usually cast along x and y axes.
Grouped Data	Grouped data refers to a range of values which are combined together so as to make trends in the data more apparent.
Heterogeneity	Heterogeneity is the state of being of incomparable magnitudes. For contrast, see **Homogeneity.**
Heuristic	A heuristic method of solving problems involve intelligent trial and error. For contrast, see **Algorithm.**
Histogram	A histogram is a bar graph with no spaces between the bars where the area of the bars are proportionate to the corresponding frequencies. If the bars have the same width then the heights are proportionate to the frequencies.
Homogeneity	Homogeneity is the state of being of comparable magnitudes. For contrast, see **Heterogeneity.**
Identity	1. An equation that is true for every possible value of the variables. For example $x^2 - 1 \equiv (x-1)(x+1)$ is an identity while $x^2 - 1 = 3$ is not, as it is only true for the values $x = \pm 2$. 2. The **identity element** of an operation is a number such that when operated on with any other number results in the other number. For example, the identity element under addition of real numbers is zero; the identity element under multiplication of 2×2 matrices is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
Independent Events	In Statistics, two events are said to be independent if they do not affect each other. That is, the occurrence of one event does not depend on whether or not the other event occurred.
Inferential Statistics	Inferential Statistics is the branch of mathematics which deals with the generalisations of samples to the population of values.

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Infinity	The symbol ∞ indicating a limitless quantity. For example, the result of a nonzero number divided by zero is infinity.
Integration	Integration is the process of finding the integral which is the antiderivative of a function.
Interpolation	An interpolation is an estimate of an unknown value which is within the range of previously observed values. For contrast, see **Extrapolation.**
Interval	An interval on a number line is a continuum of points bounded by two limits (end points). An **Open Interval** refers to an interval that excludes the end points and is denoted (a, b). For example, $(0,1)$. A **Closed Interval** in an interval which includes the end points and is denoted $[a, b]$. For example $[-1,3]$. A **Half-Open Interval** is an interval which includes one end point and excludes the other. For example, $[0, \infty)$.
Interval Scale	Interval scale refers to data where the difference between values can be quantified in absolute terms and any zero value is arbitrary. Finding a ratio of data values on this scale gives meaningless results. For example, temperature is measured on the interval scale: the difference between $19^o C$ and $38^o C$ is $19^o C$, however, $38^o C$ is not twice as warm as $19^o C$ and a temperature of $0^o C$ does not mean there is no temperature. See also Nominal, Ordinal and Ratio scales.
Inverse	1. **The inverse of an element under an operation** is another element which when operated on with the first element results in the identity. For example, the inverse of a real number under addition is the negative of that number. 2. **The inverse of a function** $f(x)$ is another function denoted $f^{-1}(x)$, which is such that $f[f^{-1}(x)] = f^{-1}[f(x)] = x$.
Irrational Number	A number that cannot be represented as an exact ratio of two integers. For example, π or the square root of 2.
Limit	The limit of a function is the value which the dependent variable approaches as the independent variable approaches some fixed value.
Line of Best Fit	The line of best fit is the line that minimises the sum of the squares of the deviations between each point and the line.
Linear Expression	An expression of the form $ax + b$ where x is a variable and a and b are constants, or in more variables, an expression of the form $ax + by + c, ax + by + cz + d$ where a, b, c and d are constants.

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Logarithm	A logarithm is the power of another number called the base that is required to make its value a third number. For example 3 is the logarithm which carries 2 to 8. In general, if y is the logarithm which carries a to x, then it is written as $y = \log_a x$ where a is called the base. There are two popular bases: base 10 and base e. 1. **The Common Logarithm (Log)**: the equation $y = \log x$ is the shortened form for $y = \log_{10} x$. 2. **The Natural Logarithm (Ln)**: The equation $y = \ln x$ is the shortened form for $y = \log_e x$
Matrix	A rectangular arrangement of numbers in rows and columns.
Method	In Statistics, the research methods are the tools, techniques or processes that we use in our research. These might be, for example, surveys, interviews, or participant observation. Methods and how they are used are shaped by methodology.
Methodology	Methodology is the study of how research is done, how we find out about things, and how knowledge is gained. In other words, methodology is about the principles that guide our research practices. Methodology therefore explains why we're using certain methods or tools in our research.
Modulus	The modulus of a complex number $z = x + iy$ is the real number $\lvert z\rvert = \sqrt{x^2 + y^2}$. For example, the modulus of $z = -7 + 24i$ is $\lvert z\rvert = \sqrt{(-7)^2 + 24^2} = 25$
Mutually Exclusive Events	Two events are said to be mutually exclusive if they cannot occur simultaneously, in other words, if they have nothing in common. For example, the event "Head" is mutually exclusive to the event "Tail" when a coin is tossed.
Mutually Exhaustive Events	Two events are said to be mutually exhaustive if their union represents the sample space.
Nominal Scale	Nominal scale refers to data which names of the outcome of an experiment. For example, the country of origin of the members of the West Indies cricket team. See also Ordinal, Interval and Ratio scales.
Normal	The normal to a curve is a line which is perpendicular to the tangent to the curve at the point of contact.
Odd Function	A function is an odd function if it satisfies the property that $f(-x) = -f(x)$. For example, $f(x) = \sin x$ and $g(x) = x^3$ are odd functions. For contrast, see **Even Function**.

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Ordinal Scale	Data is said be in the ordinal scale if they are names of outcomes where sequential values are assigned to each name. For example, if Daniel is ranked number 3 on the most prolific goal scorer at the Football World Cup, then it indicates that two other players scored more goals than Daniel. However, the difference between the 3rd ranked and the 10th ranked is not necessarily the same as the difference between the 23rd and 30th ranked players. See also **Nominal, Interval and Ratio scales**.
Outlier	An outlier is an observed value that is significantly different from the other observed values.
Parameter	In statistics, a parameter is a value that characterises a population.
Partial Derivative	The partial derivative of $y = f(x_1, x_2, x_3, \dots, x_n)$ with respect to x_i is the derivative of y with respect to x, while all other independent variables are treated as constants. The patrial derivative is denoted by $\frac{\partial f}{\partial x}$. For example, if $f(x, y, z) = 2xy + x^2 z - \frac{3x^3 y}{z}$, then $\frac{\partial f}{\partial x} = 2y + 2xz - \frac{9x^2 y}{z}$
Pascal Triangle	The Pascal triangle is a triangular array of numbers such that each number is the sum of the two numbers above it (one left and one right). The numbers in the n^{th} row of the triangle are the coefficients of the binomial expansion $(x + y)^n$.
Percentile	The *p*th percentile of in a list of numbers is the smallest value such that *p*% of the numbers in the list is below that value. See also **Quartiles.**
Permutations	Permutations refers to the number of different ways of selecting a group of r objects from a set of n object when the order of the elements in the group is of importance and the items are not replaced. If $r = n$ then the permutations is $n!$; if $r < n$ then the number of permutation is $P_r^n = \frac{n!}{(n-r)!}$.
Piecewise Continuous Function	A function is said to be piecewise continuous if it can be broken into different segments where each segment is continuous.
Polynomial	A polynomial is an algebraic expression involving a sum of algebraic terms with nonnegative integer powers. For example, $2x^3 + 3x^2 - x + 6$ is a polynomial in one variable.
Population	In statistics, a population is the set of all items under consideration.
Principal Root	The principal root of a number is the positive root. For example, the principal square root of 36 is 6 (not -6).
Principal Value	The principal value of the arcsin and arctan functions lies on the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. The principal value of the arcos function lies on

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	the interval $0 \leq x \leq \pi$.
Probability	1. The probability of an event is a measure of how likely it is for an event to occur. The probability of an event is always a number between zero and 1. 2. Probability is the study of chance occurrences.
Probability Distribution	A probability distribution is a table or function that gives all the possible values of a random variable together with their respective probabilities.
Probability Space	The probability space is the set of all outcomes of a probability experiment.
Proportion	1. A relationship between two ratios in which the first ratio is always equal to the second. Usually of the form $\frac{a}{b} = \frac{c}{d}$. 2. The fraction of a part and the whole. If two parts of a whole are in the ratio 2:7, then the corresponding proportions are $\frac{2}{9}$ and $\frac{7}{9}$ respectively.
Pythagorean Triple	A Pythagorean triple refers to three numbers, a, b & c, satisfying the property that $a^2 + b^2 = c^2$.
Quadrant	The four parts of the coordinate plane divided by the x and y axes are called quadrants. Each of these quadrants has a number designation. First quadrant – contains all the points with positive x and positive y coordinates. Second quadrant – contains all the points with negative x and positive y coordinates. The third quadrant contains all the points with both coordinates negative. Fourth quadrant – contains all the points with positive x and negative y coordinates.
Quadrantal Angles	Quadrantal Angles are the angles measuring $0^o, 90^o, 180^o$ & 270^o and all angles coterminal with these. See **Coterminal.**
Quartic	A quartic equation is a polynomial of degree 4.
Quartiles	Consider a set of numbers arranged in ascending or descending order. The quartiles are the three numbers which divide the set into four parts of equal amount of numbers. The first quartile in a list of numbers is the number such that a quarter of the numbers is below it. The second quartile is the median. The third quartile is the number such that three quarters of the numbers are below it. See also **Percentile.**
Quintic	A quintic equation is a polynomial of degree 5.

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Radian	The radian is a unit of measure for angles, where one radian is $\frac{1}{2\pi}$ of a complete rotation. One radian is the angle in a circle subtended by an arc of length equal to that of the radius of the circle. See also **Degrees**.
Radical	The radical symbol ($\sqrt{}$) is used to indicate the taking of a root of a number. $\sqrt[q]{x}$ means the q^{th} root of x; if $q = 2$ then it is usually written as $\sqrt{x}$. For example $\sqrt[5]{243} = 3$, $\sqrt[4]{16} = 2$. The radical always means to take the positive value. For example, both 5 and -5 satisfy the equation $\mathrm{x}^2 = 25$, but $\sqrt{25} = 5$.
Random Variable	A random variable is a variable that takes on a particular value when a random event occurs.
Ratio Scale	Data are said to be on the ratio scale if they can be ranked, the distance between two values can be measured and the zero is absolute, that is, zero means "absence of". See also Nominal, Ordinal and Interval Scales.
Regression	Regression is a statistical technique used for determining the relationship between two quantities.
Residual	In linear regression, the residual refers to the difference between the actual point and the point predicted by the regression line. That is the vertical distance between the two points.
Root	1. The root of an equation is the same as the solution of that equation. For example, if $y = f(x)$, then the roots are the values of x for which $y = 0$. Graphically, the roots are the x-intercepts of the graph. 2. The n^{th} root of a real number x is a number which, when multiplied by itself n times, gives x. If n is odd then there is one root for every value of x; if n is even then there are two roots (one positive and one negative) for positive values of x and no real roots for negative values of x. The positive root is called the **Principal root** and is represented by the radical sign ($\sqrt{}$). For example, the principal square root of 9 is written as $\sqrt{9} = 3$ but the square roots of 9 are $\pm\sqrt{9} = \pm 3$.
Sample	A group of items chosen from a population.
Sample Space	The set of all possible outcomes of a probability experiment. Also called probability space.
Sampling Frame	In statistics, the sampling frame refers to the list of cases from which a sample is to be taken.
Scientific Notation	A shorthand way of writing very large or very small numbers. A

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	number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10 (for example, 7000 = 7×10^3 or 0.0000019 = 1.9×10^{-6}).
Series	A series is an indicated sum of a sequence.
Sigma	1. The Greek capital letter sigma, Σ, denotes the summation of a set of values. 2. The corresponding lowercase letter sigma, σ, denotes the standard deviation.
Significant Digits	The amount of digits required for calculations or measurements to be close enough to the actual value. Some rules in determining the number of digits considered significant in a number: - The leftmost non-zero digit is the first significant digit. - Zeros between two non-zero digits are significant. - Trailing zeros to the right of the decimal point are considered significant.
Simple Event	A non-decomposable outcome of a probability experiment.
Skew	Skewness is a measure of the asymmetry of a distribution of data.
Square Matrix	A matrix with equal number of rows and columns.
Square Root	The square root of a positive real number n is the number m such that $m^2 = n$. For example, the square roots of 16 are 4 and -4.
Standard Deviation	The standard deviation of a set of numbers is a measure of the average deviation of the set of numbers from their mean.
Stationary Point	The stationary point of a function $f(x)$ is the point $P(x_o, y_o)$ where $f'(x) = 0$. There are three type of stationary points, these are: 1. Maximum point is the stationary point such that $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} \leq 0$; 2. Minimum point is the stationary point such that $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} \geq 0$; 3. Point of Inflexion is the stationary point where $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = 0$ and the point is neither a maximum nor a minimum point.
Statistic	A statistic is a quantity calculated from among the set of items in a sample.
Statistical Inference	The process of estimating unobservable characteristics of a population by using information obtained from a sample.
Symmetry	Two points A and B are symmetric with respect to a line if the line is a perpendicular bisector of the segment AB.

WORDS	MEANING
Tangent	A line is a tangent to a curve at a point A if it just touches the curve at A in such a way that it remains on one side of the curve at A. A tangent to a circle intersects the circle only once.
Theoretical Probability	The chances of events happening as determined by calculating results that would occur under ideal circumstances. For example, the theoretical probability of rolling a 4 on a fair four-sided die is ¼ or 25%, because there is one chance in four to roll a 4, and under ideal circumstances one out of every four rolls would be a 4.
Trigonometry	The study of triangles. Three trigonometric functions defined for either acute angles in the right-angled triangle are: Sine of the angle x is the ratio of the side opposite the angle and the hypotenuse. In short, $\sin x = \frac{O}{H}$; Cosine of the angle x is the ratio of the short side adjacent to the angle and the hypotenuse. In short, $\cos x = \frac{A}{H}$; Tangent of the angle x is the ratio of the side opposite the angle and the short side adjacent to the angle. In short $\tan x = \frac{O}{A}$.
Z-Score	The z-score of a value x is the number of standard deviations it is away from the mean of the set of all values. $z - score = \frac{x-\bar{x}}{\sigma}$.

♦ *ADDITIONAL NOTES FOR TEACHING AND LEARNING*

UNIT 1

MODULE 1: COLLECTING AND DESCRIBING DATA

Data Collection

It is critical that students collect data *that can* be used in subsequent Modules. The data collected should also be used as stimulus material for hypothesis testing or linear regression and correlation, by investigating relationships between variables.

Description of Data

Calculators or statistical software should be used whenever possible to display and analyse the collected data. Strengths and weaknesses of the different forms of data representation should be emphasised.

UNIT 1

MODULE 2: MANAGING UNCERTAINTY

Most concepts in this Module are best understood by linking them to the data collected and concepts learnt in Module 1. Only simple arithmetic, algebraic and geometric operations are required for this Module.

Probability Theory

The main emphasis is on understanding the nature of probability as applied to modelling and data interpretation such as the waiting times for taxis and heights of students. Many problems are often best solved with the aid of a Venn diagram, tree diagram or possibility space diagram. Therefore, *students* should be encouraged to draw diagrams *(Venn, tree or possibility space)* as aids or explanations to the solution of problems.

Concepts of possibility spaces and events may be motivated through the practical activities undertaken in *the* Data *Collection section* of *Unit 1,* Module 1.

Random Variables

Clarify the concepts of discrete and continuous random variables.

Examples: Examples of discrete random variables include the number of televisions per household and the number of people queuing at checkouts; while examples of continuous random variables include the waiting times for taxis and heights of students.

It should be emphasised that, for continuous random variables, the area under the graph of a probability density function is a measure of probability and note the important fact that $P(X = a) = 0$.

Normal Distribution

Concept of continuous random variables with particular reference to the normal distribution *should be discussed.* Students should be made aware that a normal distribution N (μ, σ^2) is uniquely defined by its mean, μ, and variance, σ^2. The shape of the normal distribution for varying values of μ, and σ^2 could then be explored.

It can be demonstrated that the binomial distribution may be approximated by the normal distribution.

Example:

Use a graphical calculator to study the graph of $(p + q)^n$ where
$p = 0.25$, $q = 0.75$ and $n = 3, 10, 25, 50, 100$.

Repeat the above activity with different values of $(p + q)^n$ where
$p < 0.25$ and $q > 0.75$ or $(p > 0.25)$ and $(q < 0.75)$ and $n = 3, 10, 25, 50, 100$.

Arithmetic, Algebraic and Geometric Operations required for this Module.

The concepts below showed be discussed.

1. Use of the operations $+, -, \times, \div$ on integers, decimals and fractions.

Arithmetic

2. Knowledge of *real* numbers.

3. Simple applications of ratio, percentage and proportion.

4. Absolute value, $|a|$.

Algebra

1. Language of sets.

2. Operations on sets: union, intersection, complement.

3. Venn diagram and set notation.

4. Basic manipulation of simple algebraic expressions including factorisation and expansion.

5. Solutions of linear equations and inequalities in one variable.

6. Solutions of simultaneous linear equations in two variables.

7. Solutions of quadratic equations.

8. Ordered relations $<, >, \leq, \geq$ and their properties.

Geometry

1. Elementary geometric ideas of the plane.

2. Concepts of a point, line and plane.

3. Simple two-dimensional shapes and their properties.

4. Areas of polygons and simple closed curves.

UNIT 1

MODULE 3: ANALYSING AND INTERPRETING DATA

Allow students to use the data collected in Modules 1 and 2 to facilitate the attainment of the objectives of this Module. Classroom discussions and oral presentations of work done by students, individually or in groups, should be stressed at all times.

Sampling Distributions *and Estimation*

Shoppers often sample a plum to determine its sweetness before purchasing any. They decide from one plum what the larger bunch or lot will taste like. A chemist does the same thing when he takes a sample of rum from a curing vat. He determines *if* it is 90 *per cent* proof and infers that all the rum in the vat is 90 *per cent* proof. If the chemist tests all of the rum, and the shopper tastes all the plums, *then* there may be none to sell. Testing all the product often destroys it and is unnecessary. To determine the characteristics of the whole we have to sample a portion'. *This is the same when sampling the population.*

Time is also a factor when managers need information quickly in order to adjust an operation or to change policy. Take an automatic machine that sorts thousands of pieces of mail daily. Why wait for an entire day's output to check whether the machine is working accurately? Instead, samples can be taken at specific intervals, and if necessary, the machine can be adjusted right away.

Sampling distributions of data collected by students working in groups should be presented in tables and graphs. The emerging patterns should be discussed and used to explore the concepts and principles.

Hypothesis Testing

Suppose a manager of a large shopping mall tells us that the average work efficiency of the employees is 90 per cent. How can we test the validity of that manager's claim or hypothesis? Using a sampling method discussed, the efficiency of a sample could be calculated. If the sample statistic came out to 93 per cent, would the manager's statement be readily accepted? If the sample statistic were 43 per cent, we may reject the claim as untrue. Using common sense the claim can either be accepted or rejected based on the results of the sample. Suppose the sample statistic revealed an efficiency of 83 per cent. This is relatively close to 90 per cent. Is it close enough to 90 per cent for us to accept or reject the manager's claim or hypothesis?

Whether we accept or reject the claim we cannot be absolutely certain that our decision is correct. Decisions on acceptance or rejection of a hypothesis cannot be made on intuition. One needs to learn how to decide objectively on the basis of sample information, whether to accept or reject a hypothesis.

Correlation and Linear Regression – Bivariate Data

Information collected in Module 1, from the section Data Analysis can be applied to the concepts of linear regression and correlation.

Students should become proficient in the use of computer *software such as SPSS, Excel, Minitab* or scientific calculators *(non-programmable)* to perform statistical calculations, as in obtaining regression estimates and correlation coefficients.

UNIT 2

MODULE 1: DISCRETE MATHEMATICS

Critical Path Analysis

The critical path in an activity network has proven to be very useful to plan, schedule and control a wide variety of activities and projects in real-world situations. These projects include construction of plants, buildings, roads, the design and installation of new systems, finding the shortest route in a connected set of roads, organising a wedding, and organising a regional cricket competition.

Staying on the critical path in an activity network designed for the construction of a building, for example, ensures that the building is completed as scheduled.

UNIT 2

MODULE 2: PROBABILITY AND DISTRIBUTION

Probability

While teaching counting principles, *introduce* the concepts of independence and mutually exclusive events.

Discrete Random Variables

Computation of expected values and variances will not entail lengthy calculations or the summation of series.

The difference between discrete and continuous random variables could be illustrated by using real life situations.

Continuous Random Variables

Students may need to be introduced to the integration of simple polynomials.

UNIT 2

MODULE 3: PARTICLE MECHANICS

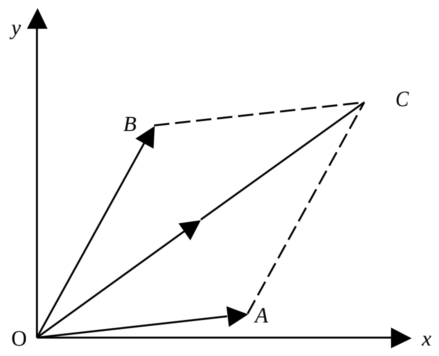
COPLANAR FORCES *AND EQUILIBRIUM*

Vectors

Be advised that students should have *practice* in dealing with vectors (see Module 2 Unit 1 of the Pure Mathematics syllabus) in order to represent a force as a vector.

Resolution of Forces

Have students consider two vectors **a**, **b** which have the same initial point O as in the figure below. Ask students to complete the parallelogram *OACB* as shown by the dotted lines, draw the diagonal *OC* and denote the vector $\overrightarrow{OC}$ by **c**.

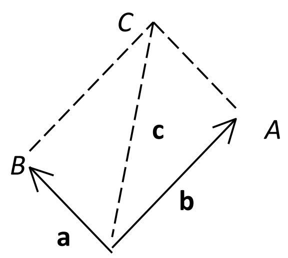


*The vector c represents the resultant of the vectors **a** and **b**. Conversely, the vectors **a, b** can be regarded as the components of **c**. In other words, starting with the parallelogram OACB, the vector* $\overrightarrow{OC}$ *is said to be resolved into vectors* $\overrightarrow{OA}$ *and* $\overrightarrow{OB}$.

Let $\overrightarrow{\mathbf{OA}} = \mathbf{a}$ *and* $\overrightarrow{\mathbf{OB}} = \mathbf{b}$ *and* $\overrightarrow{\mathbf{OC}} = \mathbf{c}$

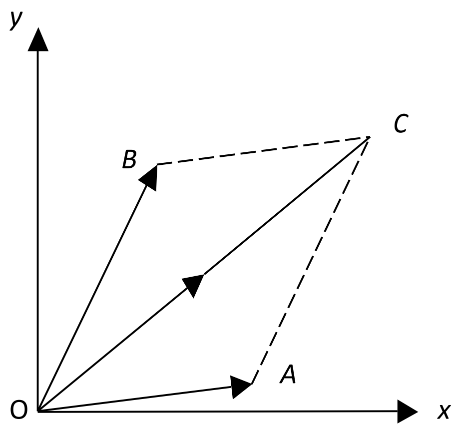
By the parallelogram $\overrightarrow{\mathbf{OC}} = \overrightarrow{\mathbf{OA}} + \overrightarrow{\mathbf{OB}} = \mathbf{a} + \mathbf{b}$

Forces acting on a particle in equilibrium are equivalent to a single force acting at a common point.



*Diagrams are not in sequence to examples, the fixed point **O** is not shown in parallelogram **OACB** and points **B** and **A** are not correctly labelled.*

See example below.



Let $\overrightarrow{OA} = \boldsymbol{a}$ *and* $\overrightarrow{OB} = \boldsymbol{b}$ *and* $\overrightarrow{OC} = \boldsymbol{c}$

By the parallelogram law $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB} = \boldsymbol{a} + \boldsymbol{b}$

Forces

Students should be made aware of the definitions used in Mechanics.

Examples include:

(a) ***body*** *is any object to which a force can be applied;*

(b) ***particle*** *is a body whose dimensions, except mass, are negligible;*

(c) ***weight*** *is the force with which the earth attracts the body. It acts at the body's centre of gravity and is always vertically downwards;*

(d) ***a light body*** *is considered to be weightless;*

(e) ***pull and push*** *(P) are forces which act on a body at the point(s) where they are applied; and,*

(f) *normal reaction (R) is a force which acts on a body in contact with a surface. It acts in a direction at right angles to the surfaces in contact.*

Drawing Force Diagrams

Students should know that drawing a clear **force diagram** is an essential first step in the solution of any problem in mechanics which is concerned with the action of forces on a body.

***Students should be aware of the* important points *as listed below* to remember when drawing force diagrams:**

1. make the diagram large enough to show clearly all the forces acting on the body and to enable any necessary geometry and trigonometry to be done;

2. show only forces which are acting on the body being considered;

3. weight always acts on the body unless the body is described as light;

4. contact with another object or surface gives rise to a normal reaction and sometimes friction;

5. attachment to another object (by string, spring, hinge) gives rise to a force on the body at the point of attachment;

6. forces acting on a particle act at the same point; and,

7. check that no forces have been omitted or included more than once.

KINEMATICS AND DYNAMICS

Definitions

Distance *is how much ground is covered by an object despite its starting or ending point.*

Displacement is the position of a point relative to a fixed origin O. It is a vector. The SI Unit is the metre (m). Other metric units are centimeter (cm), kilometer (km).

Velocity is the rate of change of displacement with respect to time. It is a **vector**. The SI Unit is **metre per second** ($m\ s^{-1}$). Other metric units are $cm\ s^{-1}$, kmh^{-1}.

Speed is the magnitude of the velocity and is a scalar quantity.

Uniform velocity is the constant speed in a fixed direction.

Average velocity – $\dfrac{\text{change in displacement}}{\text{time taken}}$

Average speed – $\dfrac{\text{total distance travelled}}{\text{time taken}}$

Acceleration is the rate of change of velocity with respect to time. It is a **vector**. The SI Unit is **metre per second square** ($m\ s^{-2}$). Other metric units are $cm\ s^{-2}$, $km\ h^{-2}$.

Negative acceleration is also referred to as retardation.

Uniform acceleration is the constant acceleration in a fixed direction.

Motion in one dimension – When a particle moves in **one dimension**, that is, along a straight line, it has only two possible directions in which to move. Positive and negative signs are used to identify the two directions.

Vertical motion under gravity – this is a special case of uniform acceleration in a straight line. The body is thrown **vertically upward**, or falling **freely downward**. This uniform acceleration is due to **gravity** and acts vertically downwards towards the centre of the earth. It is denoted by **g** and may be approximated by **9.8 m s $^{-2}$** or **10 m s $^{-2}$.**

Graphs in Kinematics

A **displacement-time** graph for a body moving in a straight line shows its displacement x from a fixed point on the line plotted against time, t. The **velocity** v of the body at time, t is given by the **gradient** of the graph since

$$\frac{dx}{dt} = v.$$

The **displacement-time** graph for a body moving with **constant velocity** is a **straight line**. The velocity, v of the body is given by the gradient of the line.

The **displacement-time** graph for a body moving with **variable velocity** is a **curve**.
The velocity at any time, t may be estimated from the gradient of the tangent to the curve at that time. The average velocity between two times may be estimated from the gradient of the chord joining them.

Velocity-time graph for a body moving in a straight line shows its velocity v plotted against time, t. The **acceleration,** a of the body at time, t is given by the **gradient** of the graph at t, since $a = \frac{dv}{dt}$.

The **displacement** in a time interval is given by the **area** under the **velocity-time** graph for that time interval

Since $x = \int_{t_1}^{t_2} v\, dt$.

The **velocity-time** graph for a body moving with **uniform acceleration** is a **straight line**. The acceleration of the body is given by the gradient of the line.

Particle Dynamics

Force is necessary to cause a body to **accelerate**. More than one force may act on a body. If the forces on a body are in **equilibrium,** then the body may be at rest or moving in a straight line at constant speed.

If forces are acting on a body, then the body will accelerate in the direction of the resultant force. Force is a **vector**; that is, it has magnitude and direction. The SI Unit is the **newton** (N). One newton is the force needed to give a body a mass of 1 kg an acceleration of 1 ms $^{-2}$.

Mass and Weight are different. The **mass** of a body is a measure of the matter contained in the body. A massive body will need a large force to change its motion. The mass of a body may be considered to be uniform, whatever the position of the body, provided that no part of the body is destroyed or changed.

Mass is a **scalar** quantity; that is, it has magnitude only. The SI Unit of mass is the **kilogram** (kg). However, for heavy objects it is sometimes more convenient to give mass in tonnes, where 1 tonne = 1000 kg.

The **weight** of a body is the force with which the earth attracts that body. It is dependent upon the body's distance from the centre of the earth, so a body weighs less at the top of Mount Everest than it does at sea level.

Weight is a **vector** since it is a force. The SI Unit of weight is the **newton** (N).
The weight, W, in newtons, and mass, m, in kilograms, of a body are connected by the relation W = mg, where g is the acceleration due to gravity, in m s^{-2}.

Newton's three laws of motion are the basis of the study of mechanics at this level.

1st Law: A body will remain at rest or continue to move in a straight line at constant speed unless an external force acts on it.

(a) If a body has an acceleration, then there must be a force acting on it.

(b) If a body has no acceleration, then the forces acting on it must be in equilibrium.

2nd Law: The rate of change of momentum of a moving body is proportional to the external forces acting on it and takes place in the direction of that force. When an external force acts on a body of uniform mass, the force produces an acceleration which is directly proportional to the force.

(a) The basic equation of motion for constant mass is

Force = mass × acceleration
(in N) (in kg) (in m s^{-2})

(b) The force and acceleration of the body are both in the same direction.

(c) A constant force on a constant mass gives a constant acceleration.

3rd Law: If a body, A exerts a force on a body, B, then B exerts an equal and opposite force on A. These forces between bodies are often called reactions. In a rigid body the internal forces occur as equal and opposite pairs and the net effect is zero. So only external forces need be considered.

The following are important points to remember when solving problems using Newton's laws of motion:

(a) Draw a clear force diagram.

(b) If there is no acceleration, that is, the body is either at rest or moving with uniform velocity, then the forces balance in each direction.

(c) If there is an acceleration:

 (i) use the symbol $\longrightarrow\!\!\blacktriangleright\!\blacktriangleright$ to represent it on the diagram;

 (ii) write, if possible, an expression for the resultant force; and,

 (iii) use Newton's 2^{nd} law, that is, write the equation of motion: F = ma.

Connected Particles

Two particles connected by a light inextensible string which passes over a fixed light smooth (frictionless) pulley are called **connected particles**. The tension in the string is the same throughout its length, so each particle is acted upon by the same tension.

Problems concerned with connected particles usually involve finding the acceleration of the system and the tension in the string.

To solve problems of this type:

(a) draw a clear diagram showing the forces on each particle and the common acceleration;

(b) write the equation of motion, that is, F = ma for each particle separately; and,

(c) solve the two equations to find the common acceleration, a, and possibly the tension, T, in the string.

Systems may include:

(a) one particle resting on a **smooth** or **rough** horizontal table with a light inextensible string attached and passing over a fixed small smooth pulley at the edge of the table and with its other end attached to another particle which is allowed to hang freely;

(b) as in (i), two light inextensible strings may be attached to opposite ends of a particle resting on a **smooth** or **rough** horizontal table and passing over fixed small smooth pulleys at either edge of the table and with their other ends attached to particles of different masses which are allowed to hang freely; and,

(c) one particle resting on a **smooth** or **rough** inclined plane and attached to a light inextensible string which passes over a fixed small smooth pulley at the top of the incline and with its other end attached to another mass which is allowed to hang freely.

WORK, ENERGY AND POWER

Work may be done either by or against a force (often gravity). It is a **scalar**. When a constant force F moves its point of application along a straight line through a distance s, the **work done** by F is **F.s.** The SI Unit of work is the **joule** (J). One joule is the work done by a force of one newton in moving its point of application one metre in the direction of the force.

Energy is the capacity to do work and is a scalar. The SI Unit of energy is the **joule** (the same as work).

A body possessing energy can do work and lose energy. Work can be done on a body and increase its energy, that is, work done = change in energy.

<u>Kinetic and Potential Energy</u> – are types of **mechanical energy**.

(a) Kinetic energy (K.E.) is due to a body's motion. The K.E. of a body of mass, m, moving with velocity, v, is $\frac{1}{2}mv^2$.

(b) Gravitational Potential Energy (G.P.E.) is dependent on height. The P.E. of a body of mass m at a height h:

(i) above an initial level is given by mgh; and,

(ii) below an initial level is given by – mgh.

The P.E. at the initial level is zero (any level can be chosen as the initial level).

<u>Mechanical Energy</u> – (M.E.) of a particle (or body) = P.E. + K.E. of the particle (or body).

M.E. is lost (as heat energy or sound energy) when we have: - resistances (friction) or impulses (collisions or strings becoming taut).

<u>Conservation of Mechanical Energy</u> – The total mechanical energy of a body (or system) will be **conserved** if:

(a) no external force (other than gravity) causes work to be done; and,

(b) none of the M.E. is converted to other forms. Given these conditions:

P.E. + K.E. = constant
or loss in P.E. = gain in K.E. or loss in K.E. = gain in P.E.

<u>Power</u> – is the rate at which a force does work. It is a **scalar**. The SI Unit of power is the **watt** (W). One watt
(W) = one joule per second (J s^{-1}). The **kilowatt** (kW), 1 kW = 1000 W is used for large quantities.

When a body is moving in a straight line with velocity v m s^{-1} under a tractive force F newtons, the power of the force is P = Fv.

Moving vehicles – The power of a moving vehicle is supplied by its engine. The **tractive force** of an engine is the pushing force it exerts.

To solve problems involving moving vehicles:

1. draw a clear force diagram, (non-gravitational resistance means frictional force);

2. resolve forces perpendicular to the direction of motion;

3. if the velocity is:

   (a) constant (vehicle moving with steady speed), then resolve forces parallel to the direction of motion; and,

   (b) not constant (vehicle accelerating), then find the resultant force acting and write the equation of motion in the direction of motion.

4. Use power = tractive force × speed. Common situations that may arise are:

   (a) vehicles on the level moving with steady speed, v;

   (b) vehicles moving on the level with acceleration, a, and instantaneous speed, v;

   (c) vehicles on a slope of angle, α moving with steady speed, v, either up or down the slope; and,

   (d) vehicles on a slope moving with acceleration, a and instantaneous speed, v, up or down the slope.

Impulse and Momentum

The **impulse** of a force F, constant or variable, is equal to the **change in momentum** it produces. If a force, F acts for a time, t, on a body of mass, m, changing its velocity from u to v then Impulse = mv – mu.

Impulse is the time effect of a force. It is a **vector** and for a constant force F acting for time, t; impulse = Ft.

For a variable force, F acting for time, t, impulse = $\int_{t_1}^{t_2} F\,dt$

The SI Unit of impulse is the **newton second** (Ns).

The **momentum** of a moving body is the product of its mass m and velocity v that is, mv. It is a **vector** whose direction is that of the velocity and the SI Unit of momentum is the **newton second** (Ns).

Conservation of Momentum: The principle of **conservation of momentum** states that the total momentum of a system is constant in any direction provided no external force acts in that direction. Initial momentum = final momentum. In this context a system is usually two bodies.

Problem solving

Problems concerning impulse and momentum usually involve finding the impulse acting or the velocity on the mass of a body of a system.

To find an impulse for such a system write the impulse equation on each body.
To find a velocity or mass for such a system write the equation of the conservation of momentum.

Direct Impact – takes place when **two spheres of equal radii are** moving along the same straight line and collide.

Direct Impact with a Wall – When a smooth sphere collides **directly with a smooth vertical wall**, the sphere's direction of motion is perpendicular to the wall. The sphere receives an impulse perpendicular to the wall.

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CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

UNIT 1 – Paper 01

STATISTICAL ANALYSIS

1 minutes 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This test consists of 45 items. You will have 1 hour and 30 minutes to answer them.

2. In addition to this test booklet, you should have an answer sheet.

3. Do not be concerned that the answer sheet provides spaces for more answers than there are items in this test.

4. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.

5. On your answer sheet, find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

   Sample Item

   The mean of 5, 7, 9, 11 and 13 is

   (A) 5
   (B) 7
   (C) 8
   (D) 9

   Sample Answer

   Ⓐ Ⓑ Ⓒ ●

   The best answer to this item is “9 ”, so (D) has been shaded.

6. If you want to change your answer, erase it completely before you fill in your new choice.

7. When you are told to begin, turn the page and work as quickly and as carefully as you can. If you cannot answer an item, go on to the next one. You may return to that item later.

8. You may do any rough work in this booklet.

9. The use of silent, non-programmable scientific calculators is allowed.

   Examination Materials

   A list of mathematical formulae and tables. **(Revised 2019)**

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

1. Which of the following is an advantage of using secondary data in research?

(A) Reliability is guaranteed.
(B) Information is up to date.
(C) Data is controlled by the researcher.
(D) Information is specific to the purpose of the research.

2. Which of the following sets of data can be classified as discrete data?

(A) The weather condition yesterday
(B) The number of rainy days in the last month
(C) The weather prediction for tomorrow
(D) The amount of rainfall (in cm) recorded last month

3. In which of the following charts or diagrams will all the original data values be displayed?

(A) Pie chart
(B) Histogram
(C) Stem-and-leaf diagram
(D) Box-and-whisker diagram

4. Which of the following relationships is true for positively skewed distributions?

(A) mean > median > mode
(B) mean > mode > median
(C) median > mean > mode
(D) mode > median > mean

Item **5** refers to the following information

A small snack bar near our school was opened to students for 50 days during the summer program of 2019. The table below summarizes the daily number of students who visited the snack bar.

Number of students	22	23	24	25	26	27	28
Number of days	14	10	8	7	6	3	2

5. The mean number of students who visited the snack bar was

(A) 7.10
(B) 23.96
(C) 22
(D) 25

Items 6–8 refer to the following stem-and-leaf diagram which illustrates the heights, in cm, of a sample of 25 seedlings.

3	**4 6 7 8 9 9**
4	**0 2 2 3 4 6 8 9**
5	**0 1 3 5 8**
6	**2 4 5**
7	**4 6**
8	**1**

Key: 6 | 5 represents 6.5

6. The median height of the seedling is

(A) 4.8
(B) 8
(C) 13
(D) 48

7. The interquartile range for the set of data is

(A) 2.05
(B) 2.3
(C) 3.95
(D) 6.0

8. The shape of the distribution is

(A) neutral
(B) symmetrical
(C) positively skewed
(D) negatively skewed

Item **9** refers to the following information.

The following table shows the number of goals scored by the top scorers in 2021.

Lewandowski	Messi	Ronaldo	Silva
41	30	29	28

9. A pie chart showing the goals scored is drawn. The angle of the sector, to the nearest degree, representing Ronaldo's goals is

(A) 29
(B) 79
(C) 82
(D) 90

Items **10–11** refer to the table below.

Class	1–5	6–15	16–20	21–25
Frequency	7	24	5	2

10. The class boundaries of the 3rd class are

(A) 15.5, 20.5
(B) 15.5, 19.5
(C) 16, 20
(D) 16.5, 20.5

11. The class density of the interval 6–15 is

(A) 2.4
(B) 9
(C) 10
(D) 24

12. The CSEC Math grades (1–6) for the 189 students in the fifth form of a secondary school are collected and placed into a frequency table. The diagram that will BEST illustrate this information is a

(A) pie chart
(B) histogram
(C) stem-and-leaf
(D) cumulative frequency curve

Items **13 and 14** refer to the information given below:

A teacher wishes to find out whether boys in primary school play more video games than girls. A researcher recommends the following sampling design.

Arrange the students into two groups, boys and girls. Then randomly select 10 students from each group using a table of random numbers.

13. The sampling design used by the researcher is MOST likely

(A) quota sampling
(B) cluster sampling
(C) simple random sampling
(D) stratified random sampling

14. The MAIN advantage of the sample design recommended is that

(A) the population is divided into two groups
(B) an equal number of boys and girls were selected
(C) it is the most unbiased way of choosing a sample
(D) every member of the population has an equal chance of being selected

---

Item **15** refers to the following information.

In a questionnaire circulated to heads of households in a community, the sanitation authority asked

How do you rate the garbage collection service in your community?

☐ **Average** ☐ **Above average**

15. Which of the following can be identified as defects with this question?

I. Not enough options given
II. The range of options is not specific
III. Not all households will generate garbage

(A) I and II only
(B) I and III only
(C) II and III only
(D) I, II and III

16. Which of the following is TRUE for mutually exclusive events?

(A) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(B) $P(A \cap B) = P(A) \times P(B)$
(C) $P(A \cup B) = P(A) + P(B)$
(D) $P(A|B) = P(A)$

---

17. An unbiased die with faces marked 1, 2, 3, 4, 5, 6 is tossed 7 times and the number facing up is noted each time. A member of the possibility space is

(A) 6, 1, 4, 2, 6, 3, 6
(B) 1, 3, 5, 7, 5, 3, 1
(C) 3, 5, 4, 2, 7, 1, 1
(D) 1, 2, 3, 4, 5, 6, 7

18. For a standard normal distribution curve, what approximate percentage of the area under the curve is covered by $\mu \pm 2\sigma$?

(A) 90%
(B) 95%
(C) 97.5%
(D) 99%

19. Which of the following statements is NOT a condition for the binomial model of probability?

(A) There are only two possible outcomes for each trail.
(B) The probability of success is the same for every trail.
(C) There are an infinite number of trails for the experiment.
(D) The trails of the experiment are independent of each other.

20. The probability that it will rain on any day in the month of March is 0.3. What is the probability that it will rain on 4 days in a given week (7 days)?

(A) 0.9028
(B) 0.3000
(C) 0.2269
(D) 0.0972

21. Events C and D are independent events. $P(C) = 0.48$ and $P(D) = 0.45$. $P(C \cup D)$ is

(A) 0.216
(B) 0.714
(C) 0.784
(D) 0.930

Item **22** refers to the following formation.

The discrete random variable Q, where Q takes on only the values 2, 4, 6, and 8, has a cumulative distribution function, F(q), as shown in the table below.

q	2	4	6	8
F(q)	0.35	0.61	0.84	1

22. The value of $P(Q = 4)$ is

(A) 0.23
(B) 0.26
(C) 0.39
(D) 0.61

23. The continuous random variable, X, has a probability density function, *f*, given by

$$f(x) = \begin{cases} 0.25, & \textit{for } 0 \le x \le 4 \\ 0, & \textit{otherwise} \end{cases}$$

$P(X < \frac{1}{2})$ is

(A) 0.125
(B) 0.250
(C) 0.500
(D) 0.875

<u>Items **24** and **25**</u> refer to the following table which shows the distribution of the faculty enrolled in by a group of 150 students at a university.

	Social Sciences	**Science and Technology**	**Humanities**
Males	26	27	15
Females	48	11	23

24. The probability that a student chosen at random is a male and is enrolled in the faculty of Science and Technology is

(A) $\frac{27}{38}$

(B) $\frac{9}{50}$

(C) $\frac{34}{75}$

(D) $\frac{19}{75}$

25. The probability that a student selected at random is enroled in the faculty of Social Sciences given that the student is a female is

(A) $\frac{8}{25}$

(B) $\frac{24}{37}$

(C) $\frac{24}{41}$

(D) $\frac{37}{75}$

26. The time taken to run the 100m dash follows a normal distribution with mean 10.7 seconds and variance 0.36 seconds. The probability, to four decimal places, that the time of a randomly chosen runner is less than 9.8 second is

(A) 0.0062
(B) 0.0668
(C) 0.9332
(D) 0.9938

27. If $X \sim N(30.25)$ and $P\ (X > x) = 0.025$, then the value of x is

(A) 20.2
(B) 21.8
(C) 38.2
(D) 39.8

28. If $X \sim Bin(125, 0.45)$, the BEST distribution to calculate $P(X > 95)$ is

(A) $X \sim Bin(56.25, 30.94)$

(B) $X \sim N(56.25, 30.94)$

(C) $X \sim N(125, 0.45)$

(D) $X \sim Bin(125, 0.45)$

29. A random variable, X, has the following probability distribution.

x	0	2	3	5
$P(X = x)$	0.3	0.2	0.1	0.4

The value of $E(X)$ is

(A) 1
(B) 2.5
(C) 2.7
(D) 5

30. If $X \sim Bin(5, p)$ and $Var(X) = 0.8$, one of the possible values for p could be

(A) 0.16
(B) 0.20
(C) 0.60
(D) 0.84

Items **31 and 32** refer to the following contingency table which summarizes the responses of a random sample of 300 persons to a survey on preference for three dancehall artists.

Gender	**Artist A**	**Artist B**	**Artist C**	**Total**
Male	54	61	48	163
Female	38	72	27	137
Total	92	133	75	300

A χ^2 test is carried out to determine whether there is an association between the gender of respondents and the preference of artists.

31. The expected number of male respondents who prefer Artist C is

(A) 12
(B) 26.08
(C) 40.75
(D) 48

32. The number of degrees of freedom is

(A) 2
(B) 6
(C) 9
(D) 12

33. The null hypothesis for a X^2 test for independence is always

(A) H_0: There is a relationship between the variables

(B) H_1: There is a relationship between the variables

(C) H_0: There is no relationship between the variables

(D) H_1: There is no relationship between the variables

Item **34** refers to the following information.

A company has a fleet of similar cars of different ages. An examination of the company records shows that the cost of replacement parts for the older cars is generally greater than that for the newer cars. The cost of the parts, in \$, for the cars is given by the variable y and the age of the cars, in years, is given by the variable x. The summary statistics of the records for 9 cars are given as

$$\Sigma x = 68.8 \quad \Sigma y = 180 \quad \Sigma xy = 1960 \quad \Sigma x^2 = 753.9$$

A regression line is used to show the relation between the cost and age of the cars.

34. The value of the regression coefficient, b is

(A) –2.56
(B) 2.56
(C) 2.64
(D) 7.64

Item **35** refers to the following information

The probability of becoming infected with COVID-19, (y), is believed to be impacted by the amount of physical separation between an infected person and an uninfected one. This is also know as physical distancing, (x), measured in feet.

35. Using regression analysis in the form $y = a + bx$, researchers estimate that the regression line is $y = 0.69 - 0.140x$. Which of the following is the correct interpretation of the value of b?

(A) If there were no physical distancing, the probability of being infected is expected to be 0.69.
(B) If there were no physical distancing, the probability of being infected is expected to be 0.104.
(C) For every additional foot of physical distancing, the probability of being infected is expected to decrease by 0.69.
(D) For every additional foot of physical distancing, the probability of being infected is expected to decrease by 0.104.

36. Which of the following values for the product moment correlation coefficient, r, indicates the weakest degree of linear correlation between the two variables?

(A) $r = 0.38$
(B) $r = 0.45$
(C) $r = 0.76$
(D) $r = 0.91$

37. A type II error usually occurs in hypothesis testing when the null hypothesis is

(A) rejected when the null hypothesis is true
(B) not rejected when the null hypothesis is true
(C) rejected when the alternative hypothesis is true
(D) not rejected when the alternative hypothesis is true

38. Tacks produced by a machine have a mean length of 2.3 cm. A random sample of 20 tacks has a mean length of 2.4 cm with a standard deviation of 0.09 cm. If a hypothesis test is to be done at the 5% significance level, the test statistic is

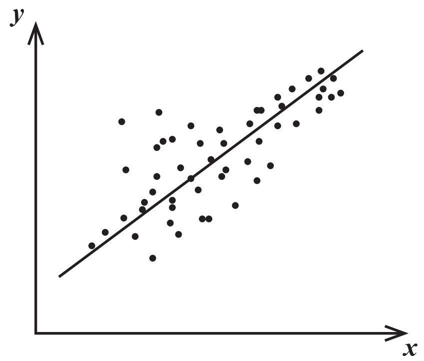
(A) 1.65
(B) 1.96
(C) 4.84
(D) 5.59

---

39. A coffee vending machine is adjusted so that, on average, it dispenses 150 ml of coffee with a standard deviation of 8.5 ml into a cup. The owner of the machine decides to check if it dispenses the correct amount. So, he takes a sample of 50 cups of coffee and performs a test. At the 5% level, what is an appropriate decision rule for this test?

(A) reject the null if the calculated $Z > 1.96$
(B) do not reject the null if the calculated Z | is < 1.96
(C) reject the null if the calculated $Z \neq 1.96$
(D) reject the null if the calculated $Z > |1.96|$

Item **40** refers to the following scatter diagram.



40. Which of the following BEST describes the above scatter diagram?

(A) There is a weak positive correlation between x and y.
(B) There is a weak negative correlation between x and y.
(C) There is a strong positive correlation between x and y.
(D) There is a strong negative correlation between x and y.

Item **41** refers to the following contingency table below which shows data collected to determine whether a person's favourite sport is dependent on gender.

GENDER	FOOTBALL	BASEBALL	BASKETBALL	GOLF	TOTAL
MALE	63	50	45	20	178
FEMALE	20	30	45	27	122
TOTAL	83	80	90	47	300

41. The expected frequency of the females who chose basketball as their favourite sport is

(A) 53.4
(B) 42.6
(C) 33.8
(D) 32.5

42. A random sample of 200 students is taken from a non-normal population with unknown variance. If the sample mean is $\bar{x}$, what is a 95% confidence interval for the population mean?

(A) $\bar{x} \pm 1.645\left(\frac{\hat{\sigma}}{\sqrt{n}}\right)$

(B) $\bar{x} \pm 1.645\left(\frac{\sigma}{\sqrt{n}}\right)$

(C) $\bar{x} \pm 1.96\left(\frac{\hat{\sigma}}{\sqrt{n}}\right)$

(D) $\bar{x} \pm 1.96\left(\frac{\sigma}{\sqrt{n}}\right)$

43. Which of the following is NOT true for a *t*-test?

(A) σ^2 is unknown
(B) n is small
(C) $v = n - 1$
(D) population is not normal

Item **44** refers to the following information

A test was conducted to determine whether a sample of 8 leaves with a mean length of 3.875 cm and a standard deviation of 1.45 cm came from a plant with leaves of mean 3.70 cm.

44. What is the value of the test statistic?

(A) 2645
(B) 0.3193
(C) 0.3414
(D) 0.3845

45. A *t*-test, at the 5% level of significance, was carried out on 15 volunteers to determine whether a new diet increased the speed of weight loss. What will be the critical region for this test?

(A) $t > 1.761$
(B) $t < -1.761$
(C) $t > 1.753$
(D) $t > 1.645$

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

Item	Specific Objective	Key		Item	Specific Objective	Key
1	1.1.1.1	A		26	1.2.4.6	B
2	1.1.1.2	C		27	1.2.4.5	D
3	1.1.2.3	C		28	1.2.4.8	B
4	1.1.2.8	A		29	1.2.2.4	C
5	1.1.2.5	B		30	1.2.3.3	B
6	1.1.2.5	A		31	1.3.4.4	C
7	1.1.2.7	A		32	1.3.4.2	A
8	1.1.2.8	C		33	1.3.4.1	C
9	1.1.2.4	B		34	1.3.5.5	B
10	1.1.2.2	A		35	1.3.5.6	D
11	1.1.2.2	A		36	1.3.5.4	A
12	1.1.2.3	A		37	1.3.2.3	C
13	1.1.1.6	A		38	1.3.2.6	D
14	1.1.1.6	B		39	1.3.2.8	D
15	1.1.1.8	C		40	1.3.5.3	C
16	1.2.1.5	C		41	1.3.4.3	B
17	1.2.1.9	A		42	1.3.1.6	C
18	1.2.4.1	B		43	1.3.3.1	D
19	1.2.5.1	C		44	1.3.3.2	B
20	1.2.3.1	D		45	1.3.3.5	A
21	1.2.1.5	B				
22	1.2.2.6	B				
23	1.2.2.7	A				
24	1.2.1.6	B				
25	1.2.1.6	C				

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

APPLIED MATHEMATICS

SPECIMEN

2022

TABLE OF SPECIFICATIONS

Paper 02

UNIT 1

Question	***Module***	***CK***	***AK***	***R***	***Total***
1	1	6	14	5	25
2	1	6	14	5	25
3	2	6	14	5	25
4	2	6	14	5	25
5	3	6	14	5	25
6	3	6	14	5	25
SUBTOTAL		**36**	**84**	**30**	**150**

TEST CODE **02105020**

SPECIMEN 2022

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 1 – Paper 02

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections. Each section consists of TWO questions.
2. Answer ALL questions.
3. Write your answers in the spaces provided in this booklet.
4. Do NOT write in the margins.
5. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.
6. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra lined page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
7. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials:

Mathematical formulae and tables **(Revised 2022)**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

“*”Barcode Area”*”
Sequential Bar Code

SECTION A

MODULE 1: COLLECTING AND DESCRIBING DATA

Answer BOTH questions.

1. (a) Differentiate between the terms 'parameter' and 'statistic'.

....

....

....

....

[2 marks]

(b) A principal wants to select 16 students from among 464 students to represent the school at an event. The following sampling methods labelled A, B, C, and D are suggested as possible ways of selecting the sample of 16 students.

W: Select equal numbers of male and female students even though there are more female students than male students.

X: Group the students into year groups and select students at random from within each group in proportion to the number of students that make up that group.

Y: Assign a unique number from 001 to 464 to each student and use a random number table to select the students.

Z: First select a student randomly from among the first *r* students on the nominal roll, thereafter, selecting every *r*th student on the roll.

(i) Identify the term which BEST describes the sampling methods A–D.

W ....

X ....

Y ....

Z ....

[4 marks]

(ii) Identify (by letter) the sampling method which would result in a sample that is MOST representative of the 464 students. Give TWO reasons for your choice.

....

....

....

....

[3 marks]

(iii) Out of the 16 students selected, there would be 4 students from the fourth form if sampling method X were used. Calculate how many of the 464 students are in fourth form.

[3 marks]

(iv) Using the Random Sampling Numbers table provided in the List of Formulae and Statistical Tables, start at the third row from the top with 720 and work across the row left to right. Select the **first** and **fourth** of the 16 students that would be selected according to sampling method Y.

....

....

[2 marks]

(c) 30 students were asked to estimate what their score would be on a recently completed test out of 60. The data is represented in the table below.

32	12	39	54	29	27	44	41	31	33
42	23	25	29	37	18	29	25	31	51
29	36	20	41	36	37	26	50	29	31

(i) Construct a stem-and-leaf diagram to illustrate the data.

[5 marks]

(ii) Comment on the shape of the distribution. Give ONE reason to support your comment.

....

....

....

[2 marks]

(iii) Calculate the 10% trimmed mean for this set of data.

[4 marks]

Total 25 marks

2. (a) The following table shows the distances travelled by some sixth form students to school on a given morning. Use the table to answer the following questions.

Distance Travelled (km)	**Frequency**
0–4	9
5–9	14
10–14	18
15–19	12
20–24	5
25–29	2

(i) State the upper boundary of the third class.

....

[1 mark]

(ii) Identify the modal class.

....

[1 mark]

(iii) Calculate the frequency density of the fourth class.

[2 marks]

GO ON TO THE NEXT PAGE

“*”Barcode Area”*”
Sequential Bar Code

(iv) Estimate the mean distance travelled.

[5 marks]

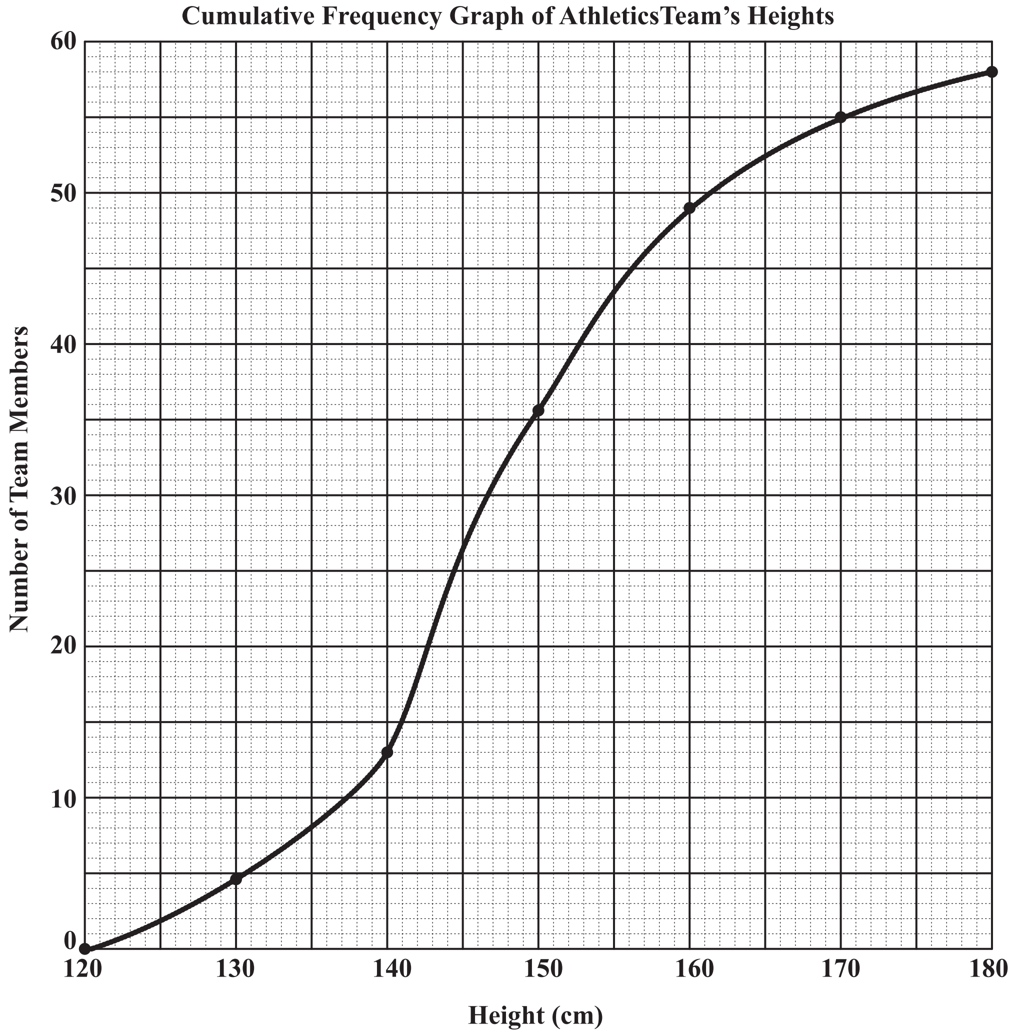
(v) Estimate the standard deviation of the distance travelled.

[5 marks]

(vi) Estimate the number of students who travelled more than 17 km.

[2 marks]

(b) The following cumulative frequency curve shows the heights of the members of the athletics team at a particular school.



GO ON TO THE NEXT PAGE

Use cumulative frequency graph, **provided on page 8**, to answer the following.

(i) Estimate the median height of the athletes.

....

....

[2 marks]

(ii) Calculate the interquartile range.

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....

....

....

[3 marks]

(iii) Estimate the number of athletes who are taller than 166 cm.

....

....

....

[2 marks]

(iv) Construct a box-and-whisker plot on the cumulative frequency graph given **on page 8** which BEST represents the data. **[2 marks]**

Total 25 marks

SECTION B

MODULE 2: MANAGING UNCERTAINTY

Answer BOTH questions.

3. (a) Sharks FC and Hurricanes FC, the two leading teams in the present football season, each have one game to play (not against each other).

If Sharks FC wins and Hurricanes FC draws or loses, Shark FC wins the championships.

If Sharks FC draws and Hurricanes FC loses, then Sharks FC wins the championship.

The following table shows the probabilities for each team winning, losing, or drawing their last game of the season.

	Win	**Draw**	**Lose**
Sharks FC	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$
Hurricanes FC	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

(i) Complete the table below to show nine possible outcomes for the last games for the **two** teams.

		Sharks FC		
		Win	**Draw**	**Lose**
Hurricanes FC	Win	$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$		
	Draw			$\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$
	Lose		$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$	

[6 marks]

GO ON TO THE NEXT PAGE

(ii) Determine the probability that Sharks FC wins the championship.

[4 marks]

(iii) Calculate the probability that Hurricanes FC draws given that Sharks wins the championships.

[4 marks]

GO ON TO THE NEXT PAGE

(b) The probability distribution of a discrete random variable, Y, is given in the following table.

y	0	1	2	3
$P(Y = y)$	$\frac{1}{3}$	p	q	$\frac{1}{4}$

The expectation is given as $E(X) = 1\frac{1}{4}$.

(i) Calculate the value of p and q.

[6 marks]

GO ON TO THE NEXT PAGE

(ii) Calculate the standard deviation of X.

[5 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

4. (a) At a regional hospital, five patients are tested for dengue fever. Doctors believe that the probability of a positive test is 0.3.

(i) List FOUR reasons why this situation may be modelled as a binomial distribution.

....

....

....

....

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....

....

....

[4 marks]

(ii) State fully the parameters of the binomial distribution.

....

....

....

[2 marks]

(b) Determine the probability that

(i) there is exactly one positive test

[4 marks]

(ii) there are less than two positive tests.

....

....

....

....

....

....

[4 marks]

(c) Calculate the expected number of positive tests.

....

....

....

[2 marks]

(d) Calculate the variance of the number of positive tests.

....

....

....

[2 marks]

(e) Calculate the probability that out of 100 tests, twenty-five or less will result positive. In your response, give TWO reasons for the use of the distribution selected.

[7 marks]

Total 25 marks

DO NOT WRITE IN THIS AREA
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SECTION C

MODULE 3: ANALYSING AND INTERPRETING DATA

Answer BOTH questions.

5. (a) The masses of mangoes in a box, can be modelled by a normal distribution with a mean of 62.2 g and a standard deviation of 3.6 g.

(i) Write a distribution for the mean mass, $\overline{X}$, of a sample of 20 mangoes.

[2 marks]

(ii) Calculate the probability that the mean mass of a random sample of 20 mangoes is less than 60 g.

[4 marks]

(iii) Calculate the probability that the mean mass of a random sample of 20 mangoes is between 63 g and 64 g.

[4 marks]

(iv) A random sample of mangoes is taken from the box. Using a 95 percent confidence interval for the mean, determine the sample size so that the sample mean will differ from the true population mean by less than 1.0 g.

[5 marks]

(v) A 90% confidence interval for the population mean, μ, is found for each sample when 60 random samples of size 20 are taken. Determine the expected number of intervals that do NOT contain μ.

[2 marks]

(b) The mean of 50 observations of a random variable X, where $X \sim \text{Bin}\left(10, \frac{1}{6}\right)$ is $\overline{X}$.

(i) Determine the distribution modelled by $\overline{X}$, stating clearly its parameters.

[3 marks]

(ii) Calculate $P(\overline{X} > 1.9)$

[4 marks]

(iii) State the percentage of the sample that will have a mean greater than 1.9.

....

[1 mark]

Total 25 marks

GO ON TO THE NEXT PAGE

6. (a) Some fruit trees are suffering from a certain disease that causes them to wither. An agronomist wishes to investigate whether there is an association between the type of treatment for the disease and the length of survival of the affected trees.

Treatment A involved no action being taken.

Treatment B involved careful removal of the diseased branches.

Treatment C involved the careful removal of the diseased branches and then the frequent spraying of the remaining branches, with an antibiotic.

The following table shows the results of the treatment given to 200 trees.

Survival	**Treatment**		
	A	**B**	**C**
< 3 months	24	32	32
≥ 3 months	30	30	52

Use a chi-squared test at the 5% level of significance to determine whether these data provide evidence of an association between the type of treatment and the survival of the trees.

(i) State the null and alternative hypotheses.

....

....

....

....

[2 marks]

(ii) Complete the table below to show the expected frequencies.

Survival	**Treatment**		
	A	**B**	**C**
< 3 months	23.8	27.3	
≥ 3 months			47.1

[3 marks]

GO ON TO THE NEXT PAGE

(iii) For this test, state the degrees of freedom and the critical value of χ^2.

....

....

....

[2 marks]

(iv) Complete the following table to determine the χ^2 test statistic, correct to three decimal places.

Observed (O)	Expected (E)	χ^2
24	23.8	0.00168
30	30.2	
32	27.3	
30	34.7	0.63660
32	36.9	
52	47.1	0.50974

[5 marks]

(v) State a valid conclusion for the test. Justify your response.

....

....

....

[2 marks]

(b) An Internet service provider runs a series of television advertisements at weekly intervals. A study was conducted to investigate the effectiveness of the advertisements. The company records the viewing figures in millions, v, for the programme in which the advertisement is shown, and the number of new customers, c, who sign up for its service the next day. The results are summarized as follows:

$$\overline{v} = 4.92, \quad \overline{c} = 104.4, \quad S_{vc} = 594.05, \quad S_{vv} = 85.44.$$

(i) From the situation given, identify the independent variable.

....

[1 mark]

(ii) Determine the equation of the regression line, c on v, in the form $c = a + bv$.

[5 marks]

GO ON TO THE NEXT PAGE

(iii) Give an interpretation of the constants a and b in relation to the effectiveness of the advertisements.

....

....

....

[2 marks]

(iv) Estimate the number of customers who will sign up with the company the day after an advertisement is shown during a programme watched by 3.7 million viewers.

[3 marks]

Total 25 marks

END OF TEST

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 1 – Paper 02

KEY AND MARK SCHEME

SPECIMEN 2022

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

SECTION A

MODULE 1: COLLECTING AND DESCRIBING DATA

Question 1.

(a) A parameter is a fixed measure or characteristic of the population **(1)** while a statistic is a characteristic of a sample **(1)**.
[2 marks] [CK]

(b) (i) W – Quota sampling **(1)**
X - Stratified random sampling **(1)**
Y - Simple random sampling **(1)**
Z - Systematic random sampling **(1)** **[4 marks] [CK]**

(ii)
- Sampling method II/Stratified Random Sampling **(1)**.
- The sampling method guarantees proportional representation equal to that of the population **(1)**.
- The sampling method is random thus eliminating bias **(1)**.

[3 marks] [R]

(iii) Let x be the number of 4^{th} form students

$\frac{x}{464} \times 16 = 4$ **(1)**

$x = \frac{4}{16} \times 464$ **(1)**

$x = 116$ students **(1)** **[3 marks] [AK]**

(iv) 211, 442, 282, 398 **[2 marks] [AK]**

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 1. (continued)

(c) (i)

stem	leaf
1	2 8
2	0 3 5 5 6 7 9 9 9 9 9
3	1 1 2 3 6 6 7 7 7
4	1 1 1 2 4
5	0 1 4

Key: 3|6 = 36

Correct stem **(1)**
Key **(1)**
All rows correct **(3)**
3 - 4 rows correct **(2)**
At least 2 rows correct **(1)** **[5 marks] [AK]**

(ii) Distribution is positively skewed **(1)**.
(Right tailed is NOT accepted for the shape of the distribution)

The mode is smaller than median / most of the data lies to the left of the data / the data is right tailed/skewed towards the right **(1)**.

1 mark for comment on shape
1 mark for any ONE reason **[2 marks] [R]**

(iii) 10% from both ends of data (SOI) **(1 mark each)**

23 - 44

1/24(23 + 2(25) + 26+ 27 +5(29) + 2(31) +
32 + 33 + 2(36) + 2(37) + 39 +3(41) +42 + 44)

$\frac{\sum x}{n}$ SOI

1/24 (792) **(1)** **(dividing his sum by 24)**

= 33 **(1)** **CORRECT ANSWER ONLY** **[4 marks][AK]**

Total 25 marks

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 2.

(a) (i) 14.5 **[1 mark] [CK]**

(ii) 10-14 **[1 mark] [CK]**

(iii) $\frac{12}{5} = 2.4$

division by 5 (1)

correct answer (1)

[2 marks] [AK]

(iv)

Distance Travelled (km)	**Frequency (f)**	**Midpoints (x)**	fx
0 - 4	9	2	18
5 - 9	14	7	98
10 - 14	18	12	216
15 - 19	12	17	204
20 - 24	5	22	110
25 - 29	2	27	54
Totals	60		700

correct fx column (1 mark) [AK]
correct midpoints (1 mark) [AK]
correct total fx column [CK]

$$mean = \frac{\sum fx}{\sum f} = \frac{700}{60} = 11.67\ km$$

correct denominator (1) [R]
candidate's correct answer (1) [AK]

[5 marks]

Question 2. (continued)

(a) (v)

Distance Travelled (km)	**Frequency (f)**	**Midpoints (x)**	**fx**	**fx^2**
0 - 4	9	2	18	36
5 - 9	14	7	98	386
10 - 14	18	12	216	2592
15 - 19	12	17	204	3468
20 - 24	5	22	110	2420
25 - 29	2	27	54	1458
Totals	60		700	10360

1 mark for correct multiplication [AK]
1 mark for correct fx^2 total [CK]

$$Variance = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2$$

$= \frac{10360}{60} - (11.67)^2$ **correct substitution into formulae (1) [AK]**

$= 36.478$

$Std.Dev = \sqrt{Var} = \sqrt{36.478} = 6.04$

1 mark for correct $\sqrt{var}$ [CK]
1 mark for correct answer [AK]

Alternate method also accepted

[5 marks] [AK]

(vi) $6 + 5 + 2 = 13\ students$

1 mark for 6 (1)
addition of subsequent values (1)
[2 marks] [R]

Question 2. (continued)

(b) (i) 147 cm

1 mark for choosing the 29th value (1) [R]
1 mark for correct corresponding height (1) [AK]
[2 marks]

(ii) Q1= 141cm **(1)** (tolerance $\pm 1cm$) **[AK]**

Q3= 156cm **(1)** (tolerance $\pm 1cm$) **[AK]**

I.Q.R = Q3 – Q1 = 156 – 141 = 15 cm **subtraction of values (1) [CK]**

[3 marks]

(iii) 58 – 53 = 5 students

1 mark for 53 (1) [AK]
1 mark for subtraction from 58 (1) [R]
[2 marks]

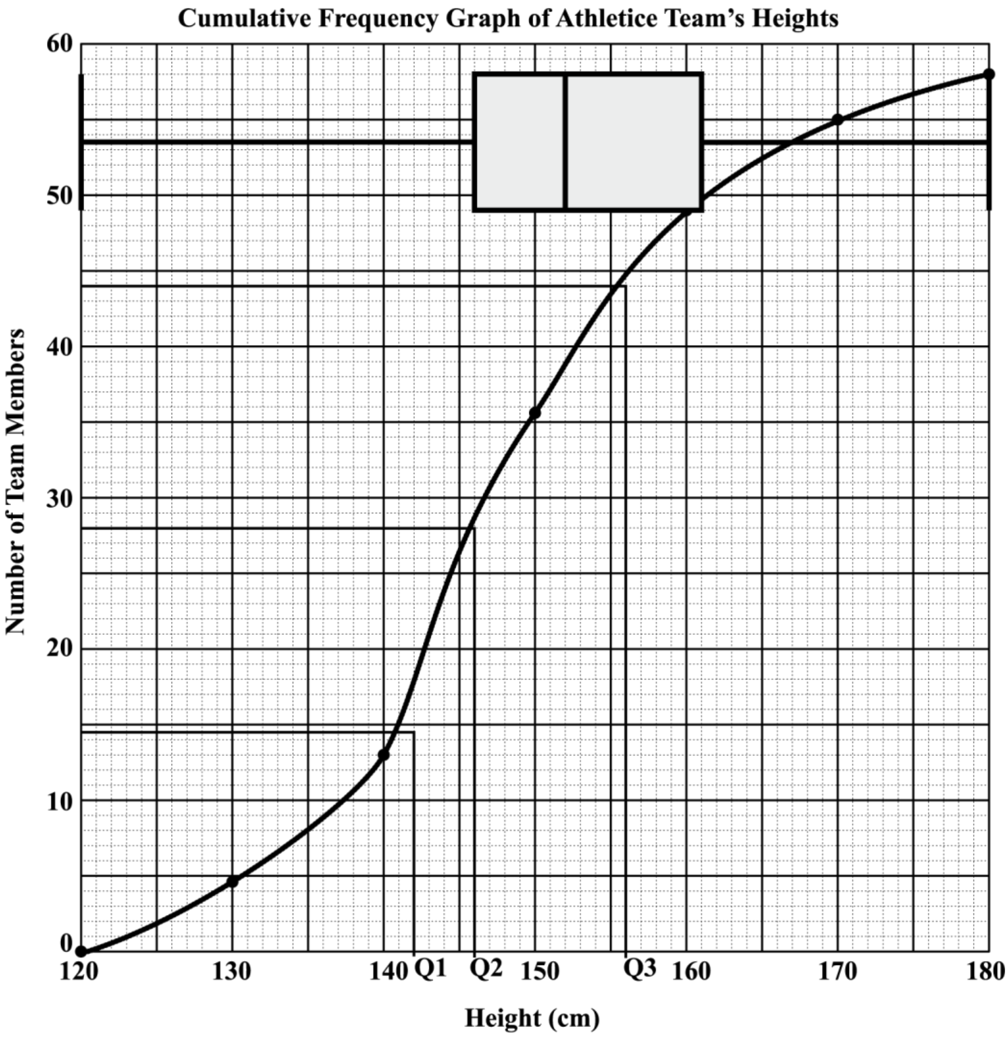
APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 2. (continued)

(b) (iv)



1 mark for whiskers (1 AK)
1 mark for box (1 AK) **[2 marks]**

Total 25 marks

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

SECTION B

MODULE 2: MANAGING UNCERTAINTY

Question 3.

(a) (i)

		Sharks FC		
		Win	*Draw*	*Lose*
Hurricanes FC	*Win*	$\frac{1}{2}\times\frac{2}{3}=\frac{2}{6}$	$\mathbf{\frac{1}{6}\times\frac{2}{3}=\frac{2}{18}}$	$\mathbf{\frac{1}{3}\times\frac{2}{3}=\frac{2}{9}}$
	Draw	$\mathbf{\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}}$	$\mathbf{\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}}$	$\frac{1}{3}\times\frac{1}{6}=\frac{1}{18}$
	Lose	$\mathbf{\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}}$	$\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$	$\mathbf{\frac{1}{3}\times\frac{1}{6}=\frac{1}{18}}$

1 for each correct probability AK
[6 marks]

(ii) P(Sharks wins championship)

$= P(SW \times HW') + P(SD \times HL)$ **(1) mark for correct interpretation [R]**

$P(HW') = 1-\frac{2}{3}=\frac{1}{6}+\frac{1}{6}=\frac{2}{6}=\frac{1}{3}$

P(Sharks wins championship)

$=\left(\frac{1}{2}\times\frac{1}{3}\right)+\left(\frac{1}{6}\times\frac{1}{6}\right)$ **1 mark each for correct partial product [AK]**

$=\frac{7}{36}$ **1 mark for correct answer only[AK]**

[4 marks]

Alternate Response

or from table $\mathbf{\frac{1}{12}+\frac{1}{12}+\frac{1}{36}=\frac{7}{36}}$

Any two correct probabilities from the table, 1 mark each [AK]
Addition of their probabilities 1 mark [R]
1 mark for correct answer [AK]

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 3. (continued)

(a) (iii) $\frac{P(HD \cap SW)}{P(SWC)}$ **(1) conditional formula stated correctly [R]**

$= \frac{\left(\frac{1}{12}\right)}{\frac{7}{36}}$ **1 mark for correct numerator [CK]**

1 mark for candidates' correct denominator [CK]

$= \frac{3}{7}$ **1 mark Correct Answer only [AK]**

[4 marks]

(b) (i) $\sum P(Y = y) = 1$ **(1) mark for correct formula [CK]**

$\frac{1}{3} + p + q + \frac{1}{4} = 1$ **(1) mark for equation {1} [R]**

$p + q = \frac{5}{12} \dots \{1\}$

$E(Y) = \sum yP(Y = y)$ **(1) mark for expectation formula CK**

$\frac{5}{4} = 0 + p + 2q + \frac{3}{4}$

$\frac{1}{2} = p + 2q \dots \{2\}$ **(1) mark for equation {2} [R]**

$working\ with\ \{2\},$ **(1) mark for solving simultaneously [R]**

$p = \frac{1}{2} - 2q \dots \{3\}$

$subtituting\ \{3\} in\ \{1\}$

$\left(\frac{1}{2} - 2q\right) + q = \frac{5}{12}$

$q = \frac{1}{12}$

$subtitute\ q = \frac{1}{12}\ in\ \{3\}$

$p = \frac{1}{3}$ **(1) mark for correct *p* [AK]**

[6 marks]

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 3. (continued)

(b) (ii) $Var(Y) = E(Y^2) - [E(Y)]^2$

$E(Y^2) = \sum y^2 P(Y = y) = 0 + \frac{1}{3} + \frac{1}{6} + \frac{9}{4} = \frac{33}{12}$

$Var(Y) = \frac{33}{12} - \left(\frac{5}{4}\right)^2 = \frac{57}{48}$

Standard Deviation (Y) = $\sqrt{\frac{57}{48}} = 1.09 \quad (3\,s.f.)$

Correct formula - 1 mark [CK]
Calculating $E(Y^2)$ - 1 mark [AK]
Correct substitution into variance formula – 1 mark [AK]
Square root of candidates' variance - 1 mark [CK]
Correct answer only 1 mark [AK]

[5 marks]

Total 25 marks

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 4.

(a) (i)
- The test can have one of two possible results - either positive or negative.
- The trials are independent, as the results of one test does not affect the results of another patient.
- Each trial has the same probability.
- There is a finite number of trials.

1 mark for each point stated **[4 marks] [R]**

(ii) $X \sim Bin(5{,}0.3)$

1 mark each for stating both parameters **[2 marks][CK]**

(b) (i) $P(X = 1) = \binom{5}{1}(0.3)^1(0.7)^4 = 0.36015$

3 marks for all values correctly substituted in formula [AK]
1 for correct answer only [AK]
[4 marks]

(ii) $P(X < 2)$

$= P(X = 0) + P(X = 1)$

1 mark for *X* = 0 + X = 1

$=\binom{5}{0}(0.3)^0(0.7)^5 + 0.36015$

1 mark for correctly calculating P (*x* = 0)$=$

$0.16807 + 0.36015$ **1 mark for summation of "candidates" values (SOI)**

$= 0.52822$ **1 mark for "candidate's" answer**

1 use of summation [R]
1 mark for *x* = 0 [AK]
1 mark for *x* < 2 [AK]
1 for correct answer only [AK]
[4 marks]

Question 4. (continued)

(c) $E(X) = np = 5(0.3) = 1.5$

1 correct substitution into formulae
1 correct answer only

[2 marks] [AK]

(d) $Var(X) = npq = 5(0.3)(0.7) = 1.05$

1 correct substitution into formulae1
1 correct answer only

[2 marks] [AK]

(e) $X \sim Bin(100,0.3)$

The value of n (100) is large.

The value of $np > 5$ (np=30)

$$X \sim N(30,21)$$

$$P(X \leqslant 25) = P\left(Z \leqslant \frac{25.5 - 30}{\sqrt{21}}\right) = P(z \leqslant -0.982)$$

$1 - \phi(0.982)$

$1 - 0.8370 = 0.163$

2 marks 1 each for stating both parameters [CK]
1 mark for application of continuity correction [CK]
1 mark for correct standardization [AK]
1 mark for correct use of phi [AK]
1 mark for correct table values [CK]
1 mark for correct answer only [AK] **[7 marks]**

Total 25 marks

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

SECTION C

MODULE 3: ANALYZING AND INTERPRETING DATA

Question 5.

(a) (i) $\bar{X} \sim N\left(62.2, \frac{12.96}{20}\right)$

12.96 or 3.6^2 **(1)**

N **(1)**

1 mark for the sigma squared/*n*
1 mark for it is N normal

[2 marks] [AK]

(ii) Less than 60 g

$P(\bar{X} < 60) = P\left(Z < \frac{60-62.2}{3.6/\sqrt{20}}\right)$ **Attempt to standardize (1) [CK]**

$= P(Z < -2.733)$ **Correct standardization (1) [AK]**

$= \phi(-2.733)$

= 1 - 0.9968 **use of ϕ for standardized value (1) [CK]**

= 0.0032 **Correct answer (1) [AK]**

[4 marks]

(iii) Between 63 g and 64 g

$P(63 < \bar{X} < 64) = P\left(\frac{63-62{,}2}{3.6/\sqrt{20}} < Z < \frac{64-62.2}{3.6/\sqrt{20}}\right)$

Stating that $\bar{X}$ is between 63 and 64 (1) [R]
Attempt to standardize (1) [CK]

$= P(0.994 < Z < 2.236)$ **correct standardization (1) [AK]**

$= \phi(2.236) - \phi(0.994)$ **use of ϕ for "his" standardized value (1) [CK]**

= 0.9873 - 0.8399

= 0.1474 **correct answer (1) [AK]**

[4 marks]

Question 5. (continued)

(a) (iv) For a 95% CI $\quad -1.96 < \frac{\bar{X} - \mu}{3.6/\sqrt{n}} < 1.96$ **SOI (1) [R]**

$-1.96 < \frac{(\bar{X} - \mu)\sqrt{n}}{3.6} < 1.96$

$-1.96 \times 3.6 < \sqrt{n} < 1.96 \times 3.6$

using $\bar{X} - \mu = 1$ **1 mark [AK]**

simplification **1 mark [AK]**

$\sqrt{n} < 1.96 \times 3.6$ **1 mark for correct substitution [AK]**

$n < 49.8$

$n = 49$ **1 mark for correct answer [AK]**

[5 marks]

(v) 60 – 90% of 60 **SOI (1) [R]**

60 – 54

6 intervals do NOT contain μ **(1) [AK]**

[2 marks]

Question 5. (continued)

(b) (i) np = $\frac{10}{6} = 1.667$

1 mark for calculating the expected value (mean) [AK]

npq = $\frac{10}{6} \times \frac{5}{6}$ = 1.389 **1 mark for calculating the variance [AK]**

$\bar{X} \sim N\left(1.667, \frac{1.389}{50}\right)$ **1 mark for stating the distribution [R]**

$\bar{X} \sim N(1.667, 0.028)$ **[3 marks]**

(ii) = P($\bar{X}$ > 1.95) **1 mark for using continuity correction [R]**

= $P\left(Z > \frac{1.95-1.667}{\sqrt{0.028}}\right)$ **1 mark for attempt to standardize (CK)**

= P(Z >1.691) 1 mark for c**orrect standardization [AK]**

= $1 - \phi(1.691)$ **Use of ϕ (1) [CK]**

= 1 - 0.9546

= 0.0454 **1 mark for correct answer (1) [AK]**

[4 marks]

(iii) 0.0454 = 4.54 %

1 mark for changing values to percentage **[1 mark]**

TOTAL 25 marks

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 6.

(a) (i) H_0: There is no association between type of treatment and the survival of the trees. **(1)**

H_1: Type of treatment and the survival of trees are associated. **(1)**

[2 marks]; AK

(ii)

	Treatment		
Survival	**A**	**B**	**C**
< 3 months	**23.8**	**27.3**	**36.9**
≥ 3 months	**30.2**	**34.7**	**47.1**

1 mark for each correct value
[3 marks] [AK]

(iii) 2 degrees of freedom **correct answer (1) [AK]**

Critical value = 7.378 **correct reading from the tables (1) [CK]**

[2 marks]

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 6. (continued)

(a) (iv)

Observed (O)	Expected (E)	$\chi^2 = (O - E)^2/E$
24	23.8	0.00168
30	30.2	**0.00132**
32	27.3	**0.80916**
30	34.7	0.63660
32	36.9	**0.65068**
52	47.1	0.50977
		2.609

$\chi^2 = \frac{(O-E)^2}{E}$ **SOI (1) [CK]**
1 mark each for correct calculations of χ^2 (3) [AK]
1 mark for test statistic (1) [AK] **[5 marks]**

(v) Since the test statistic is less than the critical value of χ^2 **(1)** accept H_0 and conclude there is no association between the type of treatment and the survival time of the fruit trees **(1)**.
[2 marks] [R]

(b) (i) Independent- v (the number of viewers) **[1 mark] [R]**

(ii) $b = \frac{S_{vc}}{S_{vv}}$ **1 mark SOI [CK]**

$b = \frac{594.05}{85.44} = 6.953$ **1 mark [AK]**

$a = \bar{c} - b\bar{v}$ **1 mark SOI [CK]**

$= 104.4 - 6.953 \times 4.92$

$= 70.192$ **1 mark for correct answer [AK]**

$a = 70.2 + 6.95v$ **1 mark for substituting values into c [AK]**
[5 marks]

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 6. (continued)

(b) (iii) a is the number of sign-ups without advertising **(1)**

b is the of extra sign-ups per million viewers of advert **(1)**

[2 marks] [R]

NB: the gradient of slope is not accepted

(iv) $c = 70.192 + (6.953 \times 3.7)$ **1 mark for multiplying *b* by 3.7 [CK]**

$= 95.92$ **1 mark for adding c to *a* [CK]**

$= 96.0$ million 1 mark for **correct answer [AK]**

[3 marks]

TOTAL 25 marks

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

APPLIED MATHEMATICS

SPECIMEN

2022

TABLE OF SPECIFICATIONS

Paper 032

UNIT 1

Question	***Module***	***CK***	***AK***	***R***	***Total***
1	1-3	-	-	18	18
2	1-3	-	30	-	30
3	1-3	-	-	12	12
SUBTOTAL		**-**	**30**	**30**	**60**

TEST CODE **02105032-CASE**

SPECIMEN 2022

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 1 – Paper 032

ALTERNATIVE TO SCHOOL BASED ASSESSMENT

CASE STUDY FOR THE ALTERNATIVE TO SCHOOOL-BASED ASSESSEMENT EXAMINATION

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. The Paper 032 paper will consist of THREE questions based on your analysis of the given case study.
2. Examine the case study carefully to prepare for your examination.

N.B. **Candidates are to receive this paper ONE week in advance of the date of the examination.**

CASE STUDY

A small production company employs 40 machine operators, working an eight-hour day, whose main function is to assemble components to be used in a further manufacturing process. Machines are placed into groups which are set to operate at different assembly speeds as shown in the following table.

Group	**A**	**B**	**C**	**D**	**E**
Assembly speed (components per minute)	6–8	9–11	12–14	15–17	18–20
Number of Components	15	15	40	20	10

Components from each group are checked at intervals and defective components are taken off the line. The owner of the company is of the opinion that the assembly speed, x components per minute, influences the number of defective components, y, found during inspection. He is also concerned about the uncertainty of the number of defective components that he should expect from each machine. As quality control manager, you are asked to investigate the uncertainty of the number of defective components produced and his assessment on whether the assembly speed of his machines influences the number of defective components produced.

SPECIMEN 2022

TEST CODE **02105032**

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 1 – Paper 032

ALTERNATIVE TO SCHOOL-BASED ASSESSMENT

1 hour 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of a case study and THREE questions. Answer ALL questions.
2. Write your answers in the spaces provided in this booklet.
3. Do NOT write in the margins.
4. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.
5. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra lined page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
6. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials

Mathematical formulae and tables **(Revised 2022)**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

"*"Barcode Area"*"
Sequential Bar Code

INSTRUCTION: Read the case and answer the questions that follow.

CASE STUDY

A small production company employs 40 machine operators, working an eight-hour day, whose main function is to assemble components to be used in a further manufacturing process. Machines are placed into groups which are set to operate at different assembly speeds as shown in the following table.

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1. (a) Explain why a chi-squared analysis is suitable for this investigation.

....

....

....

....

....

....

[3 marks]

(b) Suggest a suitable model for testing the probability of getting a given number of defective components from a sample of components produced.

....

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....

....

[3 marks]

(c) State the independent variable and the dependent variable.

....

....

....

[2 marks]

(d) Explain why it is important to use a sample for your data collection.

....

....

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....

....

[3 marks]

(e) Explain why a stratified random sample is BEST for the situation described.

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[3 marks]

(f) Describe fully how stratified random sampling will be used to select the sample for this investigation.

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[4 marks]

Total 18 marks

2. (a) Over a four-hour period, twenty machines sampled had the following number of defective components: 9, 9, 12, 13, 14, 13, 8, 17, 12, 18, 9, 13, 17, 15, 8, 12, 16, 12, 7, 19.

(i) Construct a frequency table using the data given on defective components. Classify the data into the groups in the case, A (6–8), B (9–11), C (12–14), D (15–17) and E (18–20).

[5 marks]

(ii) Construct a suitable chart or diagram to show this data.

[5 marks]

(b) For an assembly speed of 12 components per minute, it is believed that the probability of having a defective component is 0.2. Use an appropriate distribution to calculate the probability that in a sample of 15 components from this assembly speed, there will be at least 2 defective components.

[10 marks]

(c) The following data were collected after the inspection of the five groups over a four-hour period.

Number of Defective Components Found		**2**	**3**	**4**
Group	**A**	14	20	16
	B	12	15	23
	C	15	16	19
	D	27	12	11
	E	18	23	9

Use the chi squared test, at the 5% level, to analyze the data presented.

(i) State the null and alternative hypothesis

[2 marks]

(ii) Sketch a graph to display the rejection region

[3 marks]

GO ON TO THE NEXT PAGE

(iii) Calculate the chi-squared value.

[5 marks]

Total 30 marks

3. (a) State THREE key findings from the investigation. Support each finding with the results of your investigation.

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[3 marks]

(b) Outline ONE conclusion that can be made from the investigation.

....

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....

[2 marks]

(c) Outline ONE limitation of the investigation.

....

....

....

[2 marks]

(d) Outline ONE recommendation to improve the investigation.

....

....

....

[2 marks]

Total 12 marks

END OF TEST

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 1 – Paper 02

KEY AND MARK SCHEME

SPECIMEN 2022

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 1.

(a) Machines at this company operate at different assembly speeds and the machines also produce a number of defective components **(1).** The owner of this company is concerned that higher assembly speeds result in more defective components produced **(1).** The chi squared test will be used to determine whether there is a relationship or association between the variables **(1).** **[3 marks]**

(b) Since the owner is uncertain that the speed of the machine is responsible for the number of defective components produced **(1)** it will be necessity to conduct probability tests. The Binomial distribution will be used for this **(1)** since components are examined to see whether or not they are defective **(1).** **[3 marks]**

(c) **Independent variable** - the assembly speed **(1)**
Dependent variable - the number of defective components **(1)**

[2 marks]

(d) A sample will be necessary to reduce the time that the investigation will take **(1).** It will be a difficult task to check every machine **(1).** A sample will reduce the cost of the investigation **(1).** **[3 marks]**

(e) There are five groups in the population **(1).** A stratified random sample involves selecting your sample in proportion to strata sizes **(1).** This will ensure that samples are randomly selected from each group of components **(1).** **[3 marks]**

(f) A manufacturing day has 8 hours and there are 40 machines, operating at five different assembly speeds. Five groups of 8 machines will be created based on assembly speed.

Well defined strata - 1 mark
Sample Strata sizes - 1 mark
Sampling frame indicated – 1 mark
Selecting sample using a random method – 1 mark

[4 marks]

Total 18 marks [R]

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 2.

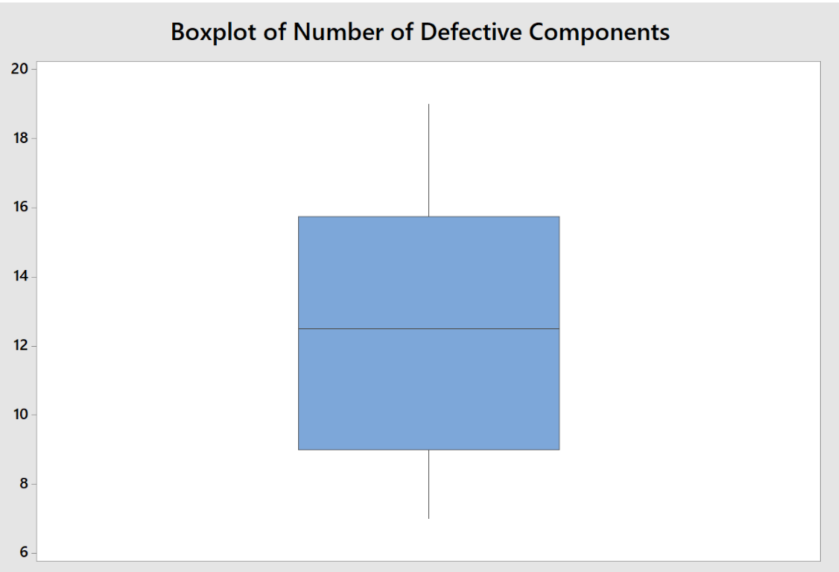
(a) (i)

Group	A (6-8)	B (9-11	C (12-14)	D (15-17)	E (18-20)
Frequency	3	3	8	4	2

Table is clearly written (unambiguous and systematic) (1 mark)
Appropriate headers (columns and rows) (1 mark)
Correct Frequencies (3 marks)
Award 3 marks for at least 3 correct
Award 2 marks for 2 correct
Award 1 marks for 1 correct

[5 marks]

(ii) **Award marks as follows**



Calculating $Q_1 = 9$ 1 mark
Calculating $Q_2 = 12.5$ 1mark
Calculating $Q_3 = 15.5$ 1 mark
Constructing the box and whisker plot 1 mark
Appropriate scale used 1 mark

[5 marks]

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

(b) Use a Binomial distribution with n = 15 and p = 0.2 **(3 marks)**

Let X be the number of defective components **(1 mark)**

Then $P(X \geq 2) = 1 - [P(X = 0) + P(X = 1)]$ = Correct $P(X \geq 2)$ **(2 marks)**

$1 - [{}^{15}C_0 \times 0.2^0 \times 0.8^{15} + {}^{15}C_1 \times 0.2 \times 0.8^{14}]$ (correct use of formula)

$= 1 - [1.801 \times 10^{-2} + 1.319 \times 10^{-1}]$ **(2 marks)**

$= 1 - 1.4995 \times 10^{-1}$ **(1 mark)**

= 0.85 (correct answer) **(1 mark)**

1 mark for using binomial
2 marks for parameters (1 mark each for the parameters (n and p))
1 mark for defining the random variable X
1 mark for correct formula (SOI)
1 mark for the $P(X \geq 2) = 1 - [P(X = 0) + P(X = 1)]$
1 mark for calculating $P(X = 0)$
1 mark for calculating $P(X = 1)$
1 mark for subtracting the probabilities from 1
1 mark for the correct answer

[10 marks]

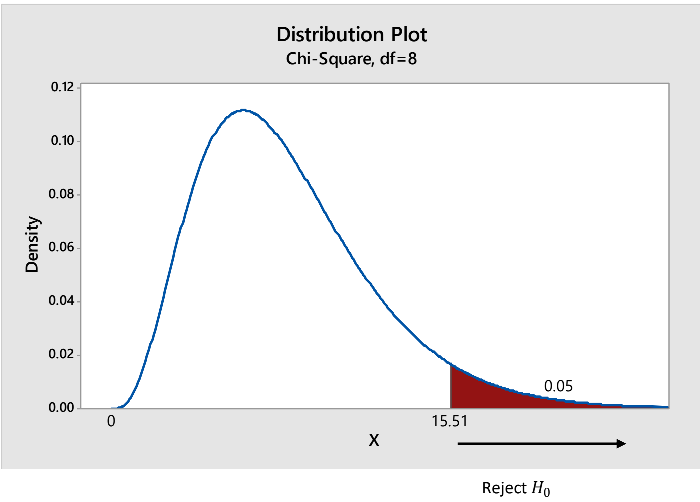
(a) (i)

H_0: There is no association between assembly speed and the number of defective components. **(1 mark)**

H_1: Assembly speed affects the number of defective components produced. **(1 mark)**

[2 marks]

(ii)



1 mark for the critical value 15.51
1 mark for the shape of the graph
1 mark for the region
[3 marks]

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

(iii)

Number of defective components found		2 O	E	3 O	E	4 O	E	Total
Group	A	14	17.2	20	17.2	16	15.6	50
	B	12	17.2	15	17.2	23	15.6	50
	C	15	17.2	16	17.2	19	15.6	50
	D	27	17.2	12	17.2	11	15.6	50
	E	18	17.2	23	17.2	9	15.6	50
Total		86		86		78		250

Expected frequencies = $\frac{\textbf{row total} \times \textbf{column total}}{\textbf{grand total}}$ **(SOI) at least 3 cells correct**

1 mark

$\chi^2 = \sum\frac{(O-E)^2}{E}$ (SOI)

A = 0.595 + 0.456 + 0.010 = 1.061

B = 1.572 + 0.281 +3.510 = 5.363

C = 0.281 + 0.837 + 0.741 = 1.859

D = 5.584 + 1.572 + 1.356 = 8.512

E = 0.037 + 1.956 + 2.792 = 4.818

(1 mark each for at least 3 correct calculations)

χ^2 = 21.61 **(1 mark)**

[5 marks]

Total 30 marks [AK]

APPLIED MATHEMATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 3.

(a) The probability of at least two defective components is 0.85 **(1 mark)**
since $P(X \geq 2) = 1 - P(X < 2)$ **(1 mark)**
The chi-squared calculated value is 21.6 **(1 mark)**
Since $\chi^2_{calc} = \sum \frac{(\mathrm{O-E})^2}{\mathrm{E}}$ **(1 mark)**

The data is positively skewed **(1 mark)**
Since $Q_3 - Q_2 < Q_2 - Q_1$ **(1 mark)**

[6 marks]

(b) Since $\chi^2_{calc} = 21.6 > 15.51 = \chi^2_{0.05}(8)$, we reject H_0 and conclude that there is an association between the assembly speed and the number of defective components.

reject H_0 $\chi^2_{calc} = 21.6 > 15.51 = \chi^2_{0.05}(8)$ 1 mark
there is an association 1 mark
[2 marks]

(c) One limitation of the finding is that the data collected was restricted to a four – hour period which would provide a snapshot of what is happening and not necessarily an accurate picture.

Restricted four -hour period 1 mark
Less accurate 1 mark
[2 marks]

(d) One recommendation is conduct regular observations over a period of time to increase the accuracy of the data

Extend time 1 mark
Increased accuracy 1 mark
[2 marks]

Total 12 marks

SPECIMEN 2022

TEST CODE **02205020**

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

MATHEMATICAL APPLICATIONS

UNIT 2 – Paper 01

1 hour 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This test consists of 45 items. You will have one hour and 30 minutes to answer them.
2. In addition to this test booklet, you should have an answer sheet.
3. Do not be concerned that the answer sheet provides spaces for more answers than there are items in this test.
4. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.
5. On your answer sheet, find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

   Sample Item

   The mean of 5, 7, 9, 11 and 13 is

   (A) 5
   (B) 7
   (C) 8
   (D) 9

   Sample Answer

   Ⓐ Ⓑ Ⓒ ●

   The best answer to this item is "9 ", so (D) has been shaded.

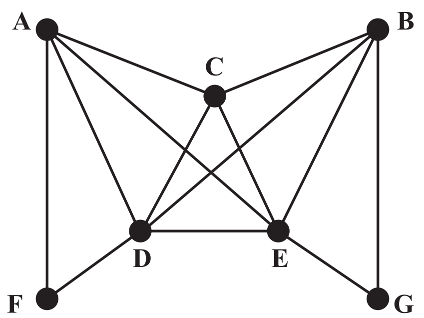
6. If you want to change your answer, erase it completely before you fill in your new choice.
7. When you are told to begin, turn the page and work as quickly and as carefully as you can. If you cannot answer an item, go on to the next one. You may return to that item later.
8. You may do any rough work in this booklet.
9. The use of silent, non-programmable scientific calculators is allowed.

   Examination Materials:

   A list of mathematical formulae and tables. **(Revised 2019)**

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

Item **1** refers to the following graph.



1. Which of the following is a path from vertex G to vertex D?

(A) GBECFAD
(B) GBECAFD
(C) GBCAEBD
(D) GEBCEAD

2. What is the contrapositive of the conditional statement "The home team misses whenever it is drizzling"?

(A) If it drizzling, then the home team misses.
(B) If the home team misses, then it is drizzling.
(C) If it is not drizzling, then the home team does not miss.
(D) If the home team wins, then it is not drizzling.

3. A proposition that is always false is a

(A) tautology
(B) contingency
(C) conjunction
(D) contradiction

4. Which of the following is De Morgan's law?

(A) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
(B) $\sim (P \wedge R) \equiv \sim P \vee \sim R$
(C) $P \vee \sim P \equiv$ True, $P \wedge \sim P \equiv$ False
(D) $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$

Item **5** refers to the following information.

Jim is drawing caricatures at a fair for 8 hours (480 minutes). He can complete a small drawing in 15 minutes and charges \$10 for that drawing. He can complete a larger drawing in 45 minutes and charges \$25 for that drawing. Jim hopes to make at least \$200 at the fair. Let x represent the number of small drawings and let y represent the number of large drawings.

5. Which of the following systems of inequalities BEST models the situation?

(A) $10x + 15y \le 480$
$45x + 25y \ge 250$

(B) $10x + 25y \le 200$
$15x + 45y \le 480$

(C) $10x + 25y > 200$
$15x + 45y < 480$

(D) $10x + 25y \ge 200$
$15x + 45y \le 480$

6. What is the name of the extra row or column that is added to balance an assignment problem?

(A) Regret
(B) Epsilon
(C) Dummy
(D) Extra

GO ON TO THE NEXT PAGE

7. Suppose you are using the Hungarian algorithm on the matrix shown below.

$$\begin{bmatrix} 4 & 7 & 5 \\ 10 & 18 & 14 \\ 12 & 8 & 19 \end{bmatrix}$$

Which of the following matrices would be the result after subtracting row minima AND subtracting column minima?

(A) $\begin{bmatrix} 0 & 3 & 1 \\ 0 & 8 & 4 \\ 4 & 0 & 11 \end{bmatrix}$

(B) $\begin{bmatrix} 4 & 7 & 5 \\ 10 & 18 & 14 \\ 12 & 8 & 19 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 3 & 0 \\ 0 & 8 & 3 \\ 4 & 0 & 10 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

8. The matrix below shows the time required for each combination of a worker and a job. The jobs are denoted by J1, J2, J3, J4 and J5: the workers by W1, W2, W3, W4 and W5.

	J1	**J2**	**J3**	**J4**	**J5**
W1	10	5	13	15	16
W2	3	9	18	13	6
W3	10	7	2	2	2
W4	7	11	9	7	12
W5	7	9	10	4	12

Each worker should perform exactly one job and the objective is to minimize the total time required to perform all jobs. What values would be in the 5th row of the matrix above after subtracting the row minimum?

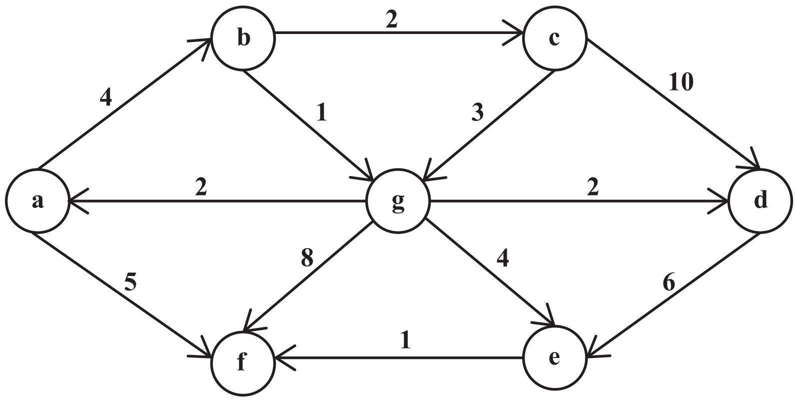
(A) 5 0 8 10 11
(B) 0 6 15 10 3
(C) 0 4 2 0 5
(D) 3 5 6 0 8

9. Activities R, S, and T are the immediate predecessors for activity W. If the earliest finishing times for the three activities are 10, 13 and 18 respectively, what will be the earliest starting time for activity W?

(A) 8
(B) 9
(C) 10
(D) 13

Item **10** refers to the following graph.

Consider the following graph.



10. If **b** is the source vertex, what is the minimum cost to reach vertex **f**?

(A) 8
(B) 9
(C) 4
(D) 6

Item **11** refers to the following information.

Let **p**, **q**, and **r** represent the following statements:

p: Sam had pizza last night.
q: Chris finished her homework.
r: Pat watched the news this morning.

11. What is the statement “Sam did not have pizza last night or Pat did not watch the news this morning.” in symbolic form?

(A) $p \wedge \sim r$
(B) $\sim p \vee q$
(C) $\sim p \vee \sim r$
(D) $\sim p \wedge \sim q$

Items **12** and **13** refer to the following information.

A movie theatre has 300 seats and charges \$7.50 for adults and \$5.50 for children. The theatre expects to make at least \$2000 for each showing. Let x represent the number of adults and y represent the number of children.

12. The system of inequalities to model the situation is

(A) $x + y \leq 300$
$7.5x + 5.5y \geq 2000$

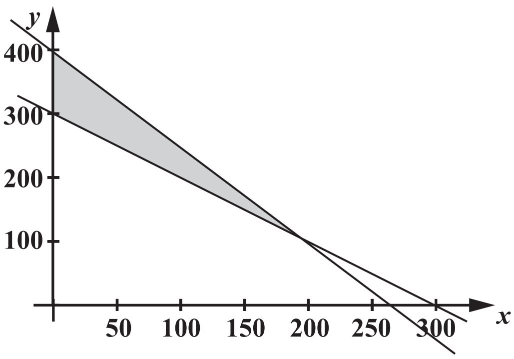
(B) $x + y < 300$
$7.5x + 5.5y \geq 2000$

(C) $x + y \leq 300$
$7.5x + 5.5y \leq 2000$

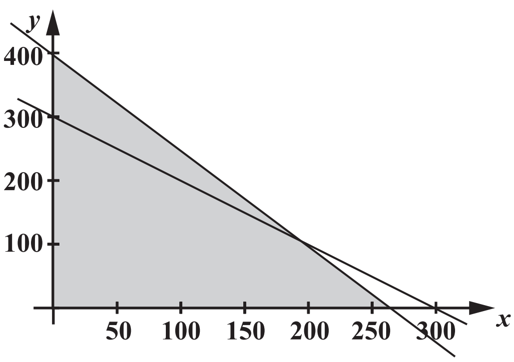
(D) $x + y > 2000$
$7.5x + 5.5y \leq 300$

13. In which of the following diagrams is the information given represented by the shaded region.

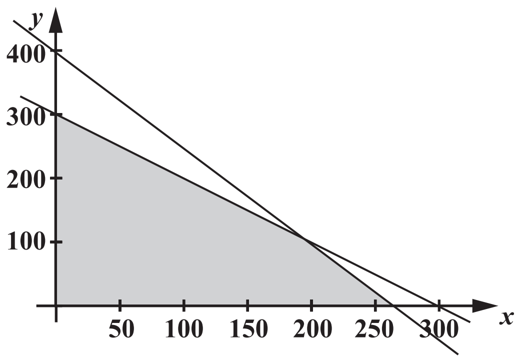
(A)



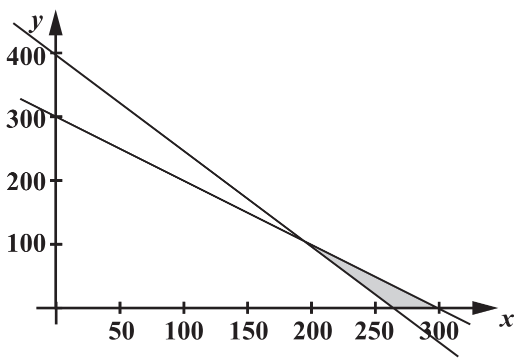
(B)



(C)



(D)



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Item **14** refers to the following information.

Given the following original cost matrix

	J1	**J2**	**J3**
T1	482	437	437
T2	402	499	432
T3	502	518	502

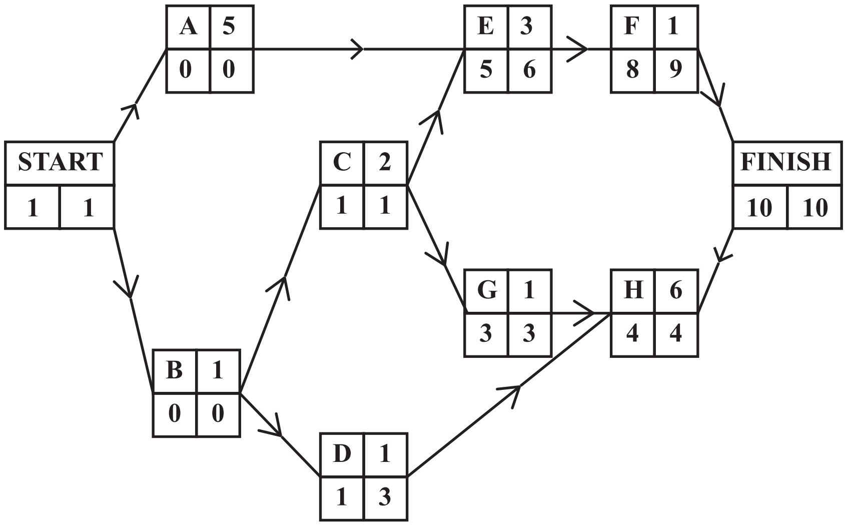
and the optimal assignment matrix

45	**0**	**0**
0	**97**	**30**
0	**16**	**0**

14. Which of the following would represent the optimal assignment?

(A) T1 → J1, T2 → J3, T3 → J2
(B) T1 → J2, T2 → J1, T3 → J3
(C) T1 → J3, T2 → J1, T3 → J2
(D) T1 → J3, T2 → J1, T3 → J1

<u>Item **15**</u> refers to the following network diagram.



15. Determine the critical path.

(A) A − E − F
(B) B − D − H
(C) B − C − G − H
(D) A − E − C − G − H

16. The events A and B are such that $P(A) = 0.44, P(B) = 0.48, P(A \cup B) = 0.71$. Events A and B are BEST described as

(A) Binomial
(B) Dependent
(C) Independent
(D) Mutually exclusive

17. The probability density function of the form $\sum_{i=1}^{N} P\left(X = x_i\right) = 1$ is applicable only to

(A) Binary variables
(B) Ordinal variables
(C) Discrete variables
(D) Continuous variables

18. If the probability density function of a continuous random variable is represented as $f(x)$, then $P(x \geq 2)$ is

(A) the height of the curve at $x = 2$
(B) the area under the curve at $x = 2$
(C) the area under the curve to the left of $x = 2$
(D) the area under the curve to the right of $x = 2$

19. In the steel industry, a manufacturer is interested in the number of flaws occurring in every 100 feet of steel sheet. The probability distribution that is most applicable to this situation is the

(A) normal distribution
(B) binomial distribution
(C) poisson distribution
(D) uniform distribution

20. How many ways can the letters of the word GERMAN be arranged so that the vowels only occupy even spaces?

(A) 36
(B) 48
(C) 96
(D) 144

21. S and T are two events such that $P\left(\overline{S}\right) = 0.4$ and $P(S \cap T) = 0.2$. Then $P\left(S \cap \overline{T}\right)$ is equal to

(A) 0.2
(B) 0.4
(C) 0.6
(D) 0.8

22. Ramey is the leading goal scorer for the Pontalia football team. The probability that he will score 0, 1, 2, or 3 goals in any of his matches is 0.20, 0.35, 0.35, and 0.10. What is probability that Ramey will score less than 3 goals in an upcoming match?

(A) 0.20
(B) 0.55
(C) 0.90
(D) 1.00

23. The daily log of a supermarket shows that an average of 10 customers are served at the 'less than 10 items' checkout counter each hour. What is the probability that 15 customers arrive at this checkout counter in 1 hour?

(A) 0.03
(B) 0.05
(C) 0.11
(D) 0.91

24. The continuous random variable X has a probability density function, f, given by

$$f(x) = \begin{cases} Ax(1-x^2) & (0 \leq x \leq 1), \\ 0 & \textit{elsewhere}, \end{cases}$$

The values of A and μ are

(A) $A = \frac{1}{4}, \mu = \frac{8}{15}$

(B) $A = \frac{1}{4}, \mu = \frac{11}{225}$

(C) $A = 4, \mu = \frac{8}{15}$

(D) $A = 4, \mu = \frac{11}{225}$

Item **25** refers to the following information.

A firm wishes to determine for the population of employees if the days with the highest number of late entries occur with equal frequencies. A random sample of 60 supervisors revealed the days of a five-day work week with the highest number of employees arriving late. The results are shown in the table below.

Day of the week	**Mon**	**Tues**	**Wed**	**Thur**	**Fri**
No. of late employees	16	10	12	10	12

25. Using a 5% significance level, which of the following is **NOT** true?

(A) $\upsilon = 4$
(B) $E_i = 12$
(C) Critical Region: $\chi^2 < 9.488$
(D) H_0: number of late entries occur with equal frequency

26. X and Y are independent random variables with the distribution given by given by X~$N(2, 25)$ and Y is $N(3, 16)$. The variance of the distribution for $2X - 3Y$ is

(A) 48
(B) 96
(C) 148
(D) 244

27. A collection of large inflatable balloons has 5% defective balloons. Selecting a non-defective balloon on the 4th try will have a geometric distribution with

(A) $x = 3, p = 0.05$
(B) $x = 4, p = 0.05$
(C) $x = 3, p = 0.95$
(D) $x = 4, p = 0.95$

Items **28 and 29** refers to the following information.

Sunny Days Nuts claims that each bag of dried mixed fruits contains 100 pieces with equal amounts of pineapple, pecan, raisin, and banana pieces. Joe randomly purchased 10 bags of the mixed fruits to investigate this claim. A count of the fruit pieces in the 10 bags found 300 pineapple, 200 pecan, 275 raisin, and 225 banana pieces.

28. What is an appropriate null hypothesis for this investigation?

(A) The average of all fruit pieces is the same.
(B) The average of all fruit pieces is different.
(C) The proportion of all fruits pieces is different.
(D) The proportion of all fruit pieces is the same.

29. The chi-squared value with $\alpha = 0.05$ and 3 degrees of freedom is 7.81. What is the value of the χ^2 statistic and the correct conclusion to be made?

(A) $\chi^2 \approx 25$; not significant at the 0.05 level
(B) $\chi^2 \approx 25$; significant at the 0.05 level
(C) $\chi^2 \approx 81.25$; not significant at the 0.05 level
(D) χ^2 81.25; significant at the 0.05 level

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30. X is a continuous random variable that represents the score obtained on a test item. The scores are defined by the probability density function:

$$f(x) = \frac{2}{9}x(3-x), 0 \leq x \leq 3$$

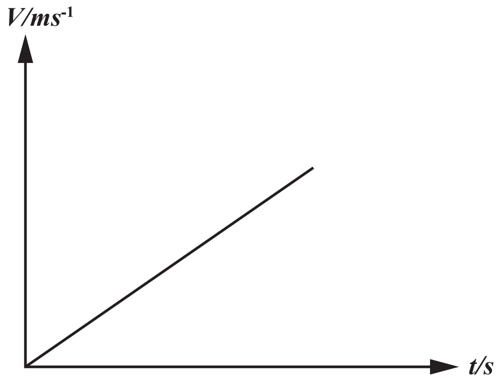
The probability that a student scores more than 1 on the test is

(A) 0.11
(B) 0.59
(C) 0.63
(D) 0.72

31. Two forces **F**1 and **F**2 have a resultant force **F**3. If **F**1 = 3**i** − 5**j** and **F**3 = −7**i** + 2**j**, then **F**2 is

(A) 4**i** − 3**j**
(B) −10**i** + 7**j**
(C) 10**i** − 7**j**
(D) −4**i** + 3**j**

Item **32** refers to the following velocity time graph which shows a particle moving in a straight line.



32. The gradient of the velocity time graph represents the

I. acceleration
II. rate of change of velocity
III. rate of change of displacement

(A) I and II only
(B) I and III only
(C) II and III only
(D) I, II and III

33. A particle is projected with a speed of u ms^{-1} at an angle of θ^0 to the horizontal. What is the distance travelled horizontally?

(A) $\frac{2u\sin\theta}{g}$

(B) $\frac{u^2\sin^2\theta}{2g}$

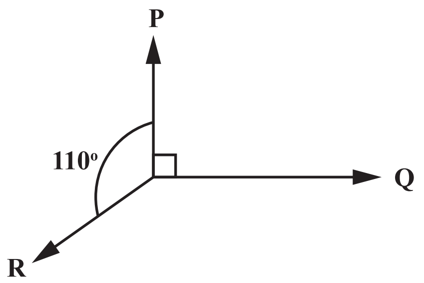
(C) $\frac{u^2\sin 2\theta}{g}$

(D) $\frac{u^2\sin\theta\cos\theta}{2g}$

34. A construction worker lifts a box of mass 15 kg to a height of 3 m. How much work is done by gravitational force?

(A) 450 J
(B) 45 J
(C) 30 J
(D) 150 J

35. A body is held in equilibrium by three forces, P, Q and R, as shown in the diagram. R has a magnitude of 45 N. What is the magnitude of the force P to one decimal place?



(A) 5.6 N
(B) 15.4 N
(C) 42.3 N
(D) 123.6 N

36. A constant force of 40 N pulls a small wooden block along a rough horizontal floor. The force acts a angle of θ^0 to the horizontal and the block moves at a constant speed of 30 ms$^{-1.}$ Given that the work done by this force is 900 J in 10 s, what is the value of θ?

(A) 4.3^0
(B) 41.4^0
(C) 48.6^0
(D) 85.7^0

37. An object of mass 4 kg lies on a rough horizontal surface where μ is the coefficient of friction between the block and surface. A force of magnitude of 20 N acts downwards on the block where $\sin a = \frac{3}{5}$. If the block remains at rest, then

(A) $\mu \leq \frac{4}{13}$

(B) $\mu \geq \frac{4}{13}$

(C) $\mu \leq \frac{3}{14}$

(D) $\mu \geq \frac{3}{14}$

38. Particles A, with mass 0.5 kg, and B, with mass, 0.4 kg are attached to the ends of a light inextensible string. The string which is taut passes over a smooth pulley. The system is released from rest and the particles move vertically. What is the magnitude of the resultant force exerted on the pulley?

(A) $0N$

(B) $\frac{20}{9}N$

(C) $\frac{40}{9}N$

(D) $\frac{80}{9}N$

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39. A golfer hits a ball with an initial speed of 30 ms^{-1} an angle of 40^0 above the hortizontal. What is the vertical component of initial velcoity of the ball, in ms^{-1}?

(A) 19.3
(B) 23.0
(C) 25.2
(D) 30.0

40. A golfer hits 2 similar balls with the same force. One ball was hit at an angle of 30^0 and the 2nd ball at an angle of 45^0. Compared to the ball hit at 30^0, the ball fired at 45^0 has

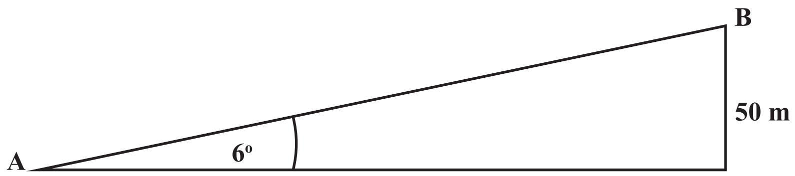
(A) shorter range, more time of flight
(B) shorter range, less time of flight
(C) longer range, less time of flight
(D) longer range, more time of flight

---

41. A car travel at a constant speed of 54 kmh^{-1}. The car's engine produce a driving force of 6000 N in order to keep the speed constant. What is the power developed by the engine?

(A) 30000 W
(B) 90000 W
(C) 324000 W
(D) 1166400 W

Item **42** refers to the following diagram.



42. A car of mass 1500 kg proceeds up a hill inclined at 6^0 to the horizontal with *B* being 50 m higher than A. If the speed of the car is constant, and the work done against resistance is 240 KJ,what is the work done by the car's engine?

(A) 71000 J
(B) 718000 J
(C) 750000 J
(D) 990000 J

43. A particle moves such that its velocity, v, is given by, $v = 2t^2 - 11t + 15$. If a and x represents the particle's acceleration and displacement respectively after t seconds which of the following are true?

I. $x = \int \left(2t^2 - 11t + 15\right) dt$

II. $\alpha = 4t - 11$

III. at t = 2.5 s the particle will be instantaneously at rest

(A) I and II only
(B) II and III only
(C) I and III only
(D) I, II and III

44. Two bowling balls, each with a mass of 9 kg are travelling toward each other. The bowling ball moving to the right has a speed of 4 ms^{-1} and the bowling ball moving to the left has a speed of 3 ms^{-1}. What is the total momentum before the collision?

(A) −27 Ns
(B) −9 Ns
(C) 9 Ns
(D) 36 Ns

45. Which of the following objects has the greatest momentum?

(A) A tractor trailer at rest
(B) A 60 kg man walking
(C) A fast bowler bowling a ball
(D) A sports car exceeding the speed limit on the highway

END OF TEST

Item	Specific Objective	Key		Item	Specific Objective	Key
1	2.1.3.1	B		26	2.2.2.2	A
2	2.1.4.4	D		27	2.2.2.5	D
3	2.1.4.3	D		28	2.2.3.3	A
4	2.1.4.4	B		29	2.2.4.5	B
5	2.1.1.1	D		30	2.2.3.1	B
6	2.1.2.1	C		31	2.3.1.2	B
7	2.1.2.4	A		32	2.3.2.1	A
8	2.1.2.4	D		33	2.3.2.3	C
9	2.1.3.4	C		34	2.3.4.3	A
10	2.1.3.3	D		35	2.3.1.6	B
11	2.1.4.1	B		36	2.3.4.1	D
12	2.1.1.3	C		37	2.3.1.5	A
13	2.1.1.2	D		38	2.3.1.5	D
14	2.1.2.1	D		39	2.3.3.3	A
15	2.1.4.7	D		40	2.3.3.3	A
16	2.2.1.4	B		41	2.3.4.4	B
17	2.2.2.1	C		42	2.3.4.3	D
18	2.2.3.1	D		43	2.3.2.5	D
19	2.2.4.1	C		44	2.3.2.7	C
20	2.2.1.2	D		45	2.3.2.7	C
21	2.2.1.2	B				
22	2.2.2.1	C				
23	2.2.2.4	A				
24	2.2.3.4	A				
25	2.2.4.4	C				

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

APPLIED MATHEMATICS

SPECIMEN

2022

TABLE OF SPECIFICATIONS

Paper 02

UNIT 2

Question	*Module*	*CK*	*AK*	*R*	*Total*
1	1	6	14	5	25
2	1	6	14	5	25
3	2	6	14	5	25
4	2	6	14	5	25
5	3	6	14	5	25
6	3	6	14	5	25
SUBTOTAL		**36**	**84**	**30**	**150**

TEST CODE **02205020**

SPECIMEN 2022

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

MATHEMATICAL APPLICATIONS

UNIT 2 – Paper 02

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections. Each section consists of TWO questions.
2. Answer ALL questions.
3. Write your answers in the spaces provided in this booklet.
4. Do NOT write in the margins.
5. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.
6. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra lined page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
7. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials:

Mathematical formulae and tables **(Revised 2022)**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

“*”Barcode Area”*”
Sequential Bar Code

SECTION A

MODULE 1: DISCRETE MATHEMATICS

Answer BOTH questions.

1. (a) (i) Construct the truth table for the proposition $\mathbf{x} \Rightarrow \mathbf{y}$.

[2 marks]

(i) Hence determine the truth value of the statement, if $3 \times 5 = 15$, then $6 + 2 = 7$

[2 marks]

GO ON TO THE NEXT PAGE

(b) Using the laws of Boolean algebra, show that $\mathbf{p \vee (p \wedge q) = p}$

[4 marks]

(c) Represent the Boolean expression

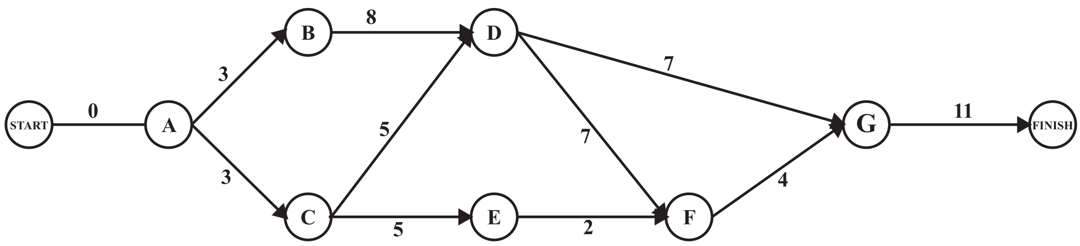
(i) $\mathbf{r \wedge (s \vee {\sim}t)}$ as a logic circuit, using only AND, OR and NOT gates

[4 marks]

(ii) $(\mathbf{w} \wedge \mathbf{x}) \vee \mathbf{y} \vee \mathbf{z}$ as a switching circuit.

[3 marks]

(d) The following diagram is an activity network relating to the assembly of an item. The number on each arc is the time taken, in minutes, to complete the activity.



(i) Construct a precedence table to represent the information in the activity network above.

[3 marks]

GO ON TO THE NEXT PAGE

(ii) Complete the following table to show the earliest start time and the latest start time of EACH activity.

Activity	**Earliest Start Time**	**Latest Start Time**
A		
B		
C		
D		
E		
F		
G		

[5 marks]

(iii) Determine the critical activities and the duration of the critical path.

....

....

[2 marks]

Total 25 marks

NOTHING HAS BEEN OMITTED.

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

2. (a) Paint-It-Right factory produces two types of paint, gloss and emulsion, in batches of 50-gallon containers. The factory takes four hours to produce a 50-gallon container of the gloss paint while it takes two hours to produce a 50-gallon container of the emulsion paint.

It takes five labour-hours to label and check a batch of gloss paint for consistency while it takes four labour-hours to check a batch of emulsion paint for consistency.

The factory has 35 hours of machine time and 55 labour-hours available each day.

Both types of paints must be produced each day.

The profit on each batch of gloss paint is \$150 and on each batch of emulsion paint is \$125.

(i) Clearly define the variables to be used in setting up this problem as a linear programing model.

[2 marks]

(ii) Define the TWO major constraints to be used in setting up this problem.

[2 marks]

(iii) Write the FOUR inequalities of the constraints that must be used in setting a linear programing model of the information given.

[3 marks]

GO ON TO THE NEXT PAGE

(iv) Write the objective function from this information that will maximize the profit of this paint producing process.

....

....

[2 marks]

(v) On the graph sheet provided **on page 9**, draw a clearly labelled graph representing the four inequalities. **[5 marks]**

(vi) On the graph plotted **on page 9**, clearly identify the feasible region to satisfy the inequalities. **[1 mark]**

Using the graph plotted **on page 9**, determine

(vii) the number of batches of each type of paint that will maximize the profit function

[2 marks]

(viii) the maximum profit that can be made on the production of each batch of these two types of paints.

....

....

[1 mark]

(b) In order to finish an obstacle course competition, teams of three persons are asked to complete three different tasks. They must work as a team; however, each task can only be completed by one team member. Each task is scored out of 30 and the team with the highest score wins.

Three friends form a team so they can enter the competition. The team of three is selected so as to maximize the combined scores of the members while assigning each member to a task.

They each perform the tasks of the course and their scores are shown in the table below.

	Task X	***Task Y***	***Task Z***
Adrian	25	28	29
Brian	28	22	27
Corey	23	24	26

Use the Hungarian algorithm to determine the task to which each is assigned. Your response **must** include the maximum combined score.

GO ON TO THE NEXT PAGE

[7 marks]

Total 25 marks

SECTION B

MODULE 2: PROBABILITY AND DISTRIBUTIONS

Answer BOTH questions.

3. (a) (i) Determine the number of different ways of arranging the letters of the word SATISFY if there are no restrictions.

[2 marks]

(ii) Calculate the probability that the two S's remain together in a word.

[3 marks]

(b) Two independent random variables, X and Y, are such that E[X] = 6, E[Y] = 8, Var[X] = 2 and Var[Y] = 4.

Determine

(i) E[X +Y]

[2 marks]

(ii) $E[3X-2Y]$

[3 marks]

(iii) $\text{Var}[3X - 2Y]$

[3 marks]

(c) A continuous random variable, X, has the probability density function, f, given by

$$f(x) = \begin{cases} k(4 - x) & 0 \le x \le 3 \\ 0 & \textit{otherwise}\,. \end{cases}$$

(i) Determine the value of k.

[3 marks]

(ii) Calculate $P(X > 1)$.

[3 marks]

(iii) Construct the distribution function $F(x)$.

[3 marks]

(iv) Hence determine the median value of X.

[3 marks]

Total 25 marks

4. (a) A cloth manufacturer estimates that faults occur randomly in the production process at a rate of 2 faults every 10 metres of cloth.

(i) Calculate the probability that there are exactly 2 faults in a 10-metre length of cloth.

[3 marks]

(ii) Calculate the probability of at least 3 faults in a 30-metre length of cloth.

[3 marks]

(iii) A person bought 5 metres of the cloth. Determine the probability of getting 2 faults in that piece of cloth.

[3 marks]

GO ON TO THE NEXT PAGE

(b) A Trivia quiz is made up of 15 multiple-choice questions, each with four options. It is assumed that each option has the same probability of being correct, and the questions are independent of each other. Let the random variable X represent the number of correct questions that a student taking the quiz is likely to obtain.

(i) State the distribution that may be modelled by this situation, giving its parameter(s).

....

....

[2 marks]

(ii) Determine the number of correct questions that a student is expected to obtain.

[2 marks]

(iii) Calculate the probability of getting exactly four questions correct.

[2 marks]

(c) A cubical die is to be tested for bias by analysing the results of 120 throws of the die. The number of times that each score was obtained is shown in the following table.

score	1	2	3	4	5	6
frequency	26	17	20	13	21	23

(i) Complete the following table to the expected frequency for each score, assuming that each score is equally likely to occur.

Score	1	2	3	4	5	6
Expected frequency						

[2 marks]

GO ON TO THE NEXT PAGE

(ii) State the null and alternate hypotheses.

....

....

....

....

[2 marks]

(iii) Carry out a goodness of fit analysis to test the hypotheses stated in (c) (ii), using a 10% level of significance.

[5 marks]

(iv) State your conclusion clearly.

....

....

[1 mark]

Total 25 marks

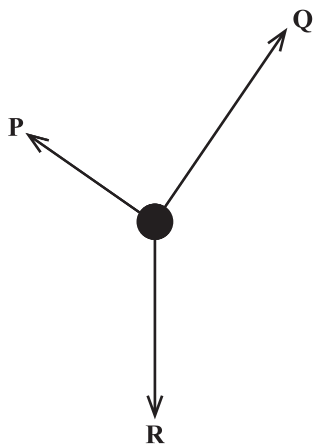
SECTION C

MODULE 3: PARTICLE MECHANICS

Answer BOTH questions.

(Where necessary, take g = 10 ms^{-2}.)

5. (a) The following diagram shows three forces P, Q and R acting a point. P is inclined at 30^{o} to the horizontal and Q is perpendicular to the to P.



Given that R = 13 N and the system of forces is in equilibrium,

(i) show the angles between the forces on the diagram above

[3 marks]

(ii) calculate the forces P and Q using Lami's theorem.

[5 marks]

GO ON TO THE NEXT PAGE

(b) A block of mass 30 kg rests on a rough plane which is inclined at an angle θ to the horizontal, as shown in the diagram. A force, T, acts on the block along the line of greatest slope. The coefficient of friction between the plane and the block is 0.35 and $\theta = 50°$.

(i) Draw a carefully labelled diagram showing all the forces acting on the block.

[3 marks]

(ii) Determine the value of T when the block is just about to slip up the plane, the system is in equilibrium and the force of friction would be acting down the plane.

[5 marks]

(iii) Determine the value of T when the block is just about to slip down the plane, the system is in equilibrium and the force of friction would be acting up the plane.

[3 marks]

(c) A truck moves in a straight line with speed u ms^{-1}. The truck retards uniformly for 15 seconds, then maintains a constant speed for 15 seconds. It comes to rest after a uniform retardation in 10 seconds. If both retardations are equal and the total distance travelled during the 40 seconds is 250 metres, calculate the value of u.

[6 marks]

Total 25 marks

6. (a) Two cars, A and B, are moving in the same direction on a smooth horizontal road. Car A has a mass of 2000 kg and the mass of car B is 1200 kg. Initially A, moving with speed 25 ms^{-1}, "is catching up with" B, whose speed is u ms^{-1}. Immediately after the cars collide, B has a speed of 20 ms^{-1}. Given that the impulse acting on B has magnitude 6000 Ns,

(i) determine the value of u

[3 marks]

(ii) calculate the loss in kinetic energy after the collision.

[5 marks]

Car B subsequently collides with a stationary car, C, of mass 1500 kg. B comes to a complete stop after the collision while C moves with a speed of x ms^{-1}.

(iii) Determine the value of x.

[3 marks]

GO ON TO THE NEXT PAGE

(b) A golfer hits a golf ball from a point, W, to a point, Z. Z is on the same horizontal level as W. The ball is projected from W at a speed of 95 ms^{-1}, and at an angle of $\alpha°$ above the horizontal.

(i) Given that the ball hits the ground at Z which is 750 m from W, calculate the two possible values of α.

[4 marks]

(ii) Given that $\alpha = 30°$, calculate the times when the ball is 100 m above the ground.

[3 marks]

(iii) Given that $\alpha = 45$, determine the magnitude and direction of the velocity of the ball at the instant when it is vertically above Z.

[7 marks]

Total 25 marks

END OF TEST

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 2 – Paper 02

KEY AND MARK SCHEME

SPECIMEN 2022

APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

SECTION A

MODULE 1:

Question 1.

(a) (i) The proposition $\mathbf{x} \Rightarrow \mathbf{y}$

x	**y**	**x ⇒ y**
T	T	T
T	F	F
F	T	T
F	F	T

OR

x	**y**	**x ⇒ y**
0	0	1
0	1	1
1	0	0
1	1	1

1 mark each for a pair of correct row entries

[2 marks] [CK]

(ii) If $3 \times 5 = 15$, then $6 + 2 = 7$

Let ***x*** *represent* $3 \times 5 = 15$ so this is true
Let ***y*** *represent* $6 + 2 = 7$, and this is false

1 mark for correct statements [CK]

Therefore, $x \Rightarrow y$ must be false.

1 mark for conclusion (R)
[2 marks]

APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

Question 1. (continued)

(b) Using the LHS

$\mathbf{p} \vee (\mathbf{p} \wedge \mathbf{q})$

$= (p \wedge 1) \vee (p \wedge q)$ identity law

$= p \wedge (1 \vee q)$ distributive law

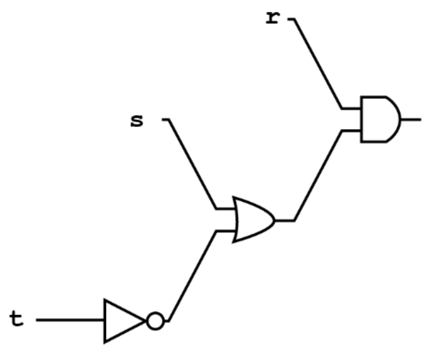
$= p \wedge 1$ identity law

$= p$ identity

1 mark for each step

[4 marks, AK]

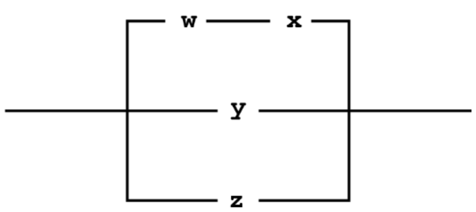
(c) (i) **r** ∧ (**s** ∨ ~ **t**) as a logic circuit, using only AND, OR and NOT gates.



1 mark for each correct gate [AK]
1 mark for correct sequencing [R]

[4 marks]

(ii)



1 mark for w and x in series [AK]
1 mark for 3 switches in parallel [AK]
1 mark for correct diagram [R]

[3 marks]

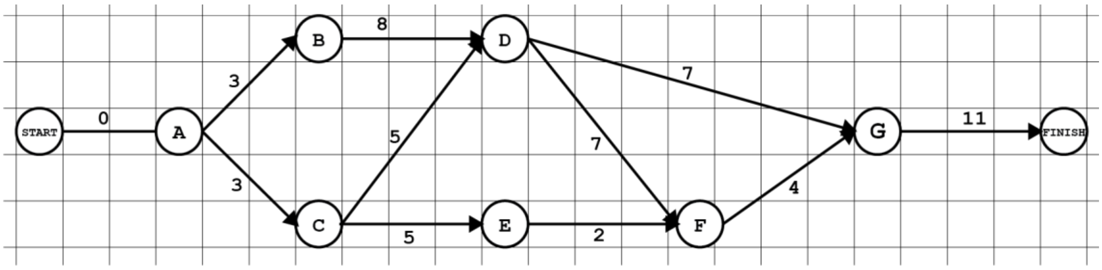
APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

Question 1. (continued)

(d) The following diagram is an activity network relating to the assembly of an item. The number on each arc is the time taken, in minutes, to complete the activity.



(i) Construct a precedence table to represent the information in the activity network above.

A	-
B	A
C	A
D	B, C
E	C
F	D, E
G	D, F

1 mark for correct entry for activity A **[CK]**
1 mark for every 3 other correct preceding events **[R]**
[3 marks]

(ii) Complete the following table to show the earliest start time and the latest start time of EACH activity.

Activity	Earliest Start Time	Latest Start Time
A	0	0
B	3	3
C	3	6
D	11	11
E	8	16
F	18	18
G	22	22

1 mark for correct entries for activity A **[CK]**
1 mark each for every 3 correct entries in earliest and late start time columns **[AK]**
[5 marks]

APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

Question 1. (continued)

(iii) Determine the critical activities and the duration of the critical path.

Critical path:	Start-A-B-D-F-G-FINISH	**(1 MARK)**	**[CK]**
Duration	= 33 MINUTES	**(1 MARK)**	**[AK]**

[2 marks]

Total 25 marks

APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

Question 2.

(a) (i) Let x represent the amount of gloss paint. **1 mark**
Let y represent the amount of emulsion paint. **1 mark**

[2 marks] [CK]

(ii) Labour hours
Machine time

[2 marks] [CK]

(iii)

$4x + 2y \le 35$	machine time	**1 mark correct coefficients AK**
$5x + 4y \le 55$	labour hours	**1 mark correct inequalities R**
$x \ge 1,\ y \ge 1$	minimum requirements	**1 mark correct constraints R**

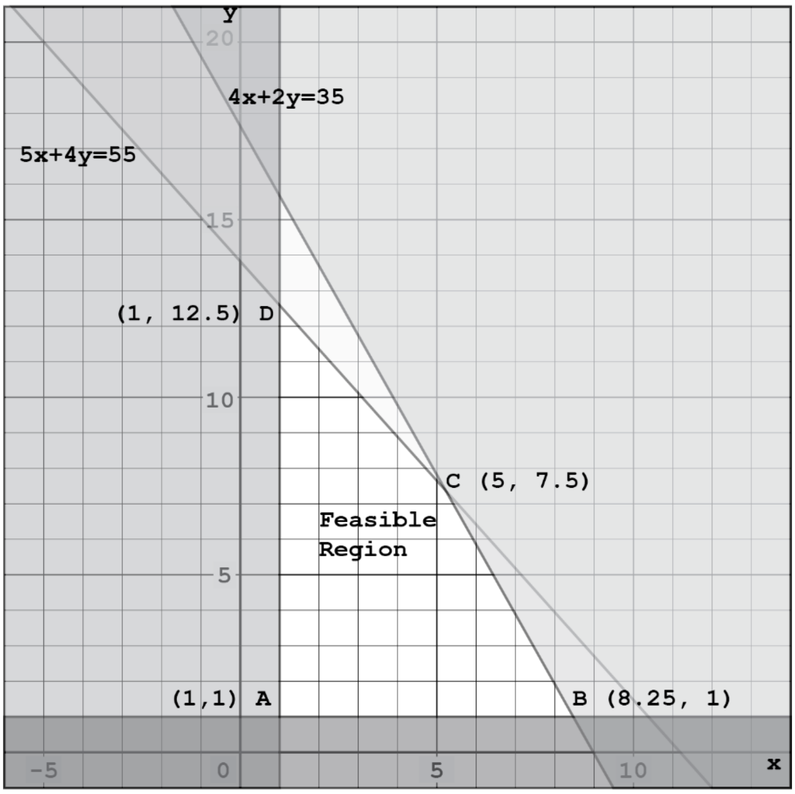
[3 marks]

(iv) Objective function $z = 150x + 125y$

correct x coefficient	**1 mark**	**R**
correct y coefficient	**1 mark**	**R**

[3 marks] [R]

(v)



1 mark for every correctly drawn and labelled inequality
1 mark for correctly identifying the feasible region
[5 marks] [AK]

Question 2. (continued)

(vi)

A (1,1)	150(1) + 125(1) = 275
B (8.25, 1)	150(8.25) + 125(1) = 1362.5
C (5, 7.5)	150(5) + 125(7.25) = 1687.5
D (1, 12.5)	150(1) + 125(12.5) = 1712.5

1 mark each for a pair of correct calculations
1 mark for identifying correct maximization values

[2 marks] [AK]

(vi) The maximum profit generated is $1712.50
1 mark for identifying maximum profit [CK]

(b)

	x	y	z	min
A	-25	-28	-29	-29
B	-28	-22	-27	-28
C	-23	-24	-26	-26

1 mark for negating all values [AK]
1 mark for correctly choosing the minimum values [AK]

	x	y	z
A	4	1	0
B	0	6	1
C	2	2	0
min	0	1	0

1 mark for correct subtraction of values [AK]

	x	y	z
A	4	0	0
B	0	5	1
c	2	1	0

1 mark for correct subtraction of values [AK]
1 mark for shading with minimum shading [R]

Allocation: Adrian – Task Y
Brian – Task X
Corey – Task Z **1 mark for the correct allocations [CK]**

Maximum score = 28+28+26 = 82 **1 mark for correct answer [AK]**

[7 marks]

Total 25 marks

APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

SECTION B

MODULE 2: PROBABILITY AND DISTRIBUTIONS

Question 3.

(a) (i) $\frac{7!}{2!} = 2520$

1 mark for division by 2! [CK]

1 mark for correct answer [AK]
[2 marks]

(ii) $\frac{6!}{2520}$ **1 mark for correct numerator [R]**

$= \frac{720}{2520}$ **1 mark for division [AK]**

$= \frac{2}{7}$ **1 mark for correct answer [AK]**

[3 marks]

(b) (i) E (X + Y)

E(X) + E(Y) = 6 + 8 **1 mark for addition of expected values [AK]**

= 14 **1 mark for correct answer [AK]**

[2 marks]

(ii) E [3X – 2Y]

= 3E[X] – 2E[Y] **1 mark for distributing E [CK]**

= 3 x 6 - 2 x 8 **1 mark for multiplication [AK]**

= 18 – 16 **1 mark for subtracting [AK]**

= 2

[3 marks]

(iii) Var [3X – 2Y]

= 9Var[X] + 4Var[Y] **1 mark for a^2Var(X)[CK]**

= 9 x 2 + 4 x 4

= 18 +16 **1 mark for addition of values [AK]**

= 34 **1 mark for correct answer [AK]**

[3 marks]

Question 3. (continued)

(c) (i) $\int_0^3 k(4-x)dx = 1$ **1 mark for integral = 1 [CK]**

$k\left(4x - \frac{x^2}{2}\right)\Big|_0^3 = 1$ **1 mark for integrating [AK]**

$k\ (12 - \frac{9}{2}) = 1$

$k = \frac{2}{15}$ **1 mark for correct answer [AK]**

[3 marks]

(ii) $= \frac{2}{15}\int_1^3 (4-x)dx$ **1 mark for substituting for k [CK]**

$= \frac{2}{15}\left(4x - \frac{x^2}{2}\right)\Big|_1^3$ **1 mark for integrating between 1 and 3 [R]**

$= \frac{2}{15}\left(12 - \frac{9}{2} - 4 + \frac{1}{2}\right)$ **1 mark for correct integration1 mark [AK]**

$= \frac{8}{15}$

[3 marks]

(iii) $P\ (X < t) = \frac{2}{15}\int_0^t (4-x)\,dx$ **1 mark [CK]**

$\frac{2}{15}\left(4x - \frac{x^2}{2}\right)\Big|_0^t$

$\frac{2}{15}\left(4t - \frac{t^2}{2}\right)$ **1 mark [AK]**

$F(x) = \begin{cases} 0 & x < 0 \\ \frac{2}{15}\left(4x - \frac{x^2}{2}\right) & 0 \le x \le 3 \\ 1 & x > 3 \end{cases}$ **1 mark [R]**

[3 marks]

(iv) $P(X < t) = 0.5$ **1 mark [R]**

$\frac{2}{15}\left(4x - \frac{x^2}{2}\right) = 0.5$

$8x - x^2 = \frac{15}{2}$

$x = \frac{8 \pm \sqrt{64-30}}{2}$ **1 mark [AK]**

$x = \frac{8 \pm 5.83}{2}$

$x = 13.83/2$ **1 mark for inadmissible [R]**

or $x = 1.085$

median $= 1.085$

[3 marks]

Total 25 marks

APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

Question 4.

(a) (i) X is the number of faults in the 10-metre length of cloth.

X~Po(2) **1 mark (SOI) [CK]**

$P(X = 2) = \frac{e^{-2}2^2}{2!}$ **1 mark for correct use of formula [AK]**

$= \frac{4e^{-2}}{2}$

$= 2e^{-2}$

$= 0.2706 \sim 0.271$ **1 mark for correct answer [AK]**

[3 marks]

(ii) In 30 metres of cloth, X~Po(6)

change of λ1 mark [R]

$P(X \text{ at least } 3) = 1 - P(X \leq 2)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$ **1 mark [AK[**

$= 1 - [e^{-6} + 6e^{-6} + 18e^{-6}]$

$= 1 - 25e^{-6}$ **1 mark [AK]**

$= 0.938$

[3 marks]

(iii) In 5 metres of cloth, X ~ Po (1) **1 mark for change of λ [R]**

$P(X = 2) = \frac{e^{-1}1^2}{2!}$

1 mark for correct use of formula [AK]

$= \frac{e^{-1}}{2}$

$= 0.1839$ **1 mark for correct answer [AK]**

[3 marks]

(b) (i) X~Bin (15, 0.25)

1 mark for binomial
1 mark for both parameters
[2 marks] [CK]

(ii) $E[X] = np = 15 \times 0.25$ **1 mark for multiplying parameters [CK]**

$= 3.75$ **1 mark for correct answer [AK]**

[2 marks]

(iii) P (at exactly 4 questions correct)

$= {}^{15}C_4 (0.25)^4(0.75)^{11}$ **1 mark correct application of formula [AK]**

$= 0.225$ **1 mark for correct answer [AK]**

[2 marks]

Question 4. (continued)

(c) (i)

score	1	2	3	4	5	6
Expected frequency	20	20	20	20	20	20

2 marks for All values correct [AK]
1 mark for at least 4 values correct [AK]

[2 marks]

(i) H_0: the results follow a uniform distribution
H_1: the results do not follow a uniform distribution.
1 mark for correct null and alternate hypothesis [AK]

Using a Chi square distribution **1 mark [R]**

At the 10% level of significance the critical region is

$\chi^2 \geq 9.236$ **5 degrees of freedom SOI** **1 mark [AK]**

correct table value **1 mark [CK]**

$\chi^2_{\text{calculated}} = \sum \frac{(O-E)^2}{E} = \frac{36}{20} + \frac{9}{20} + 0 + \frac{49}{20} + \frac{1}{20} + \frac{9}{20}$

$= \frac{104}{20}$ 1 mark for **correct formula [CK]**
$= 5.2$ 1 mark for **correct answer** [**AK**]

Since 5.2 < 9.236, the calculated value of χ^2 falls in the accepted region **(1 mark)**. Therefore, accept the hypothesis that each score is likely to occur an equal number of times **(1 mark)**.
2 marks [R]

[8 marks]

Total 25 marks

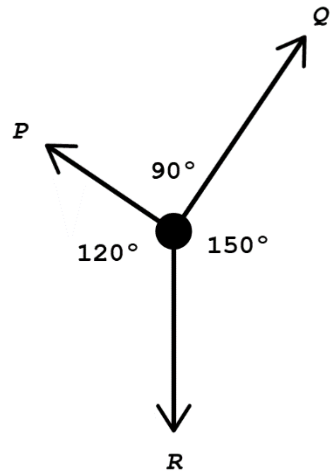
APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

Question 5.

(a) (i)



1 mark for each angle [AK]
[3 marks]

(ii) $Lami's\ theorem: \frac{P}{\sin 150^0} = \frac{Q}{\sin 120^0} = \frac{R}{\sin 90^0}$ **(SOI)**

$\frac{P}{\sin 150^0} = \frac{13}{\sin 90^0}$

$P \sin 90^0 = 13 \sin 150^0$

$P = \frac{13\ sin150^0}{\sin 90^0} = 6.5\ N$

$\frac{Q}{\sin 120^0} = \frac{13}{\sin 90^0}$

$Q \sin 90^0 = 13 \sin 120^0$

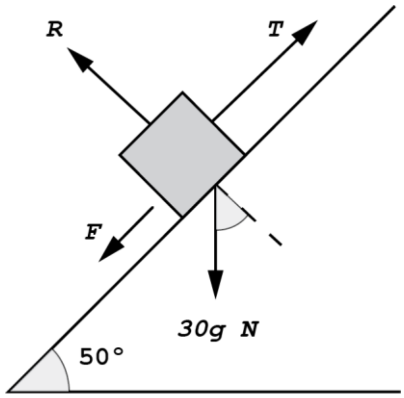
$Q = \frac{13\ sin120^0}{\sin 90^0} = 11.3\ N$

1 mark for stating Lami's (CK)
1 mark for substituting into Lami's Theorem (AK)
1 mark for calculating P (AK)
2 marks for calculating Q (2 AK)

[5 marks]

<u>Question 5. (continued)</u>

(b) (i)



[3 marks] [CK]

(ii) when the block is just about to slip up the plane the system is in equilibrium and the force of friction would be acting down the plane.

Resolving parallel to the plane:
$T = F + 30g\sin 50^0$
Resolving perpendicular to the plane:
$R = 30g\cos 50^0 = 192.8\ N$
The force of friction, $F = \mu R = 0.35 \times 192.8 = 67.5\ N$
So: $T = 67.5 + 229.8 = 297.3\ N$

1 mark for correctly resolving parallel to the plane (R)
1 mark for equation in terms of F and T (CK)
1 mark for correctly resolving perpendicular to the plane (R)
1 mark for correct F (AK)
1 mark for T (using his F) (AK)

(iii) when the block is just about to slip down the plane the system is in equilibrium and the force of friction would be acting up the plane. (SOI)

Resolving parallel to the plane:
$T = 30g\sin 50^0 - his\ F$

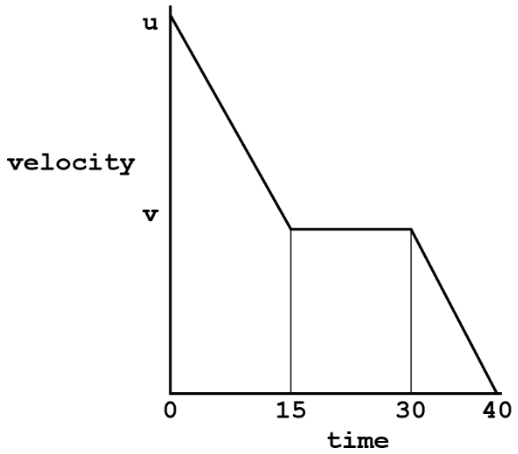
So: $T = 229.8 - 67.5 = 162.3\ N$

1 mark resolving parallel to the plane (R)
1 mark for substitution AK
1 mark for calculating T (AK)

[3 marks]

Question 5. (continued)

(c)



Both retardations are equal:

$$\frac{u-v}{15} = \frac{v-0}{10}$$

$$10u - 10v = 15v - 0$$
$$10u = 25v$$
$$v = 0.4u \quad \ldots\ldots(1)$$

1 mark for equating retardations (R)
1 mark for calculating *v* in terms of *u* (CK)

Total distance travelled is 250 m:

$$\frac{1}{2}(u+v)(15) + (v)(15) + \frac{1}{2}(10)(v) = 250$$

$$15u + 15v + 30v + 10v = 500$$
$$15u + 55v = 500 \ldots\ldots\ldots(2)$$

Substitute (1) into (2):

$$15u + 55(0.4u) = 500$$

$$37u = 500 \Rightarrow u = \frac{500}{37} = 13.5\ ms^{-1}$$

1 mark for calculating the total area (R)
1 mark for simplifying the equation (AK)
1 mark for substitution (AK)
1 mark for calculating the value of *u* (AK)

[6 marks]
TOTAL 25 marks

Question 6.

(a) (i) Impulse acting on B = change in momentum of B (SOI)

$$6000 = 1200(20) - 1200(u)$$

$$\Rightarrow u = \frac{1200(20) - 6000}{1200} = 15\ ms^{-1}$$

1 mark for formula for impulse (CK)
1 mark for substituting into formula for impulse (AK)
1 mark for calculating *u* (AK)

[3 marks]

(ii) Let the velocity of A after collision $= v\ ms^{-1}$
$impulse\ of\ A = -impulse\ on\ B$
$-6000 = 2000(v) - 2000(25)$
$\Rightarrow v = \frac{-6000 + 2000(25)}{2000} = 22\ ms^{-1}$

1 mark for relationship between impulses (R)
1 mark for substituting into formula for impulse (AK)
1 mark for calculating *v* (AK)

$loss\ in\ kinetic\ energy = initial\ K.E. - final\ K.E.$ (SOI)
$$= \frac{1}{2}(2000)(25^2) + \frac{1}{2}(1200)(15^2) - \frac{1}{2}(2000)(22^2) - \frac{1}{2}(1200)(20^2)$$

$$= 625000 + 135000 - 483000 - 240000 = 37000\ J$$

1 mark for formula for loss in K.E. (CK)
1 mark for calculating the loss in K.E. (AK)

[5 marks]

(iii) Momentum before collision = momentum after collision (SOI)
$1200(20) + 1500(0) = 1200(0) + 1500(x)$

$x = 16\ ms^{-1}$

1 mark for principle of conservation of linear momentum [CK]
1 mark for substituting [AK]
1 mark for correct answer [AK]
[3 marks]

APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

Question 6. (continued)

(b) (i) Horizontally: we use $s = ut + \frac{1}{2}at^2$

$750 = 95 \cos\alpha\, t \;\ldots..(1)$

Vertically: $v = u + at$

$-95 \sin\alpha = 95 \sin\alpha - 10t$

$t = 19 \sin\alpha \ldots\ldots(2)$

1 mark for stating equation (1) (AK)
1 mark for stating equation (2) (AK)

Substitute (2) into (1):

$750 = 95 \cos\alpha\,(19 \sin\alpha)$

$$\sin 2\alpha = \frac{1500}{1805} = 0.831$$

$2\alpha = \sin^{-1}(0.831) = 56.2^0, 123.8^0$

$\alpha = 28.1^0, 61.9^0$

1 mark for correct $\sin 2\alpha$ (CK)
1 mark for his values of α (R)

[4 marks]

(ii) Given that $\alpha = 30^0$,

calculate the times when the ball is 100 m above the ground.

Vertically: we use $s = ut + \frac{1}{2}at^2$

$$100 = 95 \sin 30^0\, t - \frac{1}{2}(10)t^2$$

$$5t^2 - 47.5t + 100 = 0$$

$$t = \frac{47.5 \pm \sqrt{47.5^2 - 4 \times 5 \times 100}}{2 \times 5}$$

$$= \frac{47.5 \pm \sqrt{2256.25 - 2000}}{10}$$

$$t = \frac{47.5 \pm \sqrt{256.25}}{10} = 6.35\, sec, 3.15\, sec$$

1 mark for using formula (CK)
1 mark for the correct quadratic equation in terms of t. (AK)
1 mark for his values of t (AK)

[3 marks]

Question 6. (continued)

(iii) Horizontally: we use $v = u + at$ and $s = ut + \frac{1}{2}at^2$

$v_1 = 95\cos 45^0 = \dfrac{95\sqrt{2}}{2} = 67.18\, ms^{-1}$

$750 = \dfrac{95\sqrt{2}}{2}t \Rightarrow t = \dfrac{150\sqrt{2}}{19}$

Vertically: $v = u + at$

$-v_2 = 95\sin 45^0 - 10t$

Substitute the value of t into above:

$-v_2 = 95\sin 45^0 - 10\left(\dfrac{150\sqrt{2}}{19}\right)$

$v_2 = 10\left(\dfrac{150\sqrt{2}}{19}\right) - \dfrac{95\sqrt{2}}{2} = 44.47\, ms^{-1}$

$Hence: v = \sqrt{\left(\dfrac{95\sqrt{2}}{2}\right)^2 + (44.47)^2} = \sqrt{6490.07} = 80.6\, ms^{-1}$

$direction\ of\ velocity = \tan^{-1}\left(\dfrac{44.47}{67.18}\right) = 33.5^0$

1 mark for calculating v_1 (R)
1 mark for calculating t (R)
1 mark for writing formula with negative v_2 (R)
1 mark for calculating v_2 (AK)
1 mark for formula for v (CK)
1 mark for calculating v (AK)

[7 marks]

TOTAL 25 marks

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

APPLIED MATHEMATICS

SPECIMEN

2022

TABLE OF SPECIFICATIONS

Paper 032

UNIT 2

Question	*Module*	*CK*	*AK*	*R*	*Total*
1	1	-	10	10	20
2	2	-	10	10	20
3	3	-	10	10	20
SUBTOTAL		**-**	**30**	**30**	**60**

TEST CODE **02205032-CASE**

SPECIMEN 2022

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 2 – Paper 032

ALTERNATIVE TO SCHOOL BASED ASSESSMENT

CASE STUDY FOR THE ALTERNATIVE TO SCHOOOL-BASED ASSESSEMENT EXAMINATION

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. The Paper 032 paper will consist of THREE questions based on your analysis of the THREE given case studies as follows:
   Question 1: DISCRETE MATHEMATICS
   Question 2: PROBABILITY AND DISTRIBUTIONS
   Question 3: PARTICLE MECHANICS

2. Examine the cases carefully to prepare for your examination.

N.B. Candidates are to receive this paper ONE week in advance of the date of the examination.

Module 1 – Discrete Mathematics

CASE STUDY

A store sells two types of laptops – A and B. The store owner pays $500 for each unit of laptop A and $800 for each unit of laptop B. The store owner has found over time, twice as many of laptop A as laptop B are sold. From past sales it is estimated that between 5 and 15 of laptop A and between 2 and 5 of laptop B may be sold each month. The plan is not to invest more than $10,000 in inventory of these laptops.

The store owner estimates that a profit of $200 can be made on the sale of one unit of laptop A while a unit of laptop B will yield a profit of $300. The store owner needs to be advised on the number of each type of laptop that should be sold to maximize the profit next month.

Module 2 – Probability and Distributions

CASE STUDY

The Ministry of Works wishes to investigate the number of car accidents in a particular stretch of highway. Police reports on the number of car accidents a day was collected for 100 weeks and tabularised below. A chi squared test was carried out at a 5 % level of significance.

Number of car accidents	0	1	2	3	4	5
Number of weeks	30	34	23	10	3	0

Module 3 – Particle Mechanics

CASE STUDY

A Form 4 physical education class is learning about golf and is interested in determining the optimum angle that a golfer needs to hit the golf ball in order to achieve the maximum possible horizontal displacement.

The experiment is carried out on a perfect day in which there are no clouds, and the wind speed is minimal. The mass of the ball is negligible and the speed at which the ball is hit is kept constant.

SPECIMEN 2022

TEST CODE **02205032**

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 2 – Paper 032

ALTERNATIVE TO SCHOOL-BASED ASSESSMENT

1 hour 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of THREE case studies and THREE questions as follows:

   Question 1: DISCRETE MATHEMATICS
   Question 2: PROBABILITY AND DISTRIBUTIONS
   Question 3: PARTICLE MECHANICS

2. ANSWER ALL questions.

3. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.

4. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra lined page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.

5. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

6. Do NOT write in the margins.

Examination Materials

Mathematical formulae and tables **(Revised 2022)**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

"*"Barcode Area"*"
Sequential Bar Code

NOTHING HAS BEEN OMITTED.

MODULE 1: DISCRETE MATHEMATICS

INSTRUCTION: Read the case and answer the questions that follow.

CASE STUDY

A store sells two types of laptops — A and B. The store owner pays \$500 for each unit of laptop A and \$800 for each unit of laptop B. The store owner has found over time, twice as many of laptop A as laptop B are sold. From past sales it is estimated that between 8 and 15 of laptop A and between 2 and 5 of laptop B may be sold each month. The plan is not to invest more than \$10,000 in inventory of these laptops.

The store owner estimates that a profit of \$200 can be made on the sale of one unit of laptop A while a unit of laptop B will yield a profit of \$300. The store owner needs to be advised on the number of each type of laptop that should be sold to maximize the profit next month.

1. (a) State ALL variables and constraints to be used in setting up the problem.

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[7 marks]

GO ON TO THE NEXT PAGE

(b) (i) On the grid provided **on page 5**, draw all the lines defined by the constraints and shade the feasible region which satisfies ALL constraints using a scale of 1 cm to represent 5 units on both axes. **[4 marks]**

(ii) State the objective function.

[1 mark]

(iii) State the optimal vertices of the graph.

[2 marks]

(iv) Calculate the solutions for each vertex.

[2 marks]

(v) Determine the optimal solution to the objective function.

[1 mark]

(c) (i) State ONE conclusion which can be made from the results of your analysis.

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[1 mark]

(ii) State ONE recommendation to improve the investigation.

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[2 marks]

Total 20 marks

NOTHING HAS BEEN OMITTED.

MODULE 2: PROBABILITY AND DISTRIBUTIONS

INSTRUCTION: Read the case and answer the questions that follow.

CASE STUDY

The Ministry of Works wishes to investigate the number of car accidents in a particular stretch of highway. Police reports on the number of car accidents a day was collected for 100 weeks and tabularised below. A chi squared test was carried out at a 5% level of significance.

Number of car accidents	0	1	2	3	4	5
Number of weeks	30	34	23	10	3	0

2. (a) Using examples from the case, outline how the given data meets THREE conditions of the Poisson distribution.

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[6 marks]

(b) (i) On the grid provided **on page 9**, construct a bar graph to present the data given.

[4 marks]

(ii) Calculate the expected value of the number of accidents correct to 1 decimal place.

[1 mark]

GO ON TO THE NEXT PAGE

(iv) Calculate the test statistic.

[4 marks]

(c) (i) State ONE valid conclusion based on the results of your calculation. Give ONE reason to support your conclusion.

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[2 marks]

(ii) State ONE limitation of the investigation. Give ONE reason to support your conclusion.

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[2 marks]

Total 20 marks

MODULE 3: PARTICLE MECHANICS

INSTRUCTION: Read the case and answer the questions that follow.

CASE STUDY

A Form 4 physical education class is learning about golf and is interested in determining the optimum angle that a golfer needs to hit the golf ball in order to achieve the maximum possible horizontal displacement.

The experiment is carried out on a perfect day in which there are no clouds, and the wind speed is minimal. The mass of the ball is negligible and the speed at which the ball is hit is kept constant.

3. (a) (i) Using examples from the case, outline how the information presented meets the TWO assumptions of projectile motion.

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[4 marks]

(ii) Identify ONE independent variable and ONE dependent variable.

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[2 marks]

(b) (i) Construct a suitable diagram to illustrate the situation described.

[3 marks]

(ii) Write an expression, in terms of t seconds, for the horizontal distance travelled by the ball.

[2 marks]

(iii) Write an equation of the vertical distance travelled.

[1 mark]

(iv) Determine the maximum horizontal range.

[4 marks]

(c) (i) State ONE conclusion which can be made from the results of your analysis. Give ONE reason to support your conclusion.

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[2 marks]

(ii) State TWO recommendations for improving the investigation.

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[2 marks]

Total 20 marks

END OF TEST

APPLIED MATHEMATICS

UNIT 2 - Paper 032

KEY AND MARK SCHEME

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 2 – Paper 032

KEY AND MARK SCHEME

SPECIMEN 2022

APPLIED MATHEMATICS

UNIT 2 - Paper 032

KEY AND MARK SCHEME

Question 1

(a) **Variables and Constraints**

x = Number of laptop A and y = Number of laptop B **(1 mark)**

$500x + 800y \leq 10000$ money to be invested (**1 mark)**

$x \geq 2y$ ratio of stock for each type of laptop **(1 mark)**

$8 \leq x \leq 15$ number of laptops **(1 mark)**

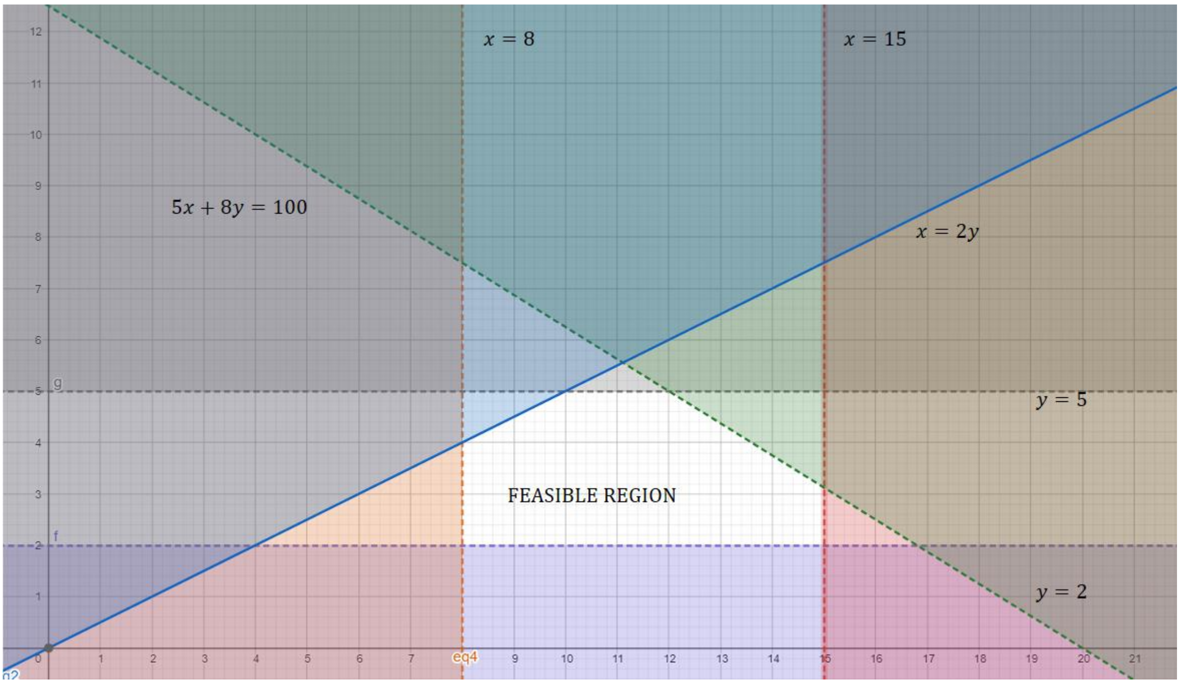
$2 \leq y \leq 5$ number of laptops B **(1 mark)**

$x, y \geq 0$ **(1 mark)**

x, y are integers **(1 mark)**

[7 marks; R]

(b) (i) **Graph**



Sketching $5x + 8y = 100$ **(1 mark)**

Sketching $x = 2y$ **(1 mark)**

Sketching the lines $x = 8$ and $x = 15$

Sketching the lines $y = 2$ and $y = 5$

(1 mark for sketching all four lines)

(1 mark for shading the correct feasible region)

[4 marks; AK]

Question 1 continued

(ii) **Objective Function**

Maximise: $P = 200x + 300y$

[1 mark; AK]

(iii) **Optimal Vertices**

(8, 2), (8, 4) (10, 5), (12, 5), (15, 2), (15, 3)

1 mark for every 3 correct values

[2 marks; AK]

(iv) **Solutions for each vertex**

200(8) + 300(2) = 2200

200(8) + 300(3) = 2500

200(10) + 300(5) = 3500

200(12) + 300(5) = 3000

200(15) + 300(2) = 3600

200(15) + 300(3) = 3900

1 mark for every 3 correct values

[2 marks; AK]

(V) **Optimal Solution**

Maximum value X = 15, Y = 3

[1 mark; AK]

(c) (i) **Conclusions**

The maximum profit is $ 3900

[1 mark; R]

(ii) **Recommendations**

A more in-depth study of the financials to determine other factors that contribute to the profit obtained can provide a more accurate estimate of the profit.

[2 marks; R]

Total 20 marks

APPLIED MATHEMATICS

UNIT 2 - Paper 032

KEY AND MARK SCHEME

Question 2

(a) **Conditions of Poisson**

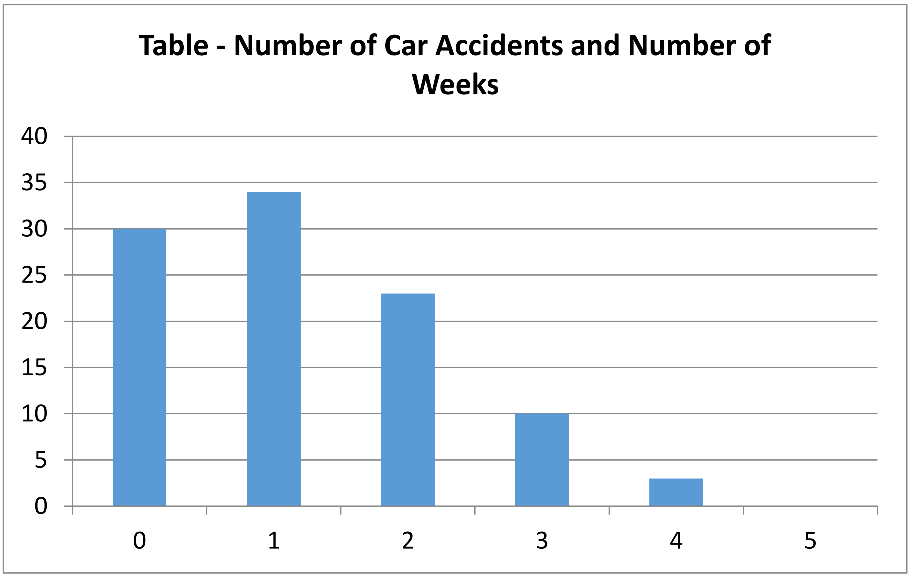
Car accidents are randomly occurring **[2 mark]**

Car accidents are independent events. **[2 mark]**

The data collected can be modelled as the average number of occurrences in a specified time. **[2 mark]**

[6 marks; R]

(b) (i) **Graph**



[4 marks; AK]

(ii) **Critical Value**

$$E(X) = \frac{\Sigma fx}{\Sigma f}$$ **[1 mark]**

$$= \frac{122}{100} = 1.2$$ **[1 mark]**

[2 marks; AK]

Question 2 continued

(iii)

Number of car accidents	0	1	2	3	4	5
Observed Frequencies	30	34	23	10	3	0
Expected Frequencies	30.1	36.1	21.7	8.7	2.6	0.8

1 mark for at least 3 correct expected frequencies

Combining expected frequencies since $E_i < 5$ **[1 mark]**

O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$
30	30.1	0.0003
34	36.1	0.122
23	21.7	0.0779
10	8.7	0.194
3	3.4	0.047

[1 mark] for all values correct

$$\chi^2_{calc} = \sum \frac{(O_i - E_i)^2}{E_i}$$

$= 0.4412$ **[1 mark]**

[4 marks; AK]

(c) (i) **Conclusions**

do not reject H_0 **[1 mark]**

Since $\chi^2_{calc} = 1.11 < 7.815 = \chi^2_{0.05}(3)$, **[1 mark]** conclude that the data does not follow a Poisson Distribution.

[2 marks; R]

(ii) **Limitations**

One limitation is that the data collected may not have been accurate **[1 mark]** as not all accidents that may have occurred would

[2 marks; R]

Total 20 marks

APPLIED MATHEMATICS

UNIT 2 - Paper 032

KEY AND MARK SCHEME

Question 3

(a) (i) **Assumptions of Projectile Motion**

The mass of the basketball in this study is **negligible**, hence the basketball can be modelled as a **particle. 2 marks**

The experiment is conducted on a cloudless day and wind speed is at a minimum thus reducing **the effects of air resistance. 2 marks**

[4 marks; R]

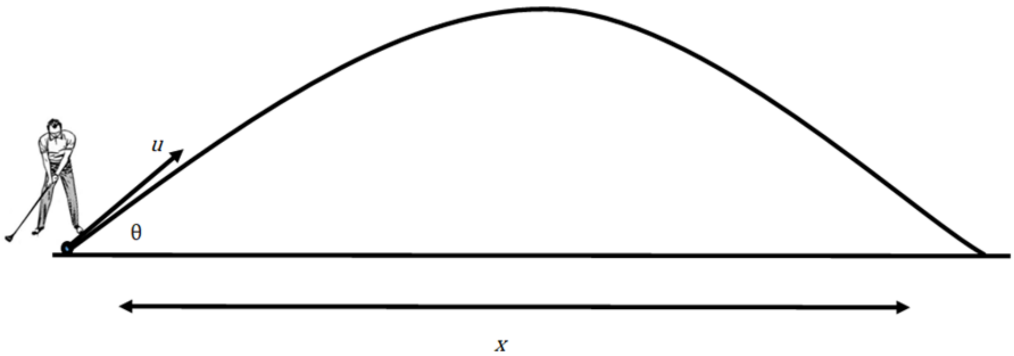
(ii) **Variables.**

Independent – angle of trajectory

Dependent – horizontal range of the ball

[2 marks; R]

(b) (i) **Diagram.**



1 mark for indicating the horizontal distance.

1 mark for indicating u and θ.

1 mark for trajectory of the golf ball.

[3 marks; AK]

(ii) **Expression for Horizontal distance travelled**

$x = (u\cos\theta)t$ **1 mark for** $(u\cos\theta)$ and **1 mark for multiplying by** t

[2 marks; AK]

(iii) **Vertical Distance**

$$y = (u\sin\theta)t - \frac{1}{2}gt^2$$

[1 mark; AK]

Question 3 continued

(iv) **Maximum horizontal range.**

When $y = 0$

$0 = (u\sin\theta)t - \frac{1}{2}gt^2$

$0 = t\left(usin\,\theta - \frac{1}{2}gt\right)$

$t = 0$ or $t = \frac{2u\sin\theta}{g}$

1 mark for obtaining the values of t

Substituting $t = \frac{2u\sin\theta}{g}$ into (i)

1 mark for substituting $t = \frac{2u\sin\theta}{g}$ into (i)

$x = (u\cos\theta)\left(\frac{2u\sin\theta}{g}\right)$

$= \frac{u^2\sin 2\theta}{g}$

1 mark for obtaining $\frac{u^2\sin 2\theta}{g}$

Maximum horizontal range $= \frac{u^2}{g}$

1 mark for $R = \frac{u^2}{g}$

[4 marks; AK]

(c) (i) **Conclusions**

The optimum angle to achieve the maximum horizontal displacement (range) is 45^0. **[1 mark]** For the range to be a maximum, $\sin 2\theta$ has a to be a maximum. The maximum value of $\sin 2\theta = 1$ and this occurs when $\theta = 45^0$. **[1 mark]**

[2 marks; R]

(ii) Recommendations

- A speed gun should be used ensure that every time the ball is hit, the speed remains the same. **[1 mark]**
- An indoor golf simulator can be used to get a more accurate determination of the angle of projection and horizontal displacement. Also, this will reduce the cost and time to conduct the experiment. **[1 mark]**

[2 marks; R]

Total 20 marks

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CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

APPLIED MATHEMATICS
(Including Statistical Analysis)

INTEGRATED MATHEMATICS

Statistical Tables

and

List of Formulae

Revised April 2022

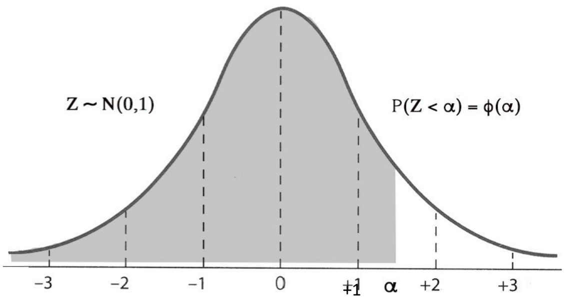
DO NOT REMOVE FROM THE EXAMINATION ROOM

Table 1: The Normal Distribution Function

If Z is a random variable, normally distributed with zero mean and unit variance, then ϕ (z) is the probability that $Z \leq z$. That is, $\phi(z) = P(Z \leq z)$.

The function tabulated below is $\phi(z)$, and is shown diagrammatically as

Standard Normal Distribution (area to the left of α)

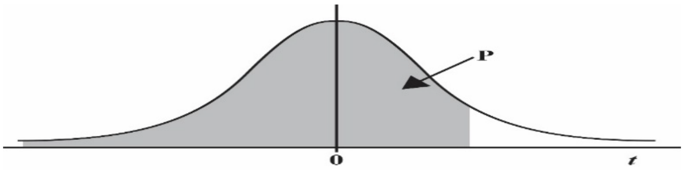


The Distribution Function, $\phi(z)$

Z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5 ADD	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6590	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0 7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9046	0.9148	0.9429	0.9441	1	2	4	1	2	4	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	1	2	3	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	1	2	3	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	1	1	2	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9760	0.9767	1	1	2	1	1	2	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	0	1	1	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	0	1	1	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	0	1	1	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	0	1	1	2	2	2
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	0	0	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Table 2: <u>*t*-Distribution</u>

If *T* has a *t*-distribution with *v* degrees of freedom then, for each pair of values of *p* and *v*, the table gives the value of *t* such that P(T ≤ *t*) = P

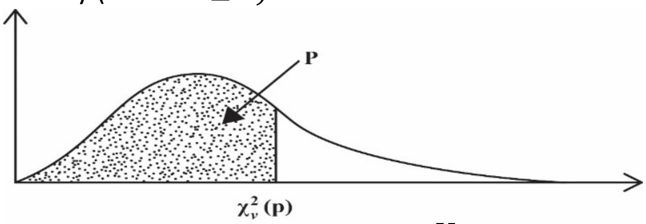


Critical Values for the *t*-distribution

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
V = 1	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	4.317	5.208	5.959
12	0.695	1.356	1.782	2.179	2.681	3.055	4.029	4.785	5.408
13	0.694	1.350	1.771	2.160	2.650	3.012	3.833	4.501	5.041
14	0.692	1.345	1.761	2.145	2.624	2.977	3.690	4.297	4.781
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Table 3: Percentage Points of the x^2 Distribution

If X is a random variable, distributed as X^2 with v degrees of freedom then p is the probability that $X \leq \chi_v^2$ (p), where the values of the percentage points χ_v^2 (p), are tabulated in the table below. p is shown diagrammatically (when $v \geq 3$) as



Critical Values for the X ²- distribution

P	.01	.025	.050	.900	.950	.975	. 990	.995	.999
v = 1	0.0001571	0.0009821	0.003932	2.706	3.841	5.024	6.635	7.879	10.83
2	0.02010	0.05064	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2971	0.4844	0.7107	7 779	9.488	11.14	13.28	14 86	18.47
5	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.52
6	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.646	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	3.053	3.816	4.575	17.28	19.68	21.92	24.72	26.76	31.26
12	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	8.260	9.591	10.85	28 41	31.41	34.17	37.57	40.00	45.31
21	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
30	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	45.44	48.76	51.74	85.53	90.53	95.02	104.4	104.2	112.3
80	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	61.76	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

Table 4: Random Sampling Numbers

18	11	36	26	88	81	11	33	64	08	23	32	00	73	04
57	33	88	37	26	10	79	91	36	03	07	52	55	84	61
72	02	11	44	25	45	92	12	82	94	35	35	91	65	78
89	83	98	71	74	22	05	29	17	37	45	65	35	54	44
44	88	03	81	30	61	00	63	42	46	22	89	41	54	47
68	60	92	99	60	97	53	55	34	01	43	40	77	90	19
87	63	49	22	47	21	76	13	39	25	89	91	38	25	19
44	33	11	36	72	21	40	90	76	95	10	14	86	03	17
60	30	10	46	44	34	19	56	00	83	20	53	53	65	29
03	47	55	23	26	90	02	12	02	62	51	52	70	68	13
09	24	34	42	00	68	72	10	71	37	30	72	97	57	56
09	29	82	76	50	97	95	53	50	18	40	89	40	83	29
52	23	08	25	21	22	53	26	15	87	93	73	25	95	70
43	78	19	88	85	56	67	56	67	16	68	26	95	99	64
45 0	69	72	62	11	12	18	25	00	92	26	82	64	3	
21	72	97	04	52	62	09	54	35	17	22	73	35	72	53
65	95	48	55	12	46	89	95	61	31	77	14	24	14	41
51	69	76	00	20	92	58	21	24	33	74	08	66	90	61
89	56	83	39	58	22	09	01	14	04	14	97	56	92	97
72	63	40	03	07	02	62	20	11	50	11	98	23	80	99

FORMULAE

PURE MATHEMATICS

For the quadratic equation: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For an arithmetic series:

$$u_n = a + (n - 1)d, \qquad S_n = \frac{n}{2}\{2a + (n - 1)d\}$$

For a geometric series:

$$u_n = a r^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1},\ r > 1, \qquad S_n = \frac{a(1 - r^n)}{1 - r},\ r < 1, \qquad S_\infty = \frac{a}{1 - r},\ |r| < 1$$

Binomial expansion:

$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 +, \ \ldots\ + b^n$, where n is a positive integer.

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(r-1)!}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots r}x^r + \ldots \text{ where } n \text{ is a real number and } |x| <|$$

Summations:

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n + 1). \qquad \sum_{r=1}^{n} r^2 = \frac{1}{6}n(n + 1)(2n+1). \qquad \sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n + 1)^2$$

Complex numbers:

$z^n = (\cos x + i \sin x)^n = \cos nx + i \sin nx$, where n is an integer and x is real

$e^{ix} = \cos x + i \sin x$ where x is real

$[r(\cos x + i \sin x)]^n = r^n(\cos nx + i \sin nx)$

Maclaurin's series:

$$e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \quad \text{for all real } x$$

$$\text{In}\,(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r!} + . \quad (-1 < x \leq 1)$$

$$\ln\,(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots - \frac{x^r}{r!} - \ldots \qquad (-1 \leq x < 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \quad \text{for all real } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + . \quad \text{for all real } x$$

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \ldots + \frac{x^r}{r!} f^r(0) + \ldots$$

Taylor's series:

$$f(x) = f(a) + f'(a)\frac{(x-a)}{1!} + f''(a)\frac{(x-a)^2}{2!} + f'''(a)\frac{(x-a)^3}{3!} + + f^r(a)\frac{(x-a)^r}{r!} + \ldots$$

The trapezium rule $\int_a^b y\,\mathrm{d}x = \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + .. + y_n - 1)\}$,

$h = \frac{b-a}{n}$, where n is the number of intervals (strips)

The Newton-Raphson iteration $x_{n+1} = x_n - \frac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

TRIGONOMETRY

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Arc length of a circle: $s = r\theta$, (θ measured in radians)

Area of a sector of a circle: $\quad \text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B$

$$\text{If } \tan\frac{a}{2} = t \text{, then } \sin\alpha = \frac{2t}{1+t^2} \text{ and } \cos\alpha = \frac{1-\mathrm{t}^2}{1+\mathrm{t}^2}$$

Trigonometric Identities:

$$cos^2\,\alpha + sin^2\,\alpha \equiv 1,\; 1 + tan^2\,a = sec^2 a, 1 + cot^2 a = cosec^2 a$$

$$\sin(\alpha \pm \beta) \equiv \sin\alpha\cos\beta \pm \cos\alpha\ \sin\beta$$

$$\cos(\alpha \pm \beta) \equiv \cos\alpha\cos\beta\ \mathrm{m}\sin\alpha\ \sin\beta$$

$$\tan(\alpha \pm \beta) \equiv \frac{\tan\alpha \pm \tan\beta}{1\,\mathrm{m}\tan\alpha\tan\beta} \qquad \alpha \pm \beta \neq (k + \frac{1}{2})\pi$$

$$cos\,2\alpha = cos^2\,\alpha - sin^2\,\alpha = 2\,cos^2\,\alpha - 1 = 1 - 2\,sin^2\,\alpha$$

$$sin\,2\alpha = 2\,sin\,\alpha\,cos\,\alpha$$

$$\sin\alpha + \sin\beta \equiv 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta \equiv 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta \equiv 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta \equiv 2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2} \quad \text{or} \quad -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

STATISTICS

Frequency distributions

$$Mean\ \overline{x} = \frac{\Sigma fx}{\Sigma f}$$

$$Standard\ Deviation\ \sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2} = \sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sum_{i=1}^{n}\frac{(x_i-\bar{x})^2}{n}$$

Median $Q_2 = \left(\frac{n+1}{2}\right)^{th}\ value$

Grouped data

Mean $(\bar{x}) = \frac{\sum fx}{\sum f}$ *where* x *=midpoint of each class, f is the frequency of each class.*

Median $= l + \left(\frac{\frac{N}{2} - f_0}{f_1}\right) w$ *where,*

l= lower limit of the median class

N = total frequency

f_0 = frequency of class preceding the median class

f_1 = frequency of median class

w = width of median class

Mode = $l + \left(\frac{f_1 - f_2}{2f_1 - f_0 - f_2}\right) w$

Where ,

l = lower limit of the modal class

f_1 = frequency of the modal class

f_0 = frequency of the class preceding the modal class

f_2 = frequency of the class succeeding the modal class

w = width of the modal class

Measures of spread or dispersion

Standard deviation for the population

s.d = $\sqrt{\frac{\sum fx^2 - \frac{(\sum fx)^2}{n}}{n}}$

where x = midpoint of each class

f = the frequency of each class

n = population size.

unbiased estimator of the variance of X is $\hat{\sigma}^2 = \frac{n}{n-1} \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

Product Moment Correlation Coefficient, r

$$r = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{\left[n\sum_{i=1}^{n} x_i{}^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right]\left[n\sum_{i=1}^{n} y_i{}^2 - \left(\sum_{i=1}^{n} y_i\right)^2\right]}}$$

Covariance Formula $= \frac{S_{x_i y_i}}{S_{x_i} S_{y_i}}$ where $S_{x_i y_i}$ is the co-variance of x and y,

$S_{x_i} S_{y_i}$ is the product of the standard deviation of x and y respectively

Regression line y on x

$y = a + bx$ passing through $(\overline{x}, \overline{y})$ where

$$\overline{x} = \frac{\sum x}{n} \text{ and } \overline{y} = \frac{\sum y}{n}$$

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$b = \frac{S_{xy}}{S_{xx}}$$,where S_{xx} is the variance of x.

$$a = \overline{y} - b\overline{x}$$

MECHANICS

Uniformly accelerated motion

$$v = u + at, \quad s = \frac{1}{2}(u + v)t, \quad s = ut + \frac{1}{2}at^2, \quad v^2 = u^2 + 2as$$

Motion of a projectile

Equation of trajectory is:

$$y = x \tan\theta - \frac{gx^2}{2V^2 \cos^2\theta}$$

$$= x \tan\theta - \frac{gx^2}{2V^2}(1 + \tan^2\theta)$$

Time of flight = $\frac{2V\sin\theta}{g}$

Greatest height = $\dfrac{V^2 \sin^2 \theta}{2g}$

Horizontal range = $\dfrac{V^2 \sin 2\theta}{g}$, maximum range = $\dfrac{V^2}{g}$ for $\theta = \dfrac{\pi}{4}$

Lami's Theorem

$\dfrac{F_1}{\sin\alpha} = \dfrac{F_2}{\sin\beta} = \dfrac{F_3}{\sin\gamma}$, where F_1, F_2, F_3 are forces acting on a particle

and α, β, γ are the angles vertically opposite F_1, F_2, F_3, respectively