REPUBLIC OF TRINIDAD AND TOBAGO
MINISTRY OF EDUCATION

Secondary Education Modernization Programme

SECONDARY SCHOOL CURRICULUM

Forms 1–3

Mathematics

Curriculum Planning and Development Division, Ministry of Education
September 2008

Published in 2008 by the
Curriculum Planning and Development Division
Ministry of Education
Rudranath Capildeo Learning Resource Centre
Mc Bean, Couva
Republic of Trinidad and Tobago

ISBN:

Printed by

Table of Contents

Minister's Foreword iii
A Note to Teachers v
Acknowledgements vii

Part 1

The National Curriculum for Forms 1–3

Part 2

Curriculum Layout and Subject Modules

Part 3

Instructional Strategies/Methods

Part 4

Assessment and Evaluation

Part 5

Glossary of Mathematical Terms

Minister's Foreword

The Government of Trinidad and Tobago, in its *Vision 2020 Draft National Strategic Plan*, has articulated a vision of "a united, resilient, productive, innovative, and prosperous nation with a disciplined, caring, fun-loving society comprising healthy, happy and well-educated people and built on the enduring attributes of self reliance, respect, tolerance, equity and integrity" (p. 9). Five developmental pillars have been identified to achieve this goal:

Developing Innovative People
Nurturing a Caring Society
Governing Effectively
Enabling Competitive Business
Investing in Sound Infrastructure and Environment

The Ministry of Education is one Ministry that is expected to play a pivotal role in *developing innovative people.* We therefore accept as one of our primary responsibilities, the establishment of an education system that will nurture imaginative, innovative, and eager learners. It must also facilitate the seamless progression of learners from early childhood education up to the tertiary level. Graduates of the system must emerge as creative, committed, and enterprising citizens who are prepared intellectually, and who have the will to become global leaders.

A critical contributor to this process is the national curriculum. These Curriculum Guides represent the core subjects of the national curriculum at the lower secondary level. They describe the formal content and process by which students at this level will gain the knowledge and skills that contribute to the achievement of our national goals. We expect that teachers will use these Guides to implement a school curriculum that is diversified, relevant, and of high quality, meeting the varied learning needs, interests, and abilities of all students. We expect, too, that students will be taught in ways that suit their own learning preferences. The curriculum will also connect them to their national heritage, help them to understand the issues facing their world today, and prepare them to meet the challenges and opportunities of the future.

On behalf of the entire education community, I congratulate and thank all those educators—curriculum personnel, teachers, editors, and others—who have worked together over the eight years of development and revision to produce these Curriculum Guides for secondary schools. The nation owes you a debt of gratitude. I urge you to continue to be shining lights in your communities as we move forward together to achieve our goals.

Esther Le Gendre
Honourable Minister of Education

A Note to Teachers

These Curriculum Guides have been developed by educators, including practising teachers, for teachers. They are intended to assist you to prepare students to meet the rapidly changing demands of life in the 21st century, while ensuring that they acquire the core of general knowledge and experience essential for later education and employment. The new curriculum that they represent is designed to guide the adoption of a more student-centred approach to instruction, and the provision of learning opportunities that are relevant to today's students and inclusive of varied learning needs and interests.

Since the beginning of the curriculum development process, we have seen profound changes in the use of technology in education and there is no doubt that similar shifts will take place in the coming years. The challenge for us as educators is to find ways to make our approach to teaching flexible, progressive, and responsive, so that we embrace and motivate change where it benefits learners. This entails becoming lifelong learners ourselves and creating environments that provide necessary community support and foster professional development.

The Guides embody the culmination of seven years of development and revision activity. The national curriculum will, however, be regularly reviewed to ensure that it continues to meet the needs of all students and matches the goals of society. Your input in this process is vital and we welcome and encourage your ongoing feedback.

Instructional decisions must be based on sound, contemporary educational theory, practice, and research. These documents will serve as important guides for the development of instructional programmes to be implemented at the school and classroom levels. They are organized in several parts. Part 1 is common to all and provides the general philosophy and aims in which every subject is anchored. Part 2 is specific to each subject and includes specific outcomes and sample activities and strategies that may be used to achieve them. The rest of the document is designed to suit the particular needs of each subject area. All the Guides include suggested assessment strategies and recommended resources.

We in the Curriculum Planning and Development Division are confident that the new National Curriculum Guides for Forms 1–3 will contribute significantly to enhanced teaching and learning experiences in our secondary schools and, consequently, the achievement of personal learning and national educational goals.

Sharon Douglass Mangroo
Director of Curriculum Development
August 2008

Acknowledgements

The Ministry of Education wishes to express its sincere appreciation to all those who contributed to the curriculum development and revision processes from 2000 to the present.

- The Inter-American Development Bank (IDB) and its staff
- The staff of the Coordinating Unit of the Secondary Education Modernization Programme (SEMPCU), past and present, provided technical assistance and planned, organized, and conducted the various exercises over the years. They include Mr. Maurice Chin Aleong, Mr. Lloyd Pujadas, Mr. Arnott West, Dr. Stephen Joseph, Ms. Patricia Sealey, Ms. Renee Figuera, and Ms. Roslyn Elias.
- Mrs Sharon Douglass Mangroo, Director of Curriculum Development, led the curriculum development sub-component and coordinated the curriculum development and revision activities.
- Dr. Robert Sargent guided the early curriculum development process.
- The principals of the pilot schools generously contributed teachers and participated in regular meetings to provide valuable feedback on field tests.
- The principals of non-pilot schools kindly released teachers to take part in writing activities.
- The staff of the School Libraries Division actively joined in workshops, facilitated research, and contributed to the infusion of information technology into the curriculum.
- Editors, past and present: Ms. Avril Ross, Ms. Lynda Quamina-Aiyejina, and Ms. Patricia Worrell devoted time, energy, and knowledge to editing the several versions of the documents.
- The Administrative staff of the Curriculum Development Division spent long hours typing and retyping the documents.
- Officers of the Divisions of Educational Services, Schools Supervision, Student Support Services, and Educational Research and Evaluation provided support as needed.
- Teachers throughout the secondary school system responded to requests for comments and other forms of feedback.
- The Curriculum Officers and members of the Curriculum Writing Teams brought their knowledge, skills and practical experiences of teaching and learning to the curriculum development workshops and skilfully synthesized all to produce these documents.

Members of the Original Curriculum Writing Team

Name	School/Office
1. Nazira Ali	Union/Claxton Bay Senior Comprehensive School
2. Valerie Ann Gilbert	Mt. Hope Junior Secondary School
3. Reginald Roberts	Belmont Junior Secondary School
4. Rita Watson-Steele	Tunapuna Government Secondary School
5. Yvonne Ramsey	Curriculum Officer
6. John Roopchan	Curriculum Officer

Members of the Curriculum Review Team

Name	School/Office
1. Patrick Alexander	Belmont Boys' Secondary School
2. Ishla Ali	ASJA Girls' College, Charlieville
3. Jane Balkaran-Siberan	Barrackpore Secondary Comprehensive School
4. Den'a Boynes-Bardouille	St. Anthony's College
5. Indra Dass	Pleasantville Senior Comprehensive School
6. Benedict Mathurasingh	Chaguanas Junior Secondary School
7. Sandra Smith	Holy Faith Convent, Couva
8. Kenneth Baisden	Curriculum Officer
9. Vimala Kamalodeen	Curriculum Officer
10. Shereen Khan	Curriculum Officer
11. Don Naranjit	Curriculum Officer
12. John Roopchan	Curriculum Officer

1

THE NATIONAL CURRICULUM FOR FORMS 1—3

Table of Contents

PART 1: THE NATIONAL CURRICULUM FOR FORMS 1–3

Background 1–1

The Curriculum Design and Development Process 1–3

Curriculum Underpinnings 1–5

Education Policies That Impact on the Curriculum 1–6

Essential Learning Outcomes 1–8

The Core Curriculum Subjects 1–12

Language Across the Curriculum 1–13

Curriculum Implementation 1–14

References 1–17

Background

From the Ministry of Education's *Corporate Plan 2008–2012* (p. 4)

The Government of Trinidad and Tobago, in its *Vision 2020 Draft National Strategic Plan*, has articulated a vision of "a united, resilient, productive, innovative, and prosperous nation with a disciplined, caring, fun-loving society comprising healthy, happy and well-educated people and built on the enduring attributes of self reliance, respect, tolerance, equity and integrity…"

Towards the achievement of this Vision, the Government has articulated five developmental pillars:

Developing Innovative People
Nurturing a Caring Society
Governing Effectively
Enabling Competitive Business
Investing in Sound Infrastructure and Environment

The Ministry of Education has been identified as one of the champions for *developing innovative people.* Central to the realization of this pillar is "A highly skilled, well-educated people aspiring to a local culture of excellence that is driven by equal access to learning opportunities."

In conjunction with other key ministries, the Ministry of Education has been charged with the realization of the following goals:

> The people of Trinidad and Tobago will be well known for excellence in innovation.
>
> Trinidad and Tobago will have a seamless, self-renewing, high-quality education system.
>
> A highly skilled, talented and knowledgeable workforce will stimulate innovation driven growth and development.
>
> The richness of our diverse culture will serve as a powerful engine to inspire innovation and creativity.

…Nationally, the reform of the education system is driven by several local, regional and international perspectives. We are committed to a seamless, self-renewing, high-quality education system underpinned by a National Model for Education. This National Model has three (3) foci as follows:

i. To ensure an alignment of the education system to government's strategic plan Vision 2020 which mandates that the education system produces caring and innovative citizens

ii. To ensure that the education system produces citizens with a sense of democracy, respect for the rights of others and elders and with the ability to contribute meaningfully to the social and economic development of the country
iii. To build a strong sense of nationalism and patriotism in our citizens. (p. 7)

The Secondary Curriculum

In its commitment to comprehensive reform and expansion of the secondary school system, the Government of the Republic of Trinidad and Tobago, in 1996, adopted the report of the National Task Force on Education as educational policy. The specific recommendations for the improvement of secondary education led to discussions with the Inter-American Development Bank (IDB) for loan funding arrangements for a programme, the Secondary Education Modernization Programme (SEMP), to modernize secondary education in Trinidad and Tobago. One of the intended outcomes of this programme was improved educational equity and quality.

The curriculum guides for Forms 1–3 in eight subject areas are among the products of the programme and contribute to this outcome.

The Curriculum Design and Development Process

In order to achieve the outcomes defined by the underpinning philosophy and goals, the Curriculum Development Division of the Ministry of Education embarked on a design and development programme consonant with accepted approaches to curriculum change and innovation.

Curriculum Design

This curriculum displays a learner-centred design. Its philosophical assumptions are mainly constructivist. Its major orientation is to curriculum as self-actualization. The curriculum is student-centred and growth oriented. It seeks to provide personally satisfying experiences for each student. As the student moves from one level to another, activities also expand to allow new insights and approaches to dealing with and integrating new knowledge.

Curriculum Development

The first stage of the curriculum development process consisted of consultations with stakeholders from a cross-section of the national community. Consultations were held with primary and secondary school teachers; principals; members of denominational school boards; members of the business community; the executive of the Trinidad and Tobago Unified Teachers' Association (TTUTA); representatives from The University of the West Indies (UWI), John S. Donaldson Technical Institute, San Fernando Technical Institute, Valsayn Teachers' College, and Caribbean Union College; parents; librarians; guidance counsellors; students; curriculum officers; and school supervisors. These consultations focussed on the philosophy, goals, and learning outcomes of education.

The result of these consultations was agreement on:

- the concept of a "core," that is, essential learning outcomes consisting of skills, knowledge, attitudes, and values that students must acquire at the end of five years of secondary schooling;
- the eight subjects to form the core;
- the desirable outcomes of secondary school education in Trinidad and Tobago.

In Stage 2 of the process, the officers of the Curriculum Development Division studied the reports of the consultations, the Education Policy Paper, the reports of the Curriculum Task Force and the Task Force for Removal of Common Entrance, as well as newspaper articles and letters to the editor on education during the preceding five years. The School Libraries Division and the Division of School Supervision assisted the Curriculum Development Division in this task. The result of the study was the identification and articulation of a set of desirable outcomes and essential exit competencies to be possessed

by all students on leaving school. All learning opportunities, all teaching and learning strategies, and all instructional plans are to contribute to the realization of these outcomes and competencies.

At Stage 3, 10 existing schools were identified to pilot the new curriculum. Teachers from eight subject areas were drawn from these schools to form curriculum writing teams for each subject. Teachers with specific subject or curriculum development skills from other schools were also included in the teams. The outputs of this phase included learning outcomes specific to each subject that contribute to the fulfilment of the national outcomes; subject content; and teaching, learning, and assessment strategies to support the outcomes.

The draft curriculum guides for Forms 1 and 2 were approved by Cabinet for introduction into schools on a phased basis in September 2003. The draft guides for Form 3 were completed and introduced in the following year. Introduction of the new guides was accompanied by professional development and training for principals and teachers. The Ministry also began to supply new and/or upgraded facilities for teaching and learning, and educational technology. At the same time, work began on a new assessment and certification system.

Curriculum Revision

As implementation proceeded, feedback was sought by the Curriculum Development Division through school visits, workshops, and reviews by UWI lecturers and other stakeholders. In 2007, a survey was conducted among teachers, followed by focus group meetings, in order to concretize feedback before embarking on the revision process. As in the original curriculum development exercise, revision—the final stage—was carried out by teams of practising teachers led by officers of the Curriculum Development Division.

Curriculum Underpinnings

The national curriculum has been informed by a wealth of available curriculum theories and processes.

The major forces that influence and shape the organization and content of the curriculum include:

1. Educational philosophy and understandings about the nature of knowledge
2. Society and culture
3. The learner and learning process
4. Learning theories
5. The nature and structure of subject matter to be learned

Thus, these areas represent the foundation on which the national curriculum is built. The philosophical concerns and educational goals that shaped the curriculum also formed the basis for the dialogue with stakeholders in which the Curriculum Development Division engaged, with the aim of developing a coherent, culturally focussed, and dynamically evolving curriculum.

An internal analysis of the education system, together with research conducted in international forums, has shown that the curriculum is core to the development of innovative people. This curriculum is aimed at attaining six essential learning outcomes. The six outcomes identified help to define universally accepted goals that have been developed and underscored by other educational jurisdictions and that have been agreed to be essential. The essential learning outcomes help to define standards of attainment for all secondary school students.

Education Policies That Impact on the Curriculum

There are several Ministry of Education policies that impact on the national secondary curriculum, though some are still in the process of formalization. These include the National Model for Primary and Secondary Education in Trinidad and Tobago, the ICT policy, Standards for the Operation of Schools, and Quality Standards. Copies of these documents may be obtained from the Ministry offices or the website at *www.moe.gov.tt*. Three other policies that have direct impact on the development and implementation of the curriculum are discussed in some detail below.

National Curriculum Policy

A Draft National Curriculum Policy has been approved by Cabinet for consultation with stakeholders. The Policy statements are summarized as follows:

1. The curriculum must articulate with the goals of national development and be supportive of the aspirations of individuals and their personal development. It must provide opportunities for every student to be equipped with the knowledge, skills, attitudes, values, and dispositions necessary for functioning in an interactive, interdependent society.
2. The curriculum must be so managed as to ensure the provision of a quality curriculum experience for all students at all levels of the system.
3. At every level of the system, there must be equitable provision of requisite facilities, resources, services, and organizational structures that are conducive to and supportive of effective learning and teaching and healthy development.
4. Continuous quality management must support all curriculum and related activities at every level of the system.
5. Ongoing research and professional development activities must equip education practitioners for continued effective practice.

Though the policy has not yet been formally issued, these statements are worthy of consideration at all stages of the curriculum cycle.

Inclusive Education Policy

The Ministry of Education is committed to "support the delivery of inclusive education in all schools by providing support and services to all learners, and by taking appropriate steps to make education available, accessible, acceptable and adaptable to all learners." An inclusive curriculum is acknowledged to be the most important factor in achieving inclusive education. In planning and teaching the school curriculum, teachers are therefore required to give due regard to the following principles:

- The National Curriculum Guides set out what most students should be taught at lower secondary school but teachers should teach the required knowledge and skills in ways that suit students' interests and abilities. This means exercising flexibility and drawing from curricula for earlier or later class levels to provide learning opportunities that allow students to make progress and experience success. The degrees of differentiation exercised will depend on the levels of student attainment.

- Varied approaches to teaching, learning, and assessment should be planned to allow all students to participate fully and effectively. Account should be taken of diverse cultures, beliefs, strengths, and interests that exist in any classroom and that influence the way students learn.

- Students with special needs should be given additional instructional support in negotiating the regular curriculum, not a different one. The guiding principle of equity is to supply students who need it with additional help to achieve set standards, but not to lower the standards.

- Continuous formative evaluation must be used to identify learning needs and to shape instruction, thus maximizing students' opportunities for achieving success. Assessment strategies must be appropriate to the way the curriculum is designed and delivered, as well as to each student's individual learning profile and stage of development.

- Suitable technology must be used in instruction to facilitate learning and enhance success.

ICT in the Curriculum

The following statements are taken from the Ministry of Education's ICT in Education Policy (pp. 28–29).

Curriculum Content and Learning Resources

- Curriculum and content must increasingly maximize the use of ICT.

- ICT must be integrated into the development and delivery of the curriculum.

- ICT integration and ICT competency measures across the curriculum shall be driven through the development and delivery of an ICT-infused curriculum.

Essential Learning Outcomes

The learning outcomes which have been deemed essential are in the areas of:

- Aesthetic Expression
- Citizenship
- Communication
- Personal Development
- Problem Solving
- Technological Competence

The achievement of these essential learning outcomes by all students is the goal that every core curriculum subject must facilitate. The core curriculum subjects, their content, and the suggested teaching, learning, and assessment strategies are the means to fulfil this end.

It is expected that by the end of the third year of secondary school, students' achievement in all six areas will result in a solid foundation of knowledge, skills, and attitudes that will constitute a platform for living in the Trinidad and Tobago society and making informed choices for further secondary education.

The essential learning outcomes are described more fully below.

Aesthetic Expression

Students recognize that the arts represent an important facet of their development, and they should respond positively to its various forms. They demonstrate visual acuity and aesthetic sensibilities and sensitivities in expressing themselves through the arts.

Students, for example:

- use various art forms as a means of formulating and expressing ideas, perceptions, and feelings;

- demonstrate understanding of the contribution of the arts to daily life, cultural identity, and diversity;

- demonstrate understanding of the economic role of the arts in the global village society;

- demonstrate understanding of the ideas, perceptions, and feelings of others as expressed in various art forms;

- demonstrate understanding of the significance of cultural resources, such as museums, theatres, galleries, and other expressions of the multicultural reality of society.

Citizenship

Students situate themselves in a multicultural, multi-ethnic environment, and understand clearly the contribution they must make to social, cultural, economic, and environmental development in the local and global context.

Students, for example:

- demonstrate understanding of sustainable development and its implications for the environment locally and globally;

- demonstrate understanding of Trinidad and Tobago's political, social, and economic systems in the global context;

- demonstrate understanding of the social, political, and economic forces that have shaped the past and present, and apply those understandings to the process of planning for the future;

- examine issues of human rights and recognize and react against forms of discrimination, violence, and anti-social behaviours;

- determine the principles and actions that characterize a just, peaceful, pluralistic, and democratic society, and act accordingly;

- demonstrate understanding of their own cultural heritage and cultural identity, and that of others, as well as the contribution of our many peoples and cultures to society.

Communication

Students use their bodies, the symbols of the culture, language, tools, and various other media to demonstrate their deeper understandings of synergies inherent in the exchange of ideas and information, and thus to communicate more effectively.

Students, for example:

- explore, reflect on, and express their own ideas, learning, perceptions, and feelings;

- demonstrate understanding of facts and relationships presented through words, numbers, symbols, graphs, and charts;

- demonstrate sensitivity and empathy where necessary in communicating various kinds of emotions and information;
- present information and instructions clearly, logically, concisely, and accurately for a variety of audiences;
- interpret and evaluate data, and express their conclusions in everyday language;
- critically reflect on and interpret ideas presented through a variety of media.

Personal Development

Students "grow from inside out," continually enlarging their knowledge base, expanding their horizons, and challenging themselves in the pursuit of a healthy and productive life.

Students, for example:

- demonstrate preparedness for the transition to work and further learning;
- make appropriate decisions and take responsibility for those decisions;
- work and study purposefully, both independently and in cooperative groups;
- demonstrate an understanding of the relationship between health and lifestyle;
- discriminate among a wide variety of career opportunities;
- demonstrate coping, management, and interpersonal skills;
- display intellectual curiosity, an entrepreneurial spirit, and initiative;
- reflect critically on ethical and other issues;
- deal effectively with change and become agents for positive, effective change.

Problem Solving

Students have a range of problem-solving strategies and apply them appropriately to situations they encounter. They demonstrate critical thinking and inquiry skills with which they process information to solve a wide variety of problems.

Students, for example:

- acquire, process, and interpret information critically to make informed decisions;
- use a variety of strategies and perspectives flexibly and creatively to solve problems;

- formulate tentative ideas, and question their own assumptions and those of others;
- solve problems individually and collaboratively;
- identify, describe, formulate, and reformulate problems;
- frame and test hypotheses;
- ask questions, observe relationships, make inferences, and draw conclusions;
- identify, describe, and interpret different points of view;
- distinguish facts from opinions.

Technological Competence

Students are technologically literate, understand and use various technologies, and demonstrate an understanding of the role of technology in their lives, in society, and in the world at large.

Students, for example:

- locate, evaluate, adapt, create, and share information using a variety of sources and technologies;
- demonstrate understanding of existing and developing technologies and use them appropriately;
- demonstrate understanding of the impact of technology on society;
- demonstrate understanding of ethical issues related to the use of technology in local and global contexts.

The Core Curriculum Subjects

The core curriculum subjects are those for which every student is required to demonstrate achievement of the stated outcomes in Forms 1–3. Additional subjects that contribute to students' holistic development and further their interests and aspirations may also be offered thereafter.

A minimum time allocation is recommended for each core subject. The principal, as instructional leader of the school, will make the final decision as to time allocation, according to the needs of the students and the resources available at any given time.

The subjects and the recommended time allocations are as follows:

Subject	**No. of Periods**	**Subject**	**No. of Periods**
English Language Arts	6	**Mathematics**	5
Science	4	**Health and Physical Education**	2
Spanish	4	**Technology Education**	4
Social Studies	4	**Visual and Performing Arts**	4

At the end of Form 3, students will be assessed for the National Certificate of Secondary Education (NCSE), Level I.

Language Across the Curriculum

The development of language skills and the ability to understand and use language correctly, competently, and effectively is fundamental to the learning outcomes expressed in the national curriculum. Language is a uniquely human capacity. Three simultaneous uses of language for learning are envisaged as students experience the national curriculum: students will learn language, they will learn through language, and they will learn about language.

Language plays a major role in learning, which occurs when students use the major modes of language—listening, speaking, reading, and writing—to achieve various purposes, among them: to communicate with others; to express personal beliefs, feelings, ideas, and so on; for cognitive development in various subjects of the curriculum; and to explore and gain insight into and understanding of literature. Language is linked to the thinking process, and its use allows students to reflect on and clarify their own thought processes and, thus, their own learning.

The national curriculum is predicated on the assumption that since students' language development takes place across the curriculum, the development process must be addressed in all subject areas. Students will develop and use patterns of language vital to understanding and expression in the different subjects that make up the curriculum.

However, the student of Trinidad and Tobago functions in a bidialectal context, that is, the natural language of the student, the Creole, differs from the target language and language of instruction, Internationally Accepted English. The philosophical position taken in the national curriculum is that both languages are of equal value and worth, and both must be respected. Students use their own language as a tool for interpreting the content of the curriculum and for mastering it. In addition, they must be taught to use the target language as effectively and effortlessly as they would their natural language.

The exponential growth in information and the use of information and communication technologies provide opportunities for students to become critical users of information. Language development and use in this context is also addressed in all subject areas.

Curriculum Implementation

Implementation of the curriculum is a dynamic process, requiring collaboration of the developers (curriculum teams) and users (teachers). In implementation, teachers are expected to use the formal curriculum, as described in the curriculum guides, to plan work and teach in a manner that accomplishes the objectives described. Teachers translate those objectives into units of study, determining the appropriate sequence and time allocation according to the learning needs of their students. The new Curriculum Guides provide sample teaching and assessment strategies, but it is also the role of the professional teacher to select and use sound teaching practices, continually assessing student learning, and systematically providing feedback to curriculum teams for use in revising and improving the guides.

A curriculum development system provides support for the tasks of curriculum implementation. The system advocated by the Ministry of Education involves stakeholders, specialist curriculum officers, principals, heads of departments, and teachers, each with specific roles and responsibilities. Some of these are outlined in the table below.

System Component	Members	Role
National Curriculum Advisory Council/ Committee	Stakeholders	• Advise on curriculum policy, goals, and standards
Curriculum Planning and Development Division (Head Office and District-based)	curriculum officers	• Plan and develop curriculum • Provide leadership in identifying curriculum goals and determining the process for development of curriculum materials • Lead writing teams (which include teachers) • Monitor implementation • Provide teacher support • Facilitate teacher professional development for curriculum implementation • Advise on processes and materials for effective implementation and student assessment • Evaluate curriculum
School Curriculum Council	Principal/Vice Principal and Heads of Departments	• Make major decisions concerning the school curriculum, such as assigning resources • Provide guidelines for Instructional Planning Teams
Instructional Planning Teams/School Instructional Committees	Teachers	• Cooperate on tasks necessary for effective implementation, such as: yearly work plans, units of study, development of materials to individualize the curriculum, identification and development of learning materials, student assessment and evaluation

Curriculum Implementation at School Level

The "School Curriculum" refers to all the learning and other experiences that the school plans for its students. It includes the formal or written curriculum, as well as the informal curriculum, which is comprised of other developmental opportunities provided by the school, such as those offered by student clubs, societies and committees, and sporting organizations (e.g., cricket team, debating society, Guides, Cadets).

The School Curriculum Council develops a School Curriculum that must be in alignment with the National Curriculum. The School Curriculum Council usually consists of the Principal and/or Vice Principal and Heads of Department. The duties of the Council include the development of school culture, goals, vision, and curriculum in alignment with the national curriculum and culture. It also provides support for curriculum work and performs evaluation functions.

In providing support for curriculum work, the Council may, for instance:

- encourage teachers to identify challenges and try new ideas;
- develop timetables to allow for development of curriculum materials, for example, year plans, units, instructional materials;
- ensure availability of learning materials;
- provide instructional leadership;
- ensure that appropriate strategies are formulated to promote student success.

In performing evaluation functions, the Council:

- monitors the curriculum (using, for example, observation, test scores, student books, formal and informal discussions with different stakeholders);
- assesses the hidden curriculum (including discipline policies, fund allocation, physical environment);
- evaluates the school programme of studies.

The roles of instructional teams and the individual teachers are described in the following tables:

Roles of School Instructional Committees
Develop/Revise/Evaluate work programmes
Determine resource needs
Identify/Develop instructional materials
Conduct classroom action research
Integrate and align curriculum
Identify and develop appropriate assessment practices
Develop reporting instruments and procedures (student and teacher performance)
Keep records

Roles of Individual Teachers
Develop/Revise instructional programme
Individualize curriculum to suit students' needs and interests
Develop/Evaluate/Revise unit plans
Develop/Select appropriate learning materials
Select appropriate teaching strategies to facilitate student success
Integrate the curriculum as far as possible, and where appropriate
Select appropriate assessment strategies
Monitor/Assess student learning and keep records
Evaluate student performance
Evaluate classroom programmes
Conduct action research
Collaborate with colleagues

References

Trinidad and Tobago. Ministry of Education. (2005a). *Draft policy for information and communications technology in education*. Port of Spain, Trinidad: Author.

Trinidad and Tobago. Ministry of Education. (2005b). *Green paper for public comment: Draft quality standards for education in Trinidad and Tobago*. Port of Spain, Trinidad: Author.

Trinidad and Tobago. Ministry of Education. (2005c). *Quest for excellence: Quality standards for education in Trinidad and Tobago: A Ministry of Education Green Paper – first revision*. Port of Spain, Trinidad: Author.

Trinidad and Tobago. Ministry of Education. (2006). *Curriculum policy: Preprimary to secondary education; draft*. Port of Spain, Trinidad: Author.

Trinidad and Tobago. Ministry of Education. (2007). *The national model for education in Trinidad and Tobago (Early childhood, primary and secondary); draft*. Port of Spain, Trinidad: Author.

Trinidad and Tobago. Ministry of Education. (2008). *Draft corporate plan 2008–2012*. Port of Spain, Trinidad: Author.

Trinidad and Tobago. Ministry of Education. Student Support Services Division. (2008). *Inclusive education policy; draft 5*. Port of Spain, Trinidad: Author.

Trinidad and Tobago. National Task Force on Education. (1994). *Education policy paper (1993-2003) (White paper)*. Port of Spain, Trinidad: Author.

Trinidad and Tobago. Task Force for the Removal of the Common Entrance Examination. (1998). *Report*. Port of Spain, Trinidad: Author.

MATHEMATICS

2

CURRICULUM LAYOUT AND SUBJECT MODULES

Table of Contents

The Mathematics Curriculum 2–1

Vision and Mission 2–1

Rationale for Teaching and Learning Mathematics 2–2

The Purpose and Structure of the Curriculum 2–3

Expected Learning Outcomes of the Mathematics Curriculum 2–6

Outcomes and Mathematics 2–6

Articulation of Expected Curriculum Outcomes With Essential Learning Outcomes 2–8

Breakdown by Essential Learning Outcomes 2–8

Modules Framework 2–12

Module 1: Capturing Number 2–12

Module 2: Data Analysis and Probability 2–13

Module 3: Keeping Tabs On Space 2–13

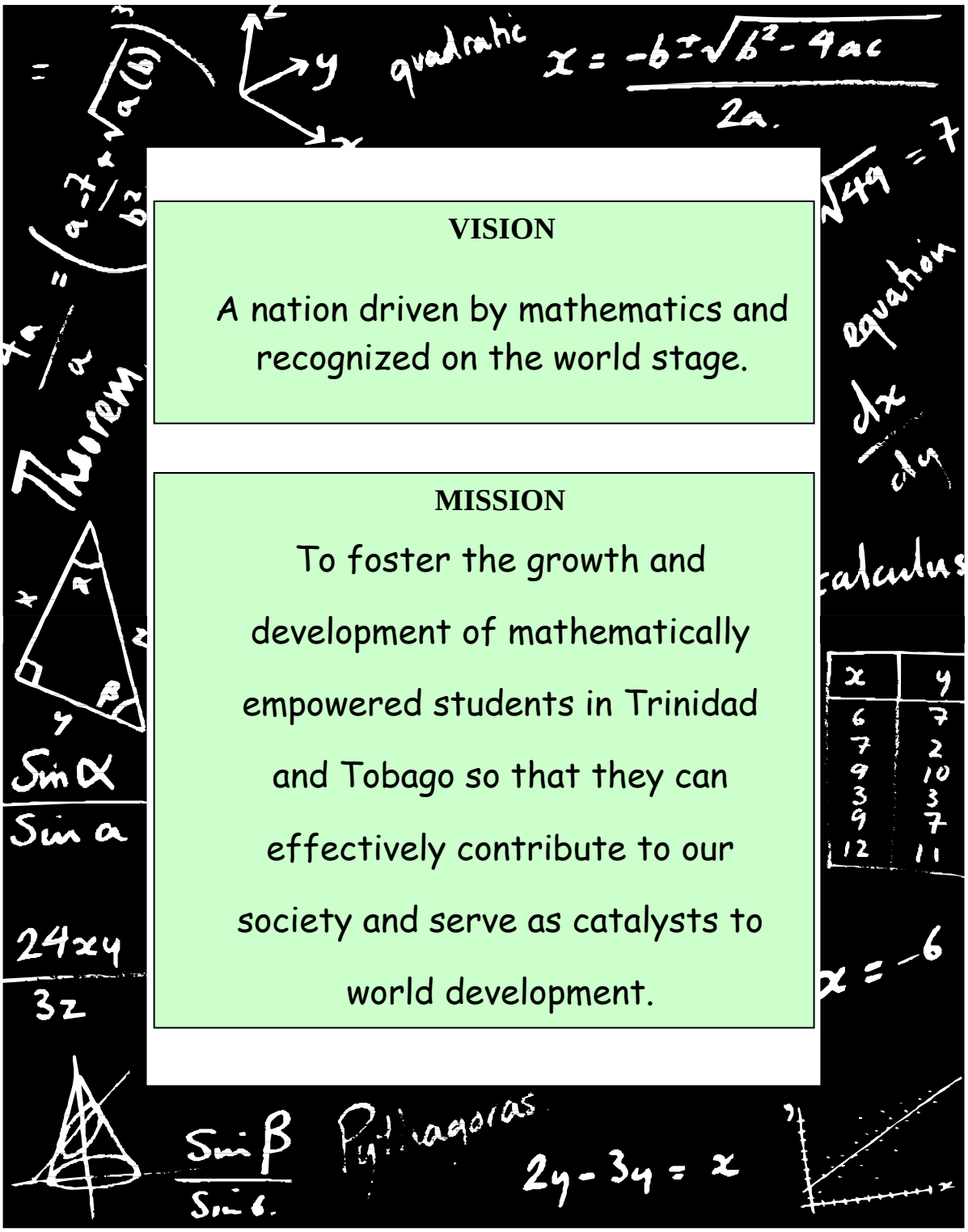
Module 4: It's All Up To Reasoning 2–14

Module 5: Measuring When It Matters 2–14

Bringing It All Together 2–16

A Final Note 2–17

THE MATHEMATICS CURRICULUM



VISION

A nation driven by mathematics and recognized on the world stage.

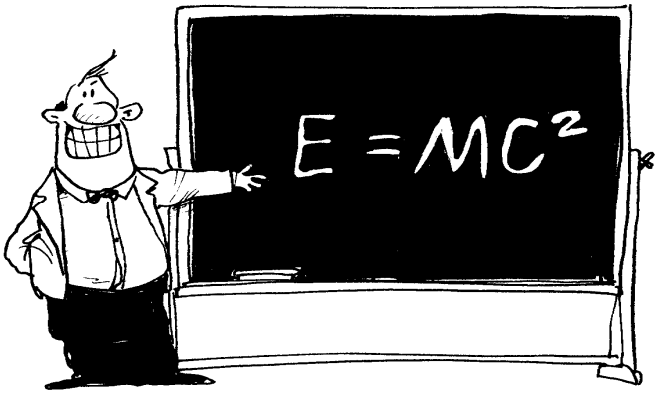
MISSION

To foster the growth and development of mathematically empowered students in Trinidad and Tobago so that they can effectively contribute to our society and serve as catalysts to world development.

RATIONALE FOR TEACHING AND LEARNING MATHEMATICS

Reports on mathematical achievement from external examination bodies, the Ministry of Education, employers, and public and private agencies have all concluded that the majority of students at both primary and secondary levels lack basic skills in numeracy. The high percentage of students who are not presently certified as being proficient in mathematics is one indicator of the problem. Many factors must be considered in addressing this situation, but the most important must be the design of a mathematics curriculum that is relevant to the needs of learners and of society.

Mathematics is an activity that is critical for the development of individuals and societies. It is the study of the properties of number, and its relationship to measurement, space, shape, statistics, and probability. Mathematics also deals with abstractions, with algebra being the strand of mathematics that presents abstraction in its purest form. The study of mathematics enables individuals to become creative and critical thinkers through the development of logical thinking, problem-solving, investigative, organizational, and argumentative skills.



Students acquire mathematical power by constructing mathematical knowledge and understanding. Mathematically empowered students can adapt to the quickening pace of change in today's society. They will have acquired basic skills, self-confidence, and self-reliance, which will prepare them to make effective contributions to their society. Through experiencing and practising the processes of communication, reasoning, making connections and representations, and recognizing patterns and relationships, students will have attained the essential learning outcomes identified for the national curriculum and, inevitably, the goals of education.

Mathematics is also essential to the study of all other subjects on both the primary and secondary schools' curriculum, and this in itself underscores its value and the role it plays in our lives.

The philosophy of education that informs this mathematics curriculum is underpinned by a belief that all children can learn, but that children learn in diverse ways. In order to ensure that all students become mathematically proficient, therefore, the curriculum is informed by current research on the nature and purpose of mathematics as well as on the pedagogy.

Since mathematics pervades our daily lives, the mathematics curriculum also reflects the various ways in which students encounter mathematics in their environment and in real-life situations. Thus, the new curriculum represents a major shift in the paradigm of understanding mathematics and how it is taught, learnt, and used. For the most part,

mathematics is conceptualized as a process that can be carried out in different ways and using multiple approaches to achieve similar ends. The curriculum focus is on doing mathematics, as far as possible, in real-life contexts. To this end, the curriculum advocates the use of performance tasks in the learning process, so as to make classroom experiences more authentic. Assessment practices should also be aligned to meaningful tasks.

THE PURPOSE AND STRUCTURE OF THE CURRICULUM

Traditionally, mathematics was accepted as the domain of an elite few who were able to master complex skills and understand abstract concepts. For many, learning mathematics was conceptualized narrowly as recalling facts and procedures that, in many instances, had no relevance to the real world, and served little purpose beyond examinations and certification.

The approach to learning mathematics taken in this curriculum should bring many benefits to the individual and to the society as a whole. Among these benefits is the basic support that mathematics lends to the other core curriculum areas in the attainment of the six essential learning outcomes, which are seen as the hallmark characteristics of all graduates. These educational learning outcomes provide the critical impetus for the thrust of the new curriculum.

The curriculum is intended to take learners through a spiralling learning process that integrates learning through content and pedagogy. A modular approach has been taken in designing the curriculum. Within this modular design, appropriate themes capture cross-sections of the traditional curriculum content, which is used primarily as a vehicle for developing the necessary competencies and understandings that will help students to attain the six essential learning outcomes.

The five modules in the curriculum are: Capturing Number; Data Analysis and Probability; Keeping Tabs on Space Using Numbers; It's All Up to Reasoning; and Measuring When it Matters.

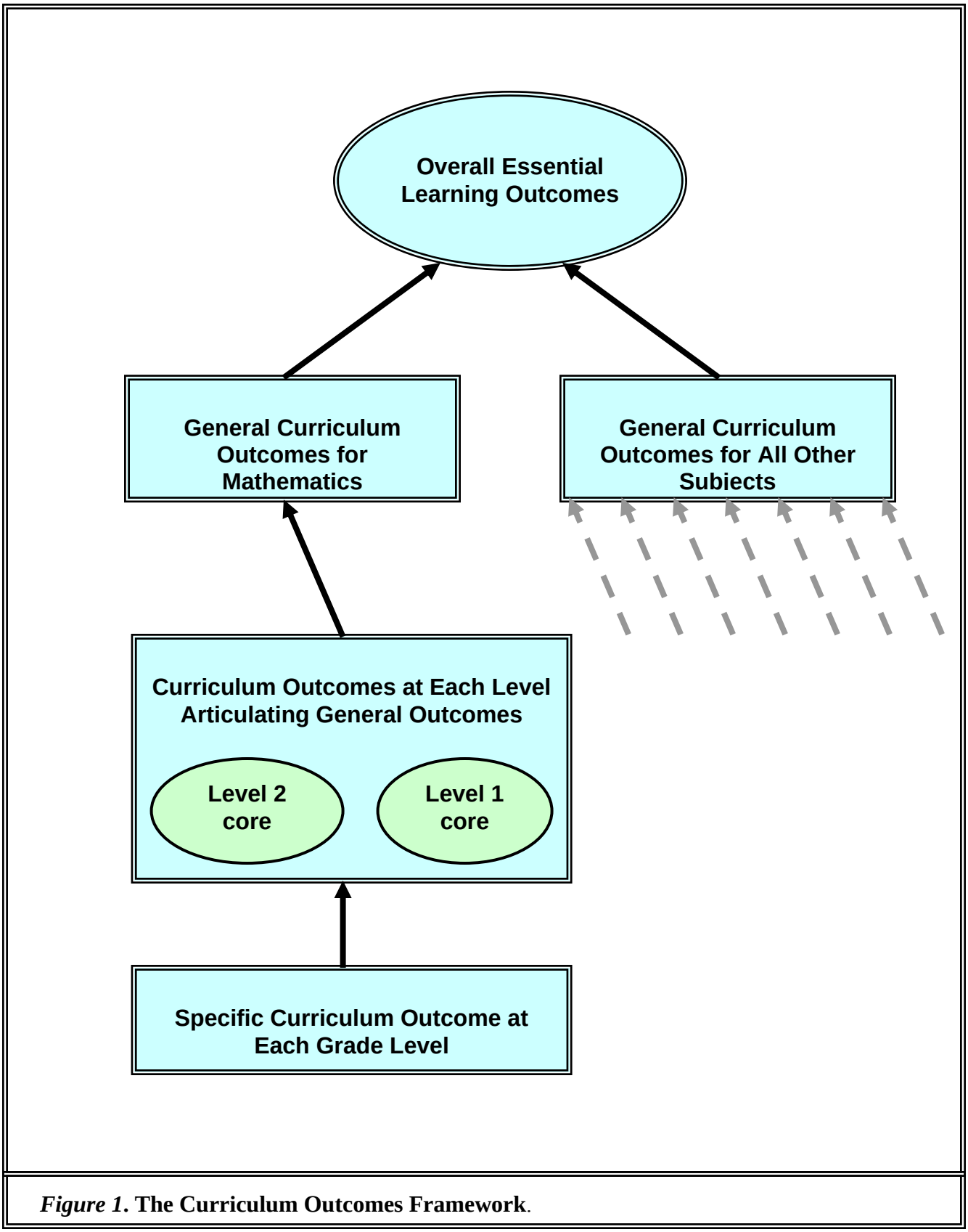


Figure 1. **The Curriculum Outcomes Framework**.

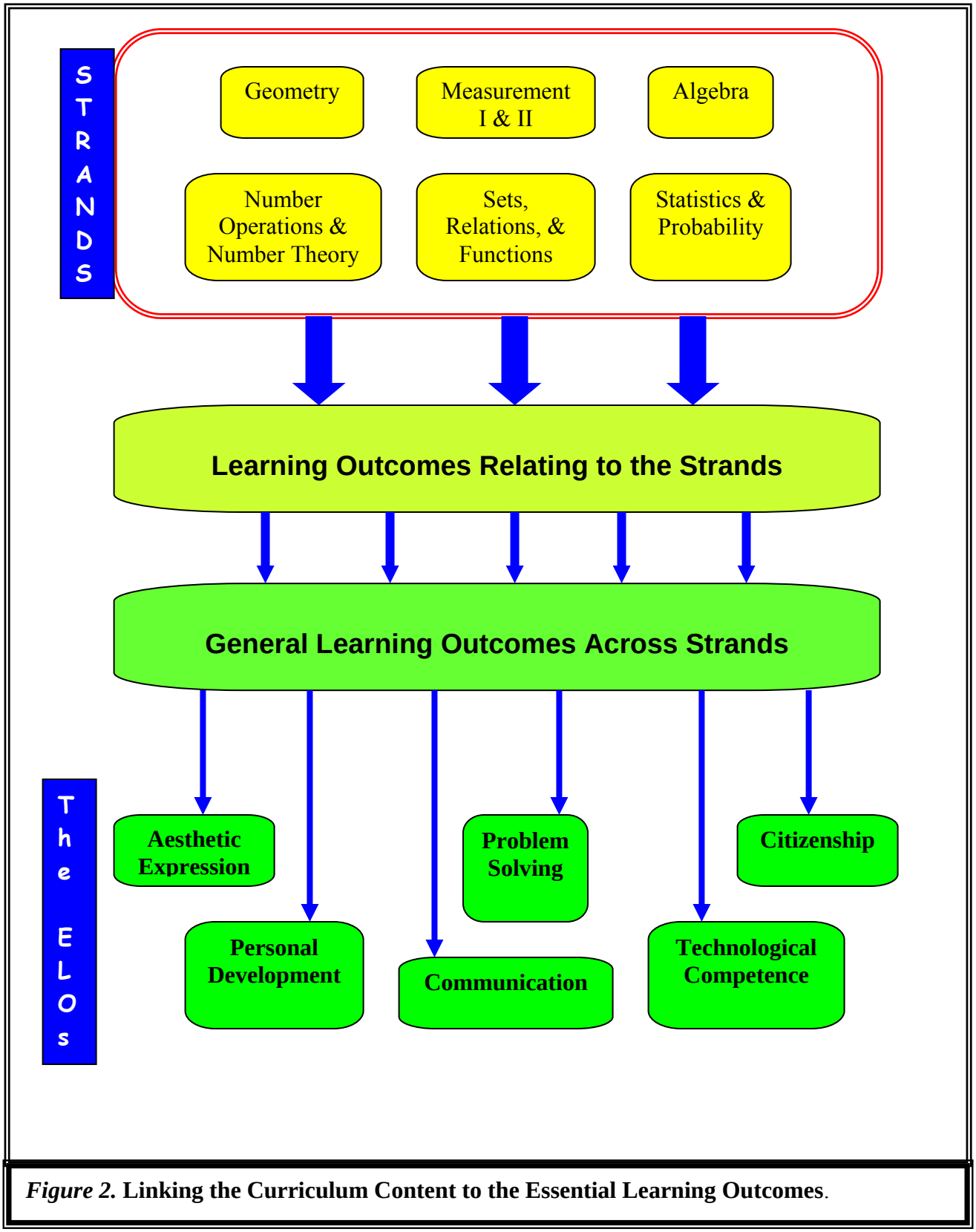


Figure 2. **Linking the Curriculum Content to the Essential Learning Outcomes**.

EXPECTED LEARNING OUTCOMES OF THE MATHEMATICS CURRICULUM

In this document, the general learning outcomes for mathematics have been organized within strands and articulate with the six essential learning outcomes specified in the national secondary curriculum. The strands and related general curriculum outcomes identified are consistent with those that have been established by the National Council of Teachers of Mathematics (NCTM) of the United States of America (USA). Those outcomes are structured as follows:

Outcomes and Mathematics

The general curriculum outcomes for mathematics have been organized within strands and articulate with the associated essential learning outcomes identified for the national curriculum. As outlined by the NCTM, these are structured as follows:

Number and Operations

1. Understand number representation
2. Understand number relationships and number systems
3. Understand operations, their relationships, and how they relate to number
4. Compute fluently
5. Make reasonable estimates

Algebra

1. Understand patterns, relations, and functions
2. Represent mathematical situations and structures using symbols
3. Analyse mathematical situations using algebraic thinking
4. Use models to represent, analyse, and interpret quantitative relationships
5. Understand, interpret, and use change in a variety of situations

Geometry

1. Use knowledge of characteristics and properties of shapes and solids to express mathematical ideas, develop arguments, and form and analyse spatial and geometric relationships
2. Specify locations using a system of coordinates
3. Make inferences about spatial relationships using coordinate geometry
4. Analyse mathematical situations using symmetry and transformations of space
5. Use mental visualization to solve problems
6. Use spatial reasoning to analyse mathematical situations and solve problems
7. Use geometrical modelling to solve problems

Measurement

1. Understand measurable attributes of objects
2. Understand units and systems of units
3. Understand and apply appropriate processes and techniques of measurement
4. Use appropriate tools and formulae to determine measurements

Data Analysis and Probability

1. Formulate, frame, and test hypotheses with data
2. Organize and reorganize data to make inferences and draw conclusions
3. Collect appropriate data for specific purposes
4. Select and use appropriate statistical techniques to analyse data
5. Use data to make predictions, to distinguish fact from opinion, and to show relationships
6. Understand risk and chance

Processes/Problem Solving

1. Build new knowledge through problem solving
2. Solve a variety of problems in a number of contexts
3. Apply and adapt a variety of appropriate strategies with the flexibility to solve problems
4. Understand risk and chance

Reasoning and Proof

1. Recognize and understand reasoning and proof as critical in communicating mathematical ideas
2. Make and investigate mathematical conjectures
3. Develop and evaluate mathematical arguments and proofs
4. Use different types of reasoning
5. Prove by different methods

Communication

1. Explore, reflect on, and express their mathematical thinking
2. Present their mathematical thinking coherently and clearly to a variety of audiences
3. Analyse and evaluate mathematical thinking of others
4. Use precise mathematical language to express ideas

Connections

1. Recognize and use connections among mathematical ideas
2. Recognize and use mathematics in a variety of contexts

Representation

1. Use representations to express mathematical ideas
2. Solve problems effectively using mathematical representations
3. Model and interpret physical, social, and mathematical phenomenon using representations

Source: National Council of Teachers of Mathematics (NCTM)

Articulation of Expected Curriculum Outcomes With Essential Learning Outcomes

The way that the expected outcomes for each module are articulated with the essential learning outcomes (ELOs) is indicated below. It should be noted that some of this curriculum's expected outcomes contribute to the achievement of more than one essential learning outcome and there may be some overlap. In such cases, objectives may appear under more than one heading. The following key is used to help users of the document to identify the relationship between the outcomes for each module and the ELOs.

Essential Learning Outcome	Designated Symbol
Aesthetic Expression	AE
Communication	C
Citizenship	CIT
Personal Development	PD
Problem Solving	PS
Technological Competence	TC

BREAKDOWN BY ESSENTIAL LEARNING OUTCOMES

Aesthetic Expression

1. Understand and appreciate symbols, patterns, relations, and functions.
2. Stimulate enjoyment in "doing mathematics" and solving problems.
3. Appreciate the usefulness of making quick estimates in real-life situations.
4. Use knowledge of characteristics and properties of shapes and solids to express mathematical ideas, develop arguments, and form and analyse spatial and geometric relationships.

5. Appreciate the role of technology in gaining a better understanding of shapes and space in general.
6. Demonstrate basic understanding of concepts and techniques of data collection, and presentation and interpretation of data.
7. Use models to represent, analyse, and interpret quantitative relationships.

Citizenship

1. Understand and appreciate the role of mathematics in past, present, and future social, political, and economic contexts.
2. Apply mathematics in cultural, environmental, and global contexts.
3. Demonstrate an appreciation of ethical issues as they relate to the use of data.
4. Develop cooperative and inquiry skills.
5. Develop qualities of sharing, collaboration, and cooperation through the use of statistics.

Communication

1. Understand and express numbers using appropriate representations.
2. Represent mathematical situations and structures using symbols.
3. Use knowledge of characteristics and properties of shapes and solids to express mathematical ideas, develop arguments, and form and analyse spatial and geometric relationships.
4. Specify locations using a system of coordinates.
5. Organize and reorganize data to make inferences and draw conclusions.
6. Recognize and understand reasoning and proof as critical in communicating mathematical ideas.
7. Develop and evaluate mathematical arguments and proofs.
8. Prove mathematical statements using different methods.
9. Explore, reflect on, and express mathematical thinking.
10. Present mathematical thinking coherently and clearly to a variety of audiences.
11. Analyse and evaluate mathematical thinking of others.
12. Use precise mathematical language to express ideas, measurements, and mathematical results.
13. Recognize and use connections among mathematical ideas.
14. Understand how mathematical ideas relate to one another.
15. Use representations to express mathematical information and ideas.

16. Model and interpret physical, social, and mathematical phenomenon using representations.
17. Formulate, frame, and test hypotheses with data.
18. Develop qualities of sharing, collaboration, and cooperation through the use of statistics.

Personal Development

1. Understand, interpret, and make use of change in a variety of situations.
2. Understand the concepts of risk and chance.
3. Explore, reflect on, and express their mathematical thinking.
4. Recognize and use connections among mathematical ideas.
5. Appreciate the role of technology in gaining a better understanding of shapes and space in general.
6. Apply mathematical thinking and spatial reasoning to solve problems that arise in other disciplines.
7. Recognize and use mathematics in a variety of contexts.

8. Compute fluently using basic understanding of laws and operations involving numbers.
9. Use different types of reasoning to solve problems.
10. Demonstrate an appreciation of ethical issues as they relate to the use of data.
11. Use data to make predictions, to distinguish fact from opinion, and to show relationships.
12. Develop qualities of sharing, collaboration, and cooperation through the use of statistics.

Problem Solving

1. Understand number relationships and number systems.
2. Understand operations, their relationships, and how they relate to number.
3. Make reasonable estimates in different situations.
4. Analyse mathematical situations using algebraic thinking.
5. Use models to represent, analyse, and interpret quantitative relationships.
6. Make inferences about spatial relationships using coordinate geometry.
7. Analyse mathematical situations using symmetry and transformations of space.

8. Use mental visualization to solve problems.
9. Use spatial reasoning to analyse mathematical situations and solve problems.
10. Use geometrical modelling to solve problems.
11. Collect appropriate data for a particular purpose.
12. Select and use appropriate statistical techniques to analyse data.
13. Build new knowledge through problem solving.
14. Solve a variety of problems in a number of different contexts.
15. Apply and adapt a variety of appropriate strategies with the flexibility to solve problems.
16. Make and investigate mathematical conjectures.
17. Use different types of reasoning.
18. Analyse and evaluate mathematical thinking of others.
19. Solve problems effectively using mathematical representations.
20. Develop qualities of sharing, collaboration, and cooperation through the use of statistics.
21. Demonstrate appropriate use of measures of central tendency in social and economic contexts.
22. Demonstrate an understanding of the use of probability in making daily decisions.

Technological Competence

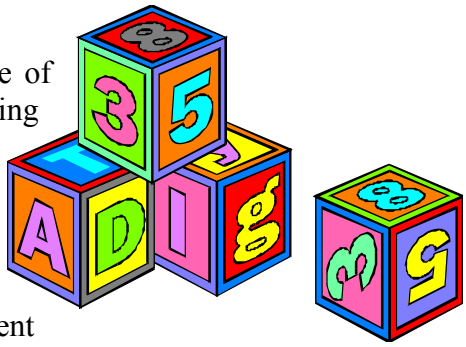
1. Compute fluently using basic understanding of laws and operations involving numbers and using the appropriate technology
2. Understand measurable attributes of objects.
3. Understand units and systems of units.
4. Understand and use a variety of appropriate processes, techniques, and tools, including technological tools, to make measurements.
5. Adapt and share mathematical information using a variety of sources and technologies.
6. Understand how to use existing and developing technologies to broaden and deepen mathematics knowledge and understanding.
7. Understand, interpret, and use change in a variety of situations.

MODULES FRAMEWORK

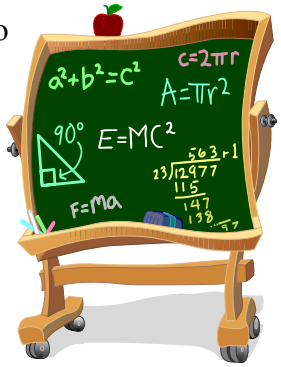
Module 1: CAPTURING NUMBER

Outcomes targeted

1. Demonstrate an understanding of relationships between large and small numbers and fluently manipulate numbers of any size including fractions, decimals, and those in exponential and/or standard forms (AE, C, PS).
2. Demonstrate a basic understanding of laws governing number and number operations (PS, PD).
3. Demonstrate a satisfactory level of competence in the use of technological tools to investigate and gain an understanding of number concepts, properties, and relationships (TC, PS).
4. Demonstrate critical thinking skills in examining ideas (PD, PS).



5. Demonstrate an understanding of numbers in different number bases and appreciate the use of base two in computer technology (TC, C).
6. Demonstrate an understanding of number relationships and number systems (PS).
7. Demonstrate an understanding of operations, their relationships, and how they relate to number (PS).
8. Make reasonable estimates (PS).
9. Understand number relationships and number systems (PS).
10. Demonstrate a basic understanding of laws governing number and number operations (PS, PD).
11. Demonstrate an understanding of percentages and its use in everyday life (C, PD).
12. Demonstrate fluency in calculations involving money as related to real life (PD, TC, PS).
13. Demonstrate a satisfactory level of understanding of problems involving rates and investments (C, PD, PS, AE, CIT).
14. Represent mathematical situations and structures using symbols (C).
15. Use precise mathematical language to express ideas (C).
16. Solve problems effectively using mathematical representations (PS).
17. Apply mathematics in cultural, environmental, and global contexts (CIT).



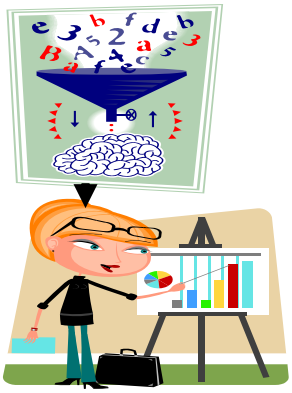
18. Demonstrate understanding of the relationship between abstract and concrete (C, PD).
19. Enhance critical thinking and enquiry skills (C, PD, PS).
20. Use precise mathematical language to express ideas (C).

21. Analyse mathematical situations using algebraic thinking (PS).
22. Develop an appreciation of the use of patterns to express mathematical ideas (C, PS).

Module 2: Data Analysis and Probability

Outcomes targeted

1. Demonstrate a basic understanding of concepts and techniques of data collection, and presentation and interpretation of data (PS, AE, TC, C).
2. Demonstrate an appreciation of ethical issues as they relate to the use of data (PD, CIT).
3. Develop qualities of sharing, collaboration, and cooperation through the use of statistics (PD, C, CIT, PS, AE).
4. Demonstrate appropriate use of measures of central tendency in social and economic contexts (PD, PS, CIT, C, TC).
5. Demonstrate enhanced knowledge and skills in the management and interpretation of data (PS, AE, PD, TC, CIT, C).
6. Understand concepts of risk and chance (PD, PS).
7. Understand what is meant by an *event* (PD, C).



8. Understand the relationship between possibility and probability (PD, PS, C).
9. Demonstrate an understanding of the use of probability in making daily decisions (PD, CIT, PS, C).

Module 3: Keeping Tabs On Space

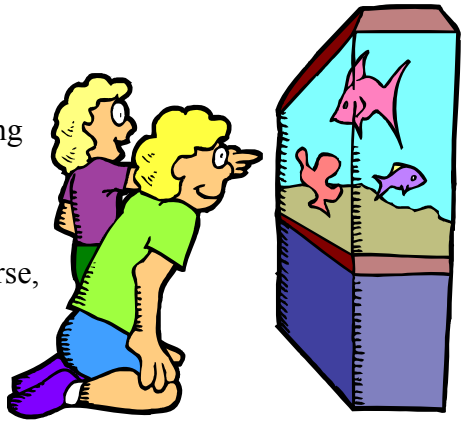
Outcomes targeted

1. Demonstrate an understanding of patterns, relations, and functions (AE, C).
2. Use a variety of representations, including symbols and simple line graphs, to express relationships between quantities (AE, C, TC).
3. Make inferences about spatial relationships using coordinate geometry (PS).
4. Use spatial reasoning to analyse mathematical situations and solve problems (PS).
5. Use knowledge of characteristics and properties of shapes and solids to express mathematical ideas, develop arguments, and form and analyse spatial and geometric relationships (AE, C).
6. Analyse mathematical situations using symmetry and transformations of space (PS).
7. Use geometrical modelling to solve problems (PS).

Module 4: IT'S ALL UP TO REASONING

Outcomes targeted

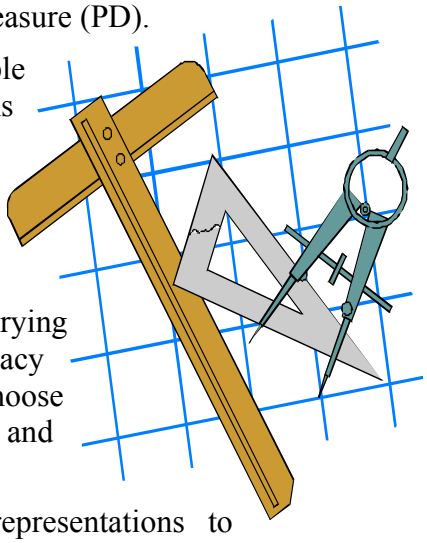
1. Demonstrate understanding of various types of relations and communicate this in various ways (PS, C).
2. Use relations to model data (C, TC).
3. Develop problem-solving skills (PS).
4. Develop cooperative and inquiry skills (CIT).
5. Solve geometric problems using spatial reasoning (PS).
6. Develop problem-solving skills (PS).
7. Communicate mathematical ideas through discourse, in writing, and graphically (C, TC).
8. Develop inquiry skills to investigate the space and objects in the environment (PS).
9. Develop cooperative and inquiry skills (CIT).



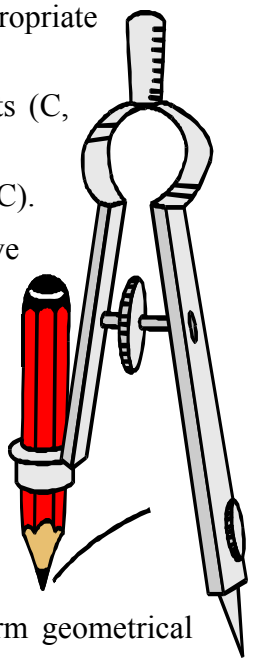
Module 5: MEASURING WHEN IT MATTERS

Outcomes targeted

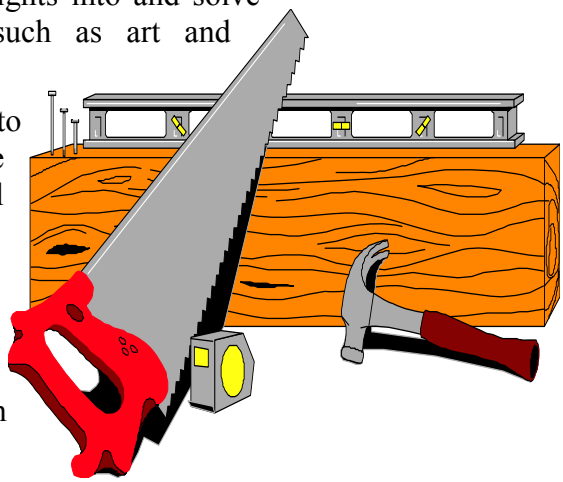
1. Understand measurable attributes of objects, units of measures, and systems of units (PD).
2. Appreciate the necessity to adopt standard units of measure (PD).
3. Understand and use the relationships between simple units and sub-units in the metric and imperial systems (PD).
4. Select and use a variety of appropriate processes, techniques, and tools, including technological tools, to make measurements (PD, PS, TC).
5. Evaluate situations and processes that require varying degrees of precision (consistency) and accuracy (correctness) when making measurements, and choose appropriate units to reflect the required precision and accuracy (PD, PS).
6. Use precise mathematical language and other representations to communicate measurements and mathematical results (C).
7. Develop confidence and inquiry skills, and enhance student motivation to explore mathematics as a multi-faceted discipline (PD).



8. Understand and appreciate the role of mathematics in past, present, and future social, political, and economic contexts (CIT).
9. Stimulate enjoyment in "doing mathematics" and solving problems in a rapidly changing technological society (AE, PD, PS, TC).
10. Understand, apply, and critically assess the effectiveness of appropriate techniques of estimations (TC, PD).
11. Use estimations to gain insight into the reasonableness of results (C, PS).
12. Use scaled models to represent authentic mathematical situations (C).
13. Apply mathematical thinking and spatial reasoning to solve problems that arise in other disciplines such as art and landscaping, and in our rapidly changing technological environment (PS, PD, CIT).
14. Describe how a change in one measurement affects the changes made in related measurable quantities, mathematical shapes, and models (C, PD).
15. Appreciate the usefulness of making quick estimates in real-life situations (AE).



16. Understand and apply knowledge, concepts, and skills to perform geometrical constructions to a specified degree of accuracy (C, PD).
17. Appreciate the value of precision and accuracy in making measurements (PD).
18. Apply creative and critical thinking, and spatial and deductive reasoning to explore relations between shapes and solids in space (PS, AE, C).
19. Use geometrical models and ideas to gain insights into and solve problems in other areas of mathematics such as art and architecture (PD, AE, PS, CIT).
20. Use precise mathematical statements to communicate logical deductions and evidence to support conjectures made about geometrical figures (C).
21. Use appropriate mensuration formulae to determine measures (PS, TC).
22. Appreciate the role of technology in gaining a better understanding of shapes and space in general (PD, AE).



BRINGING IT ALL TOGETHER

Outcomes Targeted by All Modules

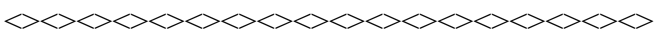
1. Use mental visualization to solve problems (C, PS).
2. Use different types of reasoning to solve problems (PS).
3. Solve problems effectively using mathematical representations (C, PS, TC).
4. Build new knowledge through problem solving (C, PD, PS).
5. Develop and evaluate mathematical arguments and proofs (C, PS).
6. Use precise mathematical language to express ideas (C, PD).
7. Recognize and understand reasoning and proof as critical in communicating mathematical ideas (C, PD, PS).
8. Recognize and use connections among mathematical ideas (C, PS).
9. Recognize and use mathematics in a variety of contexts (C, CIT, PD, PS, TC).
10. Model and interpret physical, social, and mathematical phenomenon using representations (C, CIT, PD, PS).
11. Explore, reflect on, and express their mathematical thinking (C, PS).
12. Analyse and evaluate mathematical thinking of others (C, PD, PS).
13. Appreciate the role of mathematics in economics and other fields (PD, PS).
14. Apply mathematics in cultural, environmental, and global contexts (CIT, PD, PS).
15. Adapt and share mathematical information using a variety of sources and technologies (C, CIT, PD, PS, TC).

A FINAL NOTE

The use of an outcomes-based approach to the development of this curriculum document has demanded significant changes to the traditional design structure. However, it has brought worthwhile benefits to what was previously seen as a fragmented content-based approach to curriculum implementation. In this regard, the layout of this mathematics curriculum document is expected to present more opportunities for curriculum integration and the use of authentic situations to enhance teaching and learning. With the five overlapping modules, teachers are now readily allowed the latitude to freely employ real-world activities and use multiple assessment strategies to gain a more holistic understanding of students' progress.

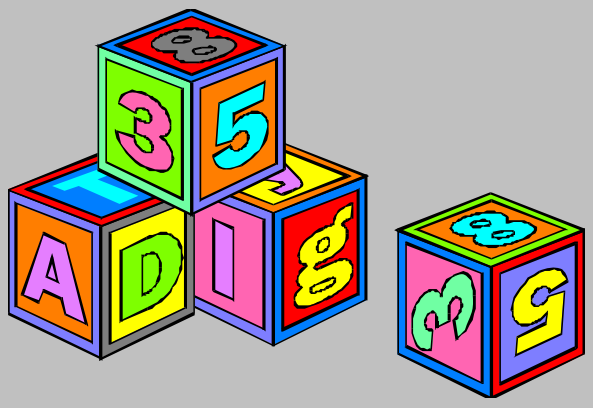
Furthermore, in meeting the standards articulated for each module of the curriculum, there are different levels of performance that need to be qualified, and these must be explained in terms of pre-defined performance indicators.

Teachers are encouraged to use the document freely as a guide to planning, implementing, and evaluating curricula that are appropriate for their contexts.

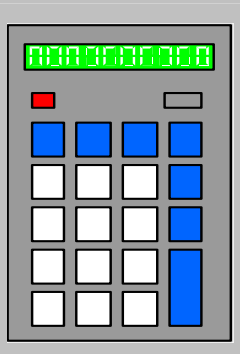


MATHEMATICS

MODULE 1



CAPTURING NUMBER



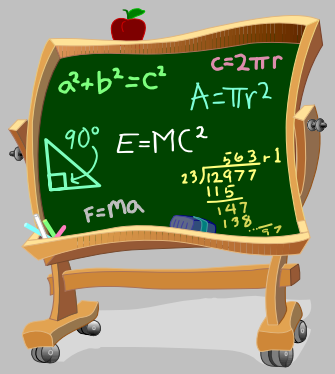


Table of Contents

MODULE 1: CAPTURING NUMBER

Rationale 2–M1–1
Aim 2–M1–1
Goals 2–M1–1

Unit 1 — Numbers Made Easy

General Outcomes 2–M1–2
Specific Outcomes by Levels
- Level 1 2–M1–3
- Level 2 2–M1–3
- Level 3 2–M1–4

Content 2–M1–4
Instructional Strategies/Methods 2–M1–5
Suggestions for Assessment 2–M1–5
Resources 2–M1–6
Sample Lessons 2–M1–6

Unit 2 — Everyday Use of Numbers

General Outcomes 2–M1–10
Specific Outcomes by Levels
- Level 1 2–M1–10
- Level 2 2–M1–10
- Level 3 2–M1–11

Content 2–M1–11
Instructional Strategies/Methods 2–M1–11
Suggestions for Assessment 2–M1–11
Resources 2–M1–12

Unit 3 — Thinking With Algebra

General Outcomes 2–M1–13
Specific Outcomes by Levels
- Level 1 2–M1–13
- Level 2 2–M1–14
- Level 3 2–M1–14

Content 2–M1–14
Instructional Strategies/Methods 2–M1–14
Suggestions for Assessment 2–M1–15
Resources 2–M1–15
Sample Lesson 2–M1–15

Bibliography/Webliography 2–M1–23

Standards 2–M1–25

CAPTURING NUMBER

Rationale

Capturing Number offers students the opportunity to understand numbers, ways of representing numbers, relationships among numbers, and number systems. It enables students to understand the meaning of operations and how they relate to one another, and to compute and make reasonable estimates.

Aim

Module 1 is designed to develop aspects of problem solving, communication, personal development, and technological competence through the use of content in Number Theory, Consumer Arithmetic, and Algebra. This is presented in three units, each of which will focus on different combinations of appropriate curriculum content.

Goals

This module supports attainment of outcomes in which students will:

1. enhance critical thinking, creative, and enquiry skills using number relationships and number patterns (PS, CIT, C);
2. appreciate the use of number in consumer arithmetic to interpret and understand systems (C, PS);
3. communicate ideas using number, symbols, and patterns (C, AE);
4. enhance problem-solving skills through development of algebraic thinking strategies and an understanding of number (PS, C);
5. enhance presentation skills (AE, C, TC, PD);
6. stimulate intellectual curiosity (PD, CIT);
7. enhance evaluation skills (PS);
8. enhance technological skills (TD).

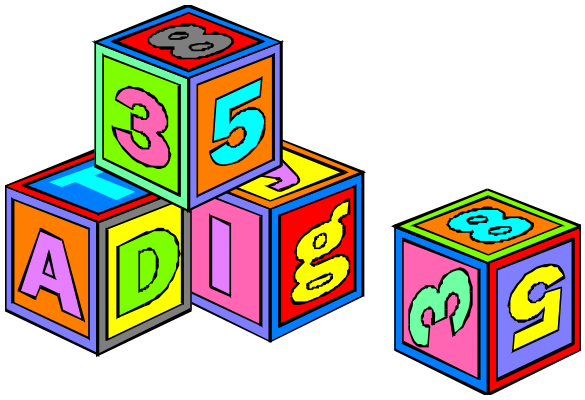
This module comprises three units:

Unit 1 – Numbers Made Easy

Unit 2 – Everyday Use of Numbers

Unit 3 – Thinking With Algebra

Unit 1



UNIT 1 — NUMBERS MADE EASY

GENERAL OUTCOMES

Students are expected to:

1. demonstrate an understanding of relationships between large and small numbers, and fluently manipulate numbers of any size including fractions, decimals, and those in exponential and/or standard forms (AE, C, PS);
2. demonstrate a basic understanding of laws governing number and number operations (PS, PD);
3. demonstrate a satisfactory level of competence in the use of technological tools to investigate and gain an understanding of number concepts, properties, and relationships (TC, PS);
4. demonstrate critical thinking skills in examining ideas (PD, PS);
5. demonstrate an understanding of numbers in different number bases and appreciate the use of base two in computer technology (TC, C);
6. demonstrate an understanding of number relationships and number systems (PS);
7. demonstrate an understanding of operations, their relationships, and how they relate to number (PS);
8. make reasonable estimates (PS);
9. understand number relationships and number systems (PS);
10. demonstrate a basic understanding of laws governing number and number operations (PS, PD).

SPECIFIC OUTCOMES BY LEVELS

Level 1

Students are expected to:

1. use the four basic operations on whole numbers (from one to one million) to solve word problems;
2. use an understanding of the order of whole numbers (from one to one million) to solve simple problems;
3. convert whole numbers (greater than one) to index form;
4. state the value of a^0 where a is an integer;
5. decompose a denary number (between one and one million) into its constituents as powers of 10;
6. trace the historical development of the denary system and other number systems;
7. use the properties of even and odd numbers (divisibility by 2), and prime and composite numbers (number of factors) to solve problems;
8. identify and describe in words, number patterns in the form of simple sequences of whole numbers, and extend to simple number patterns;
9. use HCF and LCM to solve problems;
10. express a decimal number in rational form (a/b, where a and b are whole numbers and $b \neq 0$);
11. use rational numbers in authentic situations;
12. state the relationship between rational numbers, whole numbers, and integers;
13. add, subtract, multiply, and divide rational numbers using concrete materials, visuals, and patterns;
14. apply basic operations on decimal fractions;
15. use decimal fractions in science and in other disciplines;
16. justify the reasonableness of answers in relevant problem situations using estimation techniques;
17. use proportion techniques (unitary method, unequal sharing) to solve real-world problems.

Level 2

Students are expected to:

1. represent positive and negative numbers on the number line;
2. show mastery in solving simple everyday problems involving integers (from -100 to +100) using the basic operations of addition, subtraction, multiplication, and division (without using a calculator);

3. add, subtract, multiply, and divide integers (from -1000 to +1000) using a variety of strategies and tools, including computers and calculators;
4. display (in a variety of ways) the relationship between natural numbers, whole numbers, and integers and represent them on the number line;
5. state the laws of commutativity, associativity, and distributivity;
6. state the properties of identity, inverse, and closure;
7. use number properties and the laws of commutativity, associativity, and distributivity in performing calculations;
8. recognize place value of digits in any base;
9. convert a numeral given in any base to a base 10 numeral;
10. convert a numeral given in base 10 to a numeral in any other base;
11. add, subtract, multiply, and divide numbers given in index form (positive indices only);
12. represent large and small numbers in scientific notation (from $a \times .10^{-6}$ to $a \times 10^{6}$);
13. write numbers to a given number of significant figures;
14. make computational estimates in relevant problem situation using appropriate approximation techniques.

Level 3

Students are expected to:

1. identify irrational numbers in their decimal forms (as non-terminating decimals) and locate them in the set of real numbers ($Q + Q' = R$);
2. use the calculator to approximate, to four significant figures, the value of irrational numbers between 0 and 100;
3. apply the laws and properties of numbers to the real number system;
4. display, using Venn diagrams, the relationship among the subsets of real numbers;
5. compute simple sums and differences of numbers in base 2 and base 3.

Content

- ❖ Number Operations
- ❖ Number Systems
- ❖ Number Patterns
- ❖ Indices
- ❖ Fractions
- ❖ Decimals

- ❖ Approximations and Estimations
- ❖ Ratio
- ❖ Integers
- ❖ Laws and Properties of Numbers
- ❖ Number Bases
- ❖ Representation of Numbers
- ❖ Commutative, Distributive, and Associative Laws
- ❖ Standard Form
- ❖ Significant Figures – Real Numbers
- ❖ Number Base – Binary Only

Instructional Strategies/Methods

- Activate prior knowledge
- Worksheets
- Presentations
- Research, investigations, and discussions
- Explorations/Investigations (use interactive technology that would enable students to work with different types of numbers and solve problems)
- Use of workstations
- Teacher/student demonstrations
- Collaborative learning

Suggestions for Assessment

- ✓ Pencil and paper tests
- ✓ Observation
- ✓ Worksheets
- ✓ Projects
- ✓ Performance tasks (e.g., have students classify numbers, draw diagrams, compose own items, games, etc.)
- ✓ Portfolios (students compile a portfolio comprising a selection of activities relating to number and number sense, e.g., number pattern, number chart, number puzzle)

Resources

- Calculators
- Books
- Computer and software
- Useful websites, for example, http://www.coolmaths4teachers.com
- Internet – National Council of Teachers of Mathematics (NCTM) website

---

SAMPLE LESSON #1

Unit 1: Level 2

Significant Figures

Overview

Students traditionally have a very difficult time understanding which digits are significant, especially zeroes, in a number that represents something that has been measured. This activity was designed so that students would explore and truly understand which digits are significant when dealing with numbers that represent measured values.

Learning Outcomes

Students will be able to:

- use a measuring instrument to the limit of its precision design;
- determine which digit in a measured value is the most certain;
- determine which digit in a measured value is the first uncertain value;
- understand which zeroes are significant when determining the number of significant digits in a measured number.

Materials

For each pair of students:

- 1 blue paper ruler, described below
- 1 red paper ruler, described below
- 1 yellow paper ruler, described below;
- several strips of green paper of different lengths (make sure some of them come out to where zeroes will show up in the measurements)
- 1 writing tool
- 1 piece of paper to record results

Instructional Plan

1. Using a blue paper ruler with marks of 0 on one end and 10 on the other end, measure several smaller pieces of green paper strips and record the results.
2. Using a red paper ruler the same length as the blue one above, but divided into 10 equal spacings with only marks of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 on it, measure the same green paper strips measured in Activity 1 and record the results.
3. Using a yellow paper ruler the same length as the red one above, but with 10 additional marks between each mark found on the red ruler, measure the same green paper strips measured in Activity 1 and record the results.
4. Have students analyse the results and discuss which digit in a measurement is the most certain.
5. Have students analyse the results and discuss which digit in a measurement is the first uncertain digit.
6. Discuss with students those measurements with zeroes in them to get students to understand when those zeroes are significant and when they are not.

Tying it all Together

Many students have a very difficult time understanding significant digits and why they are important in future calculations just by memorizing rules.

Using carefully selected green paper strips for students to measure, the students themselves can come up with the rules for which digits are significant and their importance in future calculations. This activity can be referred to when discussing significant figures in calculations.

Assessment

Students are provided with appropriate worksheets on which they must determine which figures are significant.

SAMPLE LESSON #2

Unit 1: Level 2

Directed Numbers

Overview: "The Postman Story"

Some students need a variety of concrete experiences before they are expected to master abstract concepts. The postman story provides an opportunity to act out integer operations prior to introducing manipulatives or paper and pencil methods.

It is important to introduce new concepts, or reteach those which have not been adequately understood, with very simple numbers. We will use combinations of 2, -2, 3, and -3.

The story line is simple. Basically, the postman brings cheques (positives) and bills (negatives). If he brings more than one item, then we are looking at "addition" of numbers.

Learning Outcomes

Students will be able to:

- appreciate the value of understanding directed numbers and how they apply to real-world situations;
- solve problems involving directed numbers in a variety of contexts.

Instructional Plan

The following examples all assume I am worth $0 to start.

2 + 3 Rewrite this as +2 + +3, to emphasize the fact that these are cheques. He brings a cheque for $2 (+2) and he also brings a cheque for $3 (+3). So I am now worth +5.

2 + (-3) He brings a cheque for $2 (+2) and he also brings a bill for $3 (-3), meaning I gained $2, but I also owe someone $3. So the net result is that I owe $1, if I use the $2 to help pay off the $3. I am now worth -1.

(-2) + 3 He brings a bill for $2 (-2), and a cheque for $3. I can use the $3 to pay for the $2 that I owe, and I have $1 left. I am now worth +1.

(-2) + (-3) He brings a bill for $2 (-2) and another bill for $3 (-3). Thus I owe a total of $5. I am now worth -5.

Teachers can actually have students act out the process of having one person deliver cheques and bills, with the numbers written out on index cards. One should also vary the scenario by starting out with a given worth, which can be positive or negative. One can also move on to more difficult numbers, including decimals, as in real money. However, the main objective is to help students absorb the concept of adding integers. One should not mask this objective by using more complicated arithmetic that will distract from this focus.

The postman story also lends itself well to subtraction of integers. Sometimes he gets confused and delivers the wrong cheques and bills. He then has to return and take away those items.

2 - 3 He brings a cheque for $2 (+2), but takes away a cheque for $3 (+3). It might clarify the process by writing +2 - +3, so that the +'s clearly signify cheque. So, I gained $2, but then lost $3, so I am now worth -3.

2 - (-3) Rewrite this as +2 -(-3). He brings a cheque for $2, and takes away a bill for $3. I no longer owe the $3 bill that the postman took away. So I gained $5. I am now worth +5.

-2 - 3 Rewrite this as -2 - +3. He brings a bill for $2 and takes away a cheque for $3. Now I am worth -5.

-2 - (-3) He brings a bill for $2 and takes away a bill for $3. I am now worth +1. Taking away bills means that I owe less, or, I am worth more.

Multiplication can also be explained with the postman story:

2(3) rewritten as 2(+3) means he brings 2 sets of $3 cheques, or, I gained $6.

2(-3) means he brings 2 sets of $3 bills, or, I owe $6.

-2(3) means he takes away 2 sets of $3, leaving me with $6 less, or -6.

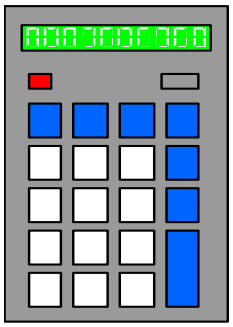
-2(-3) means he takes away 2 sets of $3 bills, so I no longer owe $6, or, I am worth $6 more (+6).

Assessment

Design and use an activity sheet involving addition and subtraction of directed numbers. The items may be presented in a chosen context or simply as a work sheet.

Source: Retrieved 26 June, 2007, from http://www.rice.edu/armadillo/Algebra/Lessons/Negative/postman.html

Unit 2



UNIT 2 — EVERYDAY USE OF NUMBERS

GENERAL OUTCOMES

Students are expected to:

1. demonstrate an understanding of percentages and their use in everyday life (C, PD);
2. demonstrate fluency in calculations involving money as related to real life (PD, TC, PS);
3. demonstrate a satisfactory level of understanding of problems involving rates and investments (C, PD, PS, AE, CIT).

SPECIFIC OUTCOMES BY LEVELS

Level 1

Students are expected to:

1. solve problems involving percentage (calculate profit and loss, percentage profit and loss, sales tax and discount);
2. solve word problems and situational problems using the simple interest concept.

Level 2

Students are expected to:

1. solve problems involving foreign exchange (e.g., discuss exchange rates and calculate conversions limited to two currencies only, at any given time);
2. solve problems involving hire purchase (calculate hire purchase price, cash price, interest);
3. describe and state the relationship among time, distance, and speed;
4. solve problems involving time, distance, and speed (**not** including graphs).

Level 3

Students are expected to:

1. solve problems involving rates—salaries, wages, overtime, and utility bills (electricity and telephone);
2. calculate the returns on different types of investments involving simple interest and compound interest (credit union, banks, stock, units, etc.);
3. calculate interest on loans (simple interest and compound interest).

Content

- ❖ Consumer Arithmetic
- ❖ Wages
- ❖ Utility Bills
- ❖ Investments and Loans

Instructional Strategies/Methods

- Activate prior knowledge
- Presentations
- Simulations and role playing (e.g., in bank settings)
- Worksheets
- Teacher/student demonstrations
- Use of workstations (e.g., create a shop/store and transact business – role playing)
- Explorations/Investigations
- Web-based activity
- Research, investigations, and discussions (e.g., students carry out transactions involving loans and savings)

Suggestions for Assessment

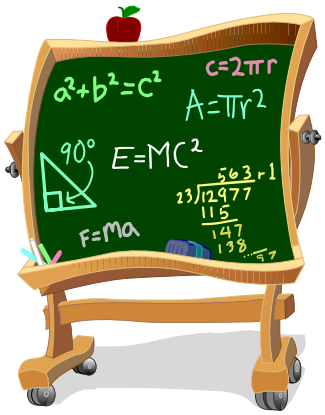
- ✓ Pencil/paper test
- ✓ Oral presentations (use newspaper advertisements to generate ideas for application to authentic situations)
- ✓ Observation
- ✓ Oral questioning (students review the different methods of calculating returns)
- ✓ Group activities
- ✓ Projects

- ✓ Presentations
- ✓ Reports (Students explore investments via the Internet and financial institutions, and present reports)

Resources

- Calculators
- Books
- Newspaper clippings
- Computer and software
- Internet – NCTM website

Unit 3



UNIT 3 — THINKING WITH ALGEBRA

General Outcomes

Students are expected to:

1. represent mathematical situations and structures using symbols (C);
2. use precise mathematical language to express ideas (C);
3. solve problems effectively using mathematical representations (PS);
4. apply mathematics in cultural, environmental, and global contexts (CIT);
5. demonstrate understanding of the relationship between abstract and concrete (C, PD);
6. enhance critical thinking and enquiry skills (C, PD, PS);
7. use precise mathematical language to express ideas (C);
8. analyse mathematical situations using algebraic thinking (PS);
9. develop an appreciation of the use of patterns to express mathematical ideas (C, PS).

Specific Outcomes by Levels

Level 1

Students are expected to:

1. use variables to represent whole numbers and create algebraic terms and expressions in relevant situations;
2. simplify algebraic expressions in 1, 2, or 3 variables using the four operations: ‘+’, ‘- ’, ‘×’, ‘÷’;

3. substitute natural numbers for variables in solving problems across the curriculum;
4. translate word problems into algebraic equations;
5. solve linear equations with one variable.

Level 2

Students are expected to:

1. substitute integers for variables in algebraic expressions;
2. solve simple linear inequalities;
3. represent solutions of linear inequalities on the number line;
4. solve linear equations involving use of the distributive law, using a variety of representations, and including fractions with **no** variable in the denominator.

Level 3

Students are expected to:

1. simplify fractional algebraic expressions, including linear expressions in the denominator;
2. multiply two binomial expressions;
3. factorize expressions involving 2, 3, and 4 terms;
4. solve linear simultaneous equations by calculation.

Content

- ❖ Variables and Constants
- ❖ Simplification of Algebraic Expressions
- ❖ Linear Equations
- ❖ Substitution
- ❖ Linear Inequalities
- ❖ Factorization
- ❖ Simultaneous Equations
- ❖ Modelling Natural Phenomena and Real-Life Situations Using Equations

Instructional Strategies/Methods

- Use of manipulatives
- Worksheets

- Activity sheets (use inequality symbols to relate to inequations, use samples from real-life situations)
- Guided instruction
- Teacher-designed activity
- Group activity (students discover the form of the factorization for the difference of two squares)

Suggestions for Assessment

- ✓ Paper/pencil test
- ✓ Presentations (have students construct sample equations and verify the solution)
- ✓ Oral assessment (have students verbalize or otherwise represent their thinking to the class)
- ✓ Project work (recording how inequalities are addressed in different contexts)
- ✓ Activity sheets
- ✓ Worksheets
- ✓ Mental quiz (difference of two squares)

Resources

- o Calculators
- o Books
- o Computer and software
- o Internet – NCTM website

SAMPLE LESSON

Unit 3: Level 2

Thinking With Algebra

Overview

Students traditionally have great difficulty with understanding abstractions in mathematics, especially as they relate to real-world problems that may be otherwise solved through guess work or trial and error. In many instances, proper use of algebraic thinking can help to provide a logical approach to solving such problems.

Learning Outcomes

Students will:

- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving;
- communicate their mathematical thinking coherently and clearly to others.

Materials

- The Mangoes Problem Activity Sheets
- The Sailors and Coconuts Problem Activity Sheets

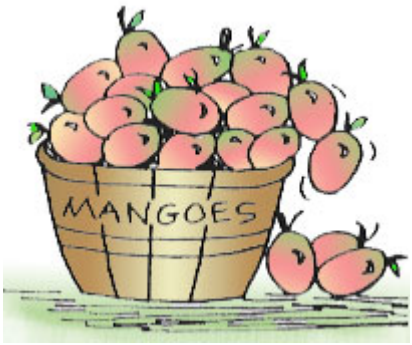
Source: National Council of Teachers of Mathematics: http://illuminations.nctm.org/

Instructional Plan

The Mangoes Problem

Read the mangoes problem to the students:

One night the King couldn't sleep, so he went down into the Royal kitchen, where he found a bowl full of mangoes. Being hungry, he took 1/6 of the mangoes.



Later that same night, the Queen was hungry and couldn't sleep. She, too, found the mangoes and took 1/5 of what the King had left.

Still later, the first Prince awoke, went to the kitchen, and ate 1/4 of the remaining mangoes.

Even later, his brother, the second Prince, ate 1/3 of what was then left.

Finally, the third Prince ate 1/2 of what was left, leaving only three mangoes for the servants.

How many mangoes were originally in the bowl?

Before students actually solve the problem, ask them to discuss, in groups, possible strategies for solving the problem. Possible strategies include:

- guess and check
- draw a picture
- work backward
- write an equation (use a variable)

Distribute the Mangoes Problem activity sheet so students may see the text of the entire problem and have a place to show their work.

[This problem can also be used for a variety of instructional purposes, including assessment, where the focus might be on assessing students' ability to use a variety of strategies, or as a task for a cooperative problem-solving group, where the goal is to use as many different strategies as possible in solving the problem.]

The following sections:

- outline each of the four previously mentioned solution methods for the mangoes problem;
- discuss how students approached the problem, and suggest two interesting generalizations of the problem.

Guess and Check

The guess-and-check strategy starts with an original guess of how many mangoes were in the bowl prior to the King's entry into the kitchen. Students then use the structure of the problem to see if their initial guess works to solve the problem correctly. If their initial guess fails to work, they make another, it is hoped "better," guess and check to see if it works. They continue this process until they make a correct guess. Some students may make wild and unreasonable guesses, so teachers should point out how to make "reasonable" first guesses and discuss the importance of making a table to collect and organize the data.

Students might realize that an initial guess has to be divisible by 6 so that the King could take one-sixth of the mangoes. For example, a student might guess that 24 mangoes were in the bowl originally. When checking this guess, however, the student will find that it results in 4, not 3, mangoes at the end. Since this outcome is too many mangoes, the student would revise his or her initial guess downward to 18, the next smallest multiple of 6. This number does, in fact, work.

Not all students will necessarily note the relevance of the fact that the initial guess is a multiple of 6. An initial guess may be 14, suggesting that students are not aware of the relevance of divisibility by 6. For their guess of 14, students may get off track and do the following computation on a calculator:

14 - 1/2 - 1/3 - 1/4 - 1/5 - 1/6

Draw a Picture

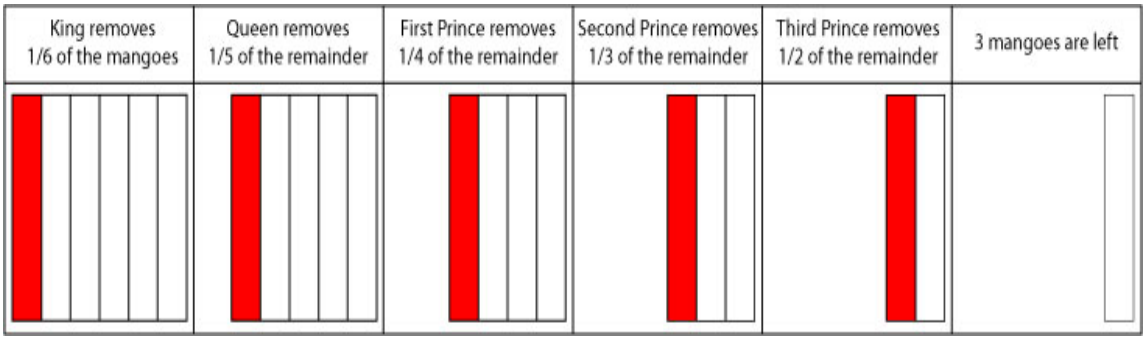
The easiest solution method to this problem is surprising in its simplicity.

Start by drawing a rectangle to represent all mangoes in the original pile prior to the removal of any of them. Since the King took one-sixth of this pile, divide the rectangle into six equal strips and "remove" one strip.

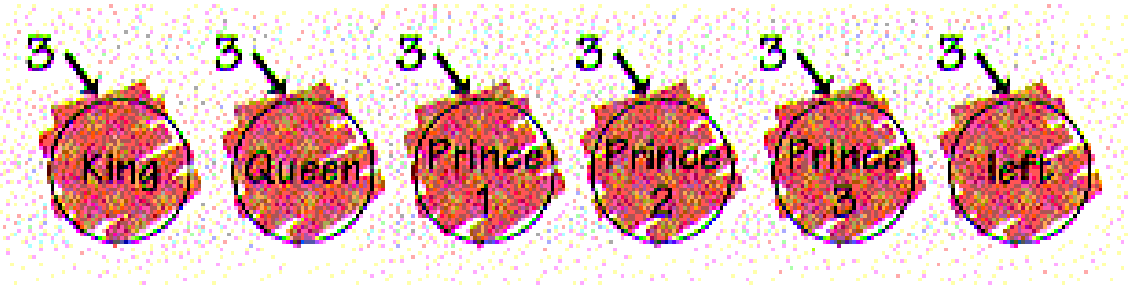
Notice that five strips remain, from which the Queen removed one-fifth, so this one-fifth is also represented by one of the original strips.

Continuing, when the first Prince removes one-fourth of what is left, the one-fourth is represented by one of the strips. Similarly, the one-third, one-half, and 3 remaining mangoes are each represented by a strip. In the final analysis, since the 3 mangoes equal one strip and originally six strips were involved, the number of original mangoes must have been $6 \times 3 = 18$.

The draw-a-picture strategy presents a concrete, visual representation of the problem.

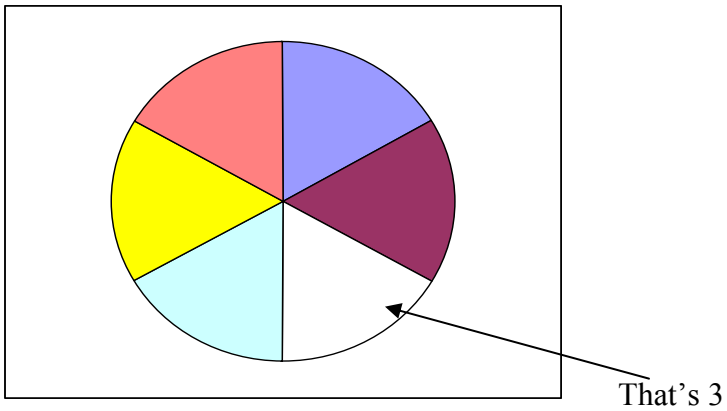


The draw-a-picture strategy may lead to some of your most interesting observations. Students may first draw six circles and shade one to represent the one-sixth the King took. They then would explain that the Queen ate one-fifth of what was left, so they would have shaded one of the remaining five circles. The process is continued until students have shaded the last of the original six circles drawn. Since the last circle represents 3 mangoes, the solution must be 3 + 3 + 3 + 3 + 3 + 3, writing each 3 above one of the circles they had shaded.



Other students may draw a picture but divide a pie into six wedges.

In the image below, one student shaded one wedge, noted five remaining wedges, and shaded one of them. She continued until she had shaded five of the six wedges. Finally, thinking about the sixth wedge, she said, “That’s three.”



Work Backward

This strategy requires three steps: start at the end of the problem (the 3 remaining mangoes); reverse each of the steps in the problem, being careful to determine the amount at this step; and work the problem from end to beginning by performing the inverse operation at each step.

Applying these steps to the mangoes problem results in the following:

1. At the end, 3 mangoes are left, representing one-half of the pile that the third Prince took. Thus, the third Prince had 6 mangoes before removing his half.
2. To determine how many the second Prince had before removing his third, we must realize that the 6 mangoes left after removal represent *two-thirds* of the pile from which he took his third. Thus, 6 is two-thirds of the number in the pile, or $6 \times 3/2 = 9$, the number in the pile before removal.
3. By continuing backward in this manner, 9 mangoes represent three-fourths of the pile before the first Prince took his, so $9 \times 4/3 = 12$ mangoes were in the pile the first Prince used.
4. Similarly, the Queen's pile was $12 \times 5/4$, or 15, and the King's must have been $15 \times 6/5$, or 18, the answer to the problem.

The following illustrates one student's method for solving this problem:

"Six represents two-thirds of something, so one-third must be three. So to get three-thirds, you must add the six (for two-thirds) to three (for one-third) and you have nine mangoes." Then, going the next-backward step, he said, "Nine needs one-fourth" (his words, meaning that since nine is three-fourths of the previous amount, it "needs" another fourth of this amount added to it), "so nine is three-fourths: divide by three (i.e., 9/3) and add this to nine, obtaining twelve." He continued quickly in this way to the final, correct solution.

Write an Equation (Use a Variable)

Some students might try this approach, especially if they are flexible in their algebraic thinking. Let x be the number of mangoes in the bowl before any are removed.

1. Since the King removed $(1/6)x$, then $x - (1/6)x$ mangoes are left after his removal. Thus, $(5/6)x$ mangoes are left.
2. The Queen removed one-fifth of $(5/6)x$, so $(5/6)x - (1/5)(5/6)x$, or $(4/6)x$, mangoes are left after her removal.
3. The first Prince removed one-fourth of $(4/6)x$ mangoes, so $(4/6)x - (1/4)(4/6)x$, or $(3/6)x$, mangoes are left after the first Prince's removal.
4. The second Prince removed one-third of $(3/6)x$, so $(3/6)x - (1/3)(3/6)x$, or $(2/6)x$, mangoes are left.
5. Finally, the third Prince removed one-half of $(2/6)x$, leaving 3 mangoes, so $(2/6)x - (1/2)(2/6)x = 1/6x = 3$. Solving $1/6x = 3$ results in $x = 18$.
6. Review the exercise to allow students to pull their ideas together.

Proposed Assessment

Assessment may be done using a similar type of problem issued to the students. The following example may be used:

Sailors and Coconuts

Pose the following problem to the students:

Three sailors were marooned on a deserted island that was also inhabited by a band of monkeys. The sailors worked all day to collect coconuts but were too tired that night to count them, They agreed to divide them equally the next morning.

During the night, one sailor woke up and decided to get his share. He found that he could make three equal piles, with one coconut left over, which he threw to the monkeys. Thereupon, he had his own share and left the remainder in a single pile.

Later that night, the second sailor awoke and, likewise, decided to get his share of coconuts. He also was able to make three equal piles, with one coconut left over, which he threw to the monkeys.

Somewhat later, the third sailor awoke and did exactly the same thing with the remaining coconuts.

In the morning, all three sailors noticed that the pile was considerably smaller, but each thought that he knew why and said nothing. When they then divided the remaining coconuts equally, each sailor received seven and one was left over, which they threw to the monkeys.

How many coconuts were in the original pile?

Distribute the "Sailors and Coconuts Problem" activity sheet so students may see the text of the entire problem and have a place to show their work.

As in the previous problem, students should use various strategies to solve this problem. (The solution is 79 coconuts.)

Source: National Council of Teachers of Mathematics: http://illuminations.nctm.org/

The Mangoes Problem - Activity Sheet

NAME _____

As a group, work to solve the following problem.

Explain your strategy and the reason you chose that strategy.

One night the King couldn't sleep, so he went down into the Royal kitchen, where he found a bowl full of mangoes. Being hungry, he took 1/6 of the mangoes.

Later that same night, the Queen was hungry and couldn't sleep. She, too, found the mangoes and took 1/5 of what the King had left.

Still later, the first Prince awoke, went to the kitchen, and ate 1/4 of the remaining mangoes.

Even later, his brother, the second Prince, ate 1/3 of what was then left.

Finally, the third Prince ate 1/2 of what was left, leaving only three mangoes for the servants.

How many mangoes were originally in the bowl?

The Sailors and Coconuts Problem – Activity Sheet

NAME _____

Work to solve the following problem using a variable (an algebraic method).

Three sailors were marooned on a deserted island that was also inhabited by a band of monkeys. The sailors worked all day to collect coconuts but were too tired that night to count them. They agreed to divide them equally the next morning.

During the night, one sailor woke up and decided to get his share. He found that he could make three equal piles, with one coconut left over, which he threw to the monkeys. Thereupon, he had his own share and left the remainder in a single pile.

Later that night, the second sailor awoke and, likewise, decided to get his share of coconuts.

He also was able to make three equal piles, with one coconut left over, which he threw to the monkeys.

Somewhat later, the third sailor awoke and did exactly the same thing with the remaining coconuts.

In the morning, all three sailors noticed that the pile was considerably smaller, but each thought that he knew why and said nothing. When they then divided the remaining coconuts equally, each sailor received seven and one was left over, which they threw to the monkeys.

How many coconuts were in the original pile?

Bibliography/Webliography

Bibliography

Gronlund, N. E. (2005). *Assessment of student achievement* (8th ed.). Boston, MA: Allyn & Bacon.

State of Delaware. Department of Education. (1995). *Mathematics curriculum framework: Content standards*. Dover, DE: Author.

Webliography

Coolmath.com
http://www.coolmath.com/teachers/

The Math Forum @ Drexel
http://mathforum.org

National Council of Teachers of Mathematics
http://www.nctm.org
http://illuminations.nctm.org/LessonDetail.aspx?id=L247
http://illuminations.nctm.org/LessonDetail.aspx?id=L264

Northwest Regional Educational Laboratory
http://www.nwrel.org/msec/just-good/10/ch7.html

Rice Center for Technology in Teaching and Learning
http://www.rice.edu/armadillo/Algebra/Lessons/Negative/postman.html

whyslopes.com
http://whyslopes.com/etc/solvingLinearEquations/stickDiagrams00a.html
http://whyslopes.com/etc/solvingLinearEquations/stickDiagrams03.html
http://whyslopes.com/etc/geometric_implications_for_algebra.html

STANDARDS

MODULE 1

Mathematics Standards

MODULE 1: CAPTURING NUMBER

1. Students will describe and apply relationships among numbers, their uses, and their representations.

At the end of Level 1

1. Model and connect physical and verbal representations of fractions, decimals, and whole numbers.
2. Demonstrate that mathematical operations can represent a variety of problem situations; explain the relative effect of operations on fractions and decimals.
3. Identify, explain, and apply number theory concepts (such as primes, multiples, factors, and composites).
4. Recognize and rewrite numbers as a base raised to a positive index.
5. Apply place value to the denary system using powers of 10, for whole numbers only. (e.g., $123 = 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$)

At the end of Level 2

1. Use concepts of negative numbers in concrete situations (such as the number line, with temperature, etc.).
2. Use, model, and identify place value and describe its relationship to magnitude.
3. Explain, derive, compare, and use relationships among operations and properties of operations.
4. Apply multiple representations of numbers, including exponents and scientific notation.
5. Represent and explain the effect of operations on positive and negative numbers.

At the end of Level 3

1. Model and connect physical and verbal representations of irrational numbers and real numbers.
2. Represent and explain the effects of operations, the commutative and associative laws, and other number properties (such as identities) on real numbers.
3. Recognize, explain, and represent the relationship among the subsets of real numbers.

2. Students will estimate and compute using mental math, estimation strategies, paper-and-pencil techniques, and technology-supported methods.

At the end of Level 1

1. Create and solve practical problems involving addition, subtraction, multiplication, and division of whole numbers.
2. Use various forms of estimation, including rounding, to determine the reasonableness of calculated answers; determine if an estimate is too high or too low.
3. Compare ratios and proportions to solve practical problems.
4. Compute percents to solve practical problems.
5. Compute and use simple interest to solve situational problems.

At the end of Level 2

1. Apply, explain, and assess the appropriateness of a variety of estimation strategies, including significant figures.
2. Apply number, identifying and using identities, inverse, and closure.
3. Apply laws of commutativity, associativity, and distributivity.
4. Apply, model, and explain procedures for computation with different kinds of rational numbers, including integers, powers, and scientific notation.
5. Use computational procedure to solve practical problems involving foreign exchange and hire purchase.

At the end of Level 3

1. Recognize the importance of binary base in computer technology and solve simple problems in same (addition and subtraction only).
2. Use computational procedure to solve problems involving rates (salaries, wages, utility bills).
3. Compute returns on various types of investments and loans (credit union, banks, units, etc.).

3. Students will make a link between concrete and abstract situations and use mathematical symbols to represent same.

At the end of Level 1

1. Identify and explain key mathematical concepts and model situations using algebraic methods.
2. Simplify algebraic expressions using the four operations: '+', '- ', '×', '÷'

3. Determine the value of a variable by evaluating formulae and algebraic expressions for given values (e.g., A = LW; given L = 4 and W = 7, A = 28).
4. Use mathematical language, notation, and symbols to represent problem situations and mathematical ideas.

At the end of Level 3

1. Identify "like" terms and use previous knowledge to simplify algebraic expressions involving rational terms.

4. Develop proficiency in applying number theory to algebraic expressions, equations, and inequations.

At the end of Level 1

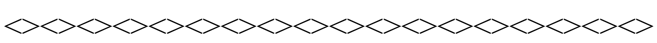
1. Use the inverse relations between addition and subtraction, multiplication and division to determine unknown quantities in equations.
2. Analyse, evaluate, and explain mathematical arguments and conclusions presented by others.

At the end of Level 2

1. Use the inverse relations between addition and subtraction, multiplication and division to determine the set of values of unknown quantities in simple inequalities.
2. Use the distributive law to simplify and solve linear equations, including those involving fractions with no variables in the denominator.

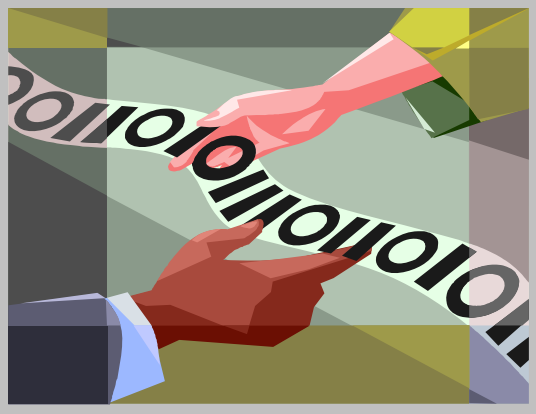
At the end of Level 3

1. Apply the distributive law to algebraic expressions and use this application to multiply two binomial expressions.
2. Determine common factors among algebraic terms and factorizing expressions involving 2, 3, and 4 terms.
3. Use methods of substitution and elimination to solve two linear simultaneous equations in two variables.



MATHEMATICS

MODULE 2



DATA ANALYSIS AND PROBABILITY

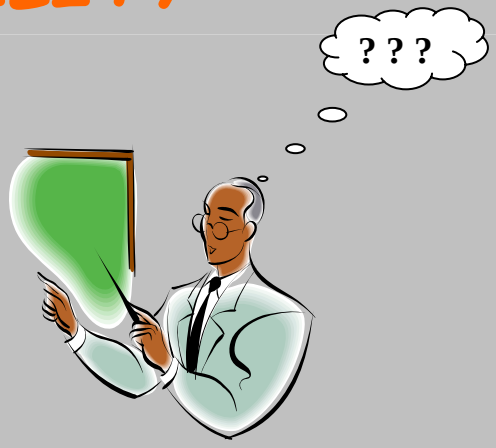


Table of Contents

MODULE 2: DATA ANALYSIS AND PROBABILITY

Rationale 2–M2–1
Aim 2–M2–1
Goals 2–M2–1

Unit 1 — Exploring Data

General Outcomes 2–M2–2
Specific Outcomes by Level
Level 1 2–M2–2
Content 2–M2–3
Instructional Strategies/Methods 2–M2–3
Suggestions for Assessment 2–M2–3
Resources 2–M2–4
Suggested Teaching and Learning Activities 2–M2–4

Unit 2 — Communicating With Data

General Outcomes 2–M2–6
Specific Outcomes by Level
Level 1 2–M2–6
Level 2 2–M2–7
Content 2–M2–7
Instructional Strategies/Methods 2–M2–7
Suggestions for Assessment 2–M2–7
Resources 2–M2–8
Suggested Teaching and Learning Activities 2–M2–8

Unit 3 — Making Informed Decisions

General Outcomes 2–M2–11
Specific Outcomes by Level
Level 3 2–M2–11
Content 2–M2–12
Instructional Strategies/Methods 2–M2–12
Suggestions for Assessment 2–M2–12
Resources 2–M2–12
Suggested Teaching and Learning Activities 2–M2–13
Sample Lesson 2–M2–14

Bibliography/Webliography 2–M2–16

Standards 2–M2–17

DATA ANALYSIS AND PROBABILITY

Rationale

Data Analysis and Probability offers rich opportunities for mathematical inquiry. It helps students to formulate and solve practical problems.

Data collection, organization, and analysis are very useful in many fields, including research, politics, business, and consumerism. The presentation of data in a visually appealing way is important in communicating information. Students also need to have knowledge about averages and probability, so that they can analyse data and make informed decisions as good citizens and consumers.

Aim

This module seeks to facilitate aspects of Personal Development, Problem Solving, Communication, and Technological Competence, as well as Aesthetic Expression and Citizenship, through the use of content in Statistics.

Goals

This module will support outcomes in which students:

1. enhance problem-solving skills through development of data management skills (PS, PD, C, AE);
2. enhance critical thinking and enquiry skills (CIT, PS, PD, C);
3. develop an appreciation of the use of data in interpreting and understanding systems (AE, PD, PS, CIT);
4. enhance knowledge of statistical methods and tools (PS, PD);
5. enhance communication skills through expressing ideas verbally as well as through visual displays (C, AE, PD, TC);
6. enhance aesthetic expression through the use of visually appealing displays (AE, PD);
7. enhance the ability to make simple everyday decisions and/or predictions in a social, political, or economic situation (CIT, C, PD, PS);
8. stimulate curiosity (PD);
9. enhance technological skills (TC, PD).

This module comprises three units:

Unit 1 – Exploring Data

Unit 2 – Communicating With Data

Unit 3 – Making Informed Decisions

Unit 1

UNIT 1 — EXPLORING DATA

This unit will be taught at only one level.

General Outcomes

Students are expected to:

1. demonstrate a basic understanding of concepts and techniques of data collection, and presentation and interpretation of data (PS, AE, TC, C);
2. demonstrate an appreciation of ethical issues as they relate to the use of data (PD, CIT);
3. develop qualities of sharing, collaboration, and cooperation through the use of statistics (PD, C, CIT, PS, AE).

Specific Outcomes by Levels

Level 1

Students will be able to:

1. identify various sources of discrete data (e.g., newspapers);
2. distinguish between discrete and continuous data;
3. collect discrete data using techniques such as direct observation, surveys, and counting;

4. record discrete data;
5. organize ungrouped discrete data into frequency tables;
6. display discrete data using simple bar charts, pie charts, and pictographs;
7. make inferences and draw conclusions from organized data;
8. display ethical behaviour when dealing with sensitive issues.

Content

- ❖ Discrete Data
- ❖ Tally Marks
- ❖ Frequency Tables
- ❖ Bar Charts
- ❖ Pictographs
- ❖ Pie Charts
- ❖ Ethical Issues

Instructional Strategies/Methods

- ▪ Cooperative learning groups
- ▪ Project-based activities
- ▪ Research
- ▪ Demonstrations
- ▪ Exploration

Suggestions for Assessment

- ✓ Oral questioning
- ✓ Quizzes
- ✓ Journals
- ✓ Project reports
- ✓ Presentation of work done orally and using the multimedia projector
- ✓ Portfolios of work done on the computer (printed)
- ✓ Guided discussion

Resources

- Data records
- Library
- Computer lab
- Internet
- Multimedia projector
- Calculators
- Software programs (*Microsoft Word*, *Textease*)

SUGGESTED TEACHING AND LEARNING ACTIVITIES

This unit can be used in a variety of projects in which students are given the opportunity to collect, organize, and display data to solve real-life problems, conduct investigations or studies, or simulate real-life situations. Students will also make inferences, draw conclusions, and present these for discussion.

---

1. Understanding Transportation

Unit 1: Level 1

Guide students to:

1. collect data on the modes of transportation used by classmates to travel to school (whether by private car, taxi, maxi-taxi, school bus, or by walking);

   (The teacher should be sensitive to social issues arising from students' different experiences, so that no embarrassment is felt by any child. If sensitive situations arise, these should be used as teaching opportunities to present ethical issues.)

2. organize data in a frequency table;

   (It is suggested that the teacher guide students to identify repeated data and explain more effective ways of presenting them, e.g., using a frequency table.)

3. use prepared charts and pictographs to display their data. Students should also be allowed to prepare their own simple charts and pictographs.

Students can make inferences about the transport situation in the country, for example, are there enough buses arriving on time to transport students? Does the number of students travelling by taxi imply that fares are too high?

---

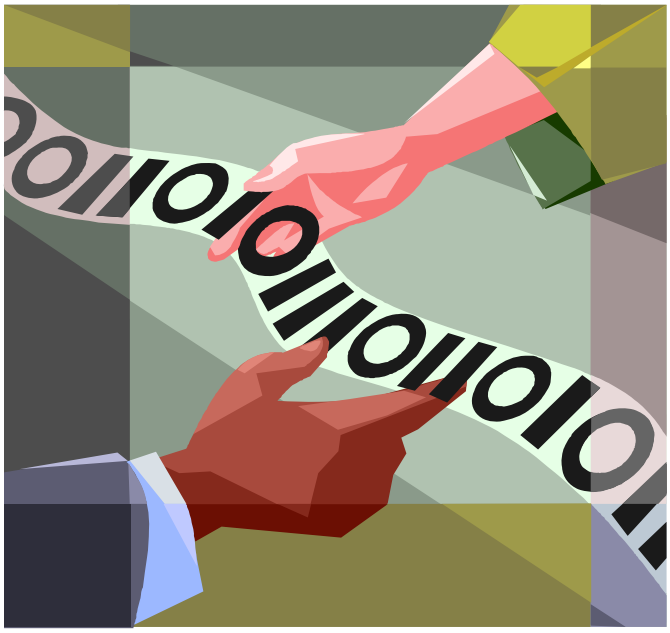
2. Class Quiz

Unit 1: Level 1

1. Plan a class quiz in which the students can score marks out of 10.
2. Guide students to:
   (i) gather the data and list them in rank order;
   (ii) make a frequency table and present information on a screen using a simple software program such as *Microsoft Word* or *Textease* and the multimedia projector.

1. Students may also display the information using bar charts, pictographs, and/or pie charts.
2. Encourage students to keep a portfolio of all work presented.
3. Guide class discussions to examine the results and state what they learnt from the experience.

Unit 2



UNIT 2 — COMMUNICATING WITH DATA

General Outcomes

Students are expected to:

1. demonstrate appropriate use of measures of central tendency in social and economic contexts (PD, PS, CIT, C, TC);
2. demonstrate enhanced knowledge and skills in the management and interpretation of data (PS, AE, PD, TC, CIT, C).

Specific Outcomes by Levels

Level 1

Students are expected to:

1. calculate the mean, mode, median for raw data;
2. find the mode for data taken from a frequency table;
3. present the findings of a simple statistical investigation.

Level 2

Students are expected to:

1. calculate the mean, mode, and median using ungrouped data from frequency tables;
2. interpret data, using the mean, and make convincing arguments based on data analysis and previous experience;
3. construct, read, and interpret information from line graphs.

Content

- ❖ Frequency Table (Ungrouped Data)
- ❖ Mean
- ❖ Mode
- ❖ Median
- ❖ Line Graphs

Instructional Strategies/Methods

- ▪ Project-based activities
- ▪ Cooperative groups
- ▪ Guided discussion
- ▪ Demonstrations
- ▪ Research, including use of the computer
- ▪ Socratic questioning
- ▪ Guided discussion

Suggestions for Assessment

- ✓ Oral questioning
- ✓ Quizzes
- ✓ Journals
- ✓ Project reports
- ✓ Oral and multimedia presentations of work done
- ✓ Portfolios—printouts of work done prepared on the computer

Resources

- Data records
- Libraries
- Charts
- Computer lab
- Internet
- Multimedia projector
- Calculators

SUGGESTED TEACHING AND LEARNING ACTIVITIES

There are several activities that can be used to bring about the learning outcomes proposed for this unit. The following are only some of the activities that may be used:

1. Chocolate Chips

Unit 2: Level 1

Students are placed into 5 groups. Each group is given 1 pack of chocolate chips.

1. Each group reports on the total number of chocolate chips in their pack.
2. The teacher notes totals on the board.
3. Students then use the data and calculate the mean, mode, and median number of chips per bag.
4. Students present their findings to the class.

2. Selecting the West Indies Cricket Team

Unit 2: Level 1

Calculate the mean batting score of potential West Indian batsmen vying for a place on the West Indies team during a given period. Students can then draw conclusions from their data, for example: Who should be included in the West Indies team? Which batsmen should be omitted from the West Indies team?

A similar approach may be used for selecting the bowlers on the West Indies team.
Batting and bowling scores may be sourced from:
http://content-wi.cricinfo.com/westindies/content/player

3. Counting Sweets

Unit 2: Level 2

Students are challenged to determine the mean, mode, and median number of sweets per bag. The teacher/student draws a table of corresponding totals per bag on the blackboard.

The students are then asked to:

1. calculate the mean, mode, and median number of sweets in each bag;
2. use a line graph to communicate information.

Resources

- Bags of sweets
- Plates

---

4. Road Fatalities

Unit 2: Level 2

Using the Internet, students can use preferred search engines to conduct an item search to obtain data on the statistics for road deaths in Trinidad and Tobago. This can be saved in a table in Microsoft Word. Students can insert clips from Clip Art or pictures to enhance a poster presentation of their data. The data can be represented using a line graph (Microsoft Excel may also be used to create the line graph from a table using the Chart Wizard Button).

As young citizens, students can also write letters to the editor of a local newspaper, discussing their ideas for reducing the number of road deaths in the country.

---

5. Using the Internet

Unit 2: Level 2

Students can access the Internet to source data on a variety of issues/topics. Data on issues such as crime, research, sports results, and so on, may be used to excite interest and keep current.

Computer software, such as Microsoft Excel, can be used to create a bar chart or pie chart to display this data.

The techno-savvy teacher can also create a Web page for the mathematics class (maybe through the use of a site like “geocities.com”) and post notes on lessons or homework assignments, or self-tests on topics already covered.

---

6. Using the Computer as a Motivational Tool

Unit 2: Level 2

Microsoft Word, Microsoft Excel, and the Internet are effective classroom tools for conducting demonstrations and investigations, completing assignments, generating data, summarizing data, and producing pictures, diagrams, and graphs. Using the computer to teach a unit of mathematics will enhance students' interest in the subject and encourage them to view it from a different perspective. Students view the computer as a fascinating technology that they desire to master. *The use of the computer will not be the focus of teaching the concepts but, rather, it will be used to empower students with the ability to explore the mathematical skills and analytical processes involved in statistics.*

Grouping: Students in the class are paired for work on the computer. These dyads may be constituted as follows:

- a mathematically challenged student with a confident mathematics student; or
- a computer literate student with a student who lacks basic computer skills.

Three classroom sessions can be dedicated to exploring the very basics of Microsoft, and Excel in particular.

During this stage, students can be taught the following:

- Construction of a frequency table – using columns and rows in Excel
- Construction, interpretation, and analysis of bar charts and pie charts – by selecting the table and using the *Chart Wizard button* to select the *Chart Type (Microsoft Excel)*
- Calculation of mean, median, or mode using the *Paste Function Button* and selecting either *average* or *median* or *mode*, and then following the instructions in the pop-up window.

Students can be taught the basics first on the whiteboard and then apply the knowledge to Microsoft Excel.

Unit 3

UNIT 3 — MAKING INFORMED DECISIONS

This unit will be taught only at Level 3.

GENERAL OUTCOMES

Students are expected to:

1. understand concepts of risk and chance (PD, PS);
2. understand what is meant by an *event* (PD, C);
3. understand the relationship between possibility and probability (PD, PS, C);
4. understand how to use probability in making daily decisions (PD, CIT, PS, C).

SPECIFIC OUTCOMES BY LEVEL

Level 3

Students are expected to:

1. distinguish between risk and chance;
2. explain what is meant by an *event*;
3. relate possibility and probability;
4. use concepts of certainty, fairness, and chance to discuss probability;
5. investigate the derivation of the probability formula;
6. recognize that the probability of an event occurring lies between 0 and 1;
7. determine the probability of an event that is certain to take place;
8. determine the probability of an event that will not take place;
9. obtain and use the probability formula.

Content

- ❖ Risk and Chance
- ❖ Certain Events
- ❖ Impossible Events
- ❖ Possibility
- ❖ Probability

Instructional Strategies/Methods

- ▪ Cooperative learning
- ▪ Project-based activities
- ▪ Research
- ▪ Guided discussion
- ▪ Socratic questioning

Suggestions for Assessment

- ✓ Oral questioning
- ✓ Quizzes
- ✓ Journals
- ✓ Project reports
- ✓ Performance tasks
- ✓ Exhibitions
- ✓ Oral and multimedia presentations
- ✓ Portfolios

Resources

- Dice
- Coins
- Writing materials
- Internet Resources

SUGGESTED TEACHING AND LEARNING ACTIVITIES

In teaching this module, the following activities may be useful:

1. Discussion – Risk and Chance

The teacher initiates and discusses games of chance with the class. This activity can facilitate discussions on the ethics of gambling and its effect on family life and society.

Discuss risk and chance by asking questions like:

1. Is it important to walk with an umbrella in the rainy season?

   ("Yes, since there is a chance that rain will fall.")

2. Is it important to wear a helmet while riding a bicycle?

   ("Yes. I risk hurting myself since there is a chance that I may fall.")

3. Which of two farmers would you buy tomatoes from, if one uses pesticides once per week on his crop, and the other uses pesticides twice per week on his crop?

   ("I will buy food containing the least amount of pesticides. There is less risk to one's health.")

Discuss the possibility or chance of events that are certain to occur, as well as events that are impossible.

Introduce probability, which is the numerical value assigned to the possibility of an event occurring.

2. Coin Tossing

Students can work individually or in groups to generate data for use in this exercise. The student is required to:

1. toss a coin 100 times;
2. record, in a table, the number of times each face turned up (heads or tails).

The teacher can lead discussion on experimental and theoretical probability as they relate to the exercise.

Further discussion and use of data collected can assist in deriving the probability formula.

3. Rolling Dice

This exercise can be done individually or in groups.

Students will:

- roll a die 100 times;
- record the number of times each number from 1 to 6 turns up.

The teacher leads discussion on the experimental and theoretical probability of the event.

Further use of the data collected can help to derive the probability formula.

Helpful Website

http://illuminations.nctm.org/LessonDetail.aspx?ID=L470

SAMPLE LESSON

Unit 3: Level 3

Collecting and Displaying Data

Overview: *"Making a Rain Catcher"*

Making mathematics real and applicable for students is one of the best ways to hold their interest. This activity allows students to benefit from actually designing a rain gauge and using it to collect data.

Lesson Outcomes

Students will be able to:

- record the data;
- plot a line graph;
- compare local results to that of other countries;
- calculate the average precipitation at their home.

Materials

- Ruler
- Plastic bottle
- Scissors
- Extra fine point marker
- Tape
- Water
- Graph paper
- Computer with Internet access

Instructional Plan

Students are asked to:

- cut the top off a plastic soda bottle so that the width is the same as the base;
- tape a ruler on the side of the bottle and, using an extra point permanent marker, mark off each centimetre;
- put some rocks at the bottom of the bottle (this will prevent the rain catcher from tipping or blowing away);
- turn the top upside down and tape it inside the bottle so as to create a funnel;
- pour some water into the bottle to the first marking, so that everyone starts at the same level;
- place their rain catcher in a not-so-busy area in their yard at home;
- check their rain catcher every morning;
- record the water level when there is some water in it and bring it to school (empty the water out so a new recording can be made the next morning);
- record their rain level with the day's date (this may be done at school). After a week of recording, students plot their data on a line graph.

The class can also add up all of the rain levels and calculate the average precipitation for the period measured.

Extension: This activity may be extended so that students can use the Internet to compare the average precipitations of their school community with those around the world.

Assessment

- ❖ Project work to design and make an instrument to collect and measure rainfall
- ❖ Oral and written presentations communicating their results

Bibliography/Webliography

Bibliography

Chambers, D. (Ed.). (1987). *Active mathematics teaching*. Madison, WI: Department of Public Instruction.

Good, T., and Grouws, D. (1977). Teaching effects: A process-product study of fourth grade mathematics classrooms. *Journal of Teacher Education*, *28*, 49–54.

Good, T., Grouws, D., and Ebmeier, H. (1983). *Active mathematics teaching*. New York: Longman.

Ryder, R. J., Burton, J. L., & Silberg, A. (2006). Longitudinal study of direct instruction effects from first through third grade. *Journal of Educational Research, 99*(3), 179–191.

Schweinhart, L. J., Weikart, D. P., & Larner, M. B. (1986). Consequences of three preschool curriculum models through age 15. *Early Childhood Research Quarterly, 1*(1), 15–45.

Webliography

ALL BLACKS
http://www.nzallblacks.net/stats.asp [rugby]

Cricinfo
http://stats.cricinfo.com/statsguru/engine/current/stats/index.html
http://www.cricinfo.com/link_to_database/NATIONAL/NZ/

National Council of Teachers of Mathematics
http://illuminations.nctm.org/

nzrugby.co.nz
http://www.nzrugbynet.co.nz/NZRFU/Resource+Library/Rugby+Almanack/2003+Rugby+Almanack/RS_Default_Topic.htm

Thats cricket
http://thatscricket.oneindia.in/statistics/

Associations/Organizations

Association for Supervision and Curriculum Development
http://www.ascd.org/portal/site/ascd

Delaware Department of Education
http://www.doe.state.de.us/

National Council of Teachers of Mathematics
http://www.nctm.org/

New Jersey Department of Education
http://www.state.nj.us/education/

Pennsylvania Department of Education
http://www.pde.state.pa.us/

STANDARDS

MODULE 2

Mathematics Standards

MODULE 2: DATA ANALYSIS AND PROBABILITY

1. Students will collect, organize, display, and analyse data in order to make decisions and predictions; determine the theoretical and experimental probability of an event to make predictions and solve problems; and interpret, evaluate, and communicate information obtained from a wide variety of sources.

At the end of Level 1

1. Identify data that is discrete and understand the difference between discrete and continuous data.
2. Collect and record data obtained by observation, survey, and counting.
3. Organize ungrouped data into frequency tables.
4. Display discrete data using simple bar charts, pie charts, and pictographs.
5. Make inferences and draw conclusions with organized data.
6. Calculate the mean, mode, and median for raw data.
7. Find the mode for data taken from a frequency table.
8. Present a report of a statistical investigation.

At the end of Level 2

1. Calculate the mean, mode, and median with ungrouped data taken from frequency tables, by hand and with use of technology (see suggested Teaching and Learning Activities).
2. Interpret data, using the mean, and formulate convincing arguments on a number of issues, based on data analysis and previous experiences.
3. Construct, read, and interpret information from line graphs.

At the end of Level 3

1. Use concepts of risk and chance.
2. Use concepts of certainty, fairness, and chance to discuss probability.
3. State the probability of an event that is certain to happen.
4. State the probability of an event that will never happen.
5. Recognize that the probability of an event occurring lies between 0 and 1.
6. State the formula for finding the probability of an event occurring.
7. Calculate the probability of an event occurring.
8. Make predictions based on experimental and theoretical probabilities.
9. Conduct simple probability experiments using concrete materials and represent the results using fractions.

2. Students will apply reasoning skills to make sense of solutions to mathematical problems in given situations.

At the end of Level 1

1. Explain why an inference or a conclusion is reasonable.

At the end of Level 2

1. Justify and defend the validity of solutions.

At the end of Level 3

1. Explain why a calculated probability is reasonable.

3. Students will read, listen, and discuss to obtain mathematical information; analyse and use the information; and present and justify mathematical ideas in written, oral, and visual formats.

At the end of Level 1

1. Read, listen, and conduct discussions to obtain mathematical information.
2. Represent and communicate mathematical ideas using oral, written, graphic, and pictorial representations.
3. Explain mathematical ideas and solutions to others using correct mathematical vocabulary.

At the end of Level 2

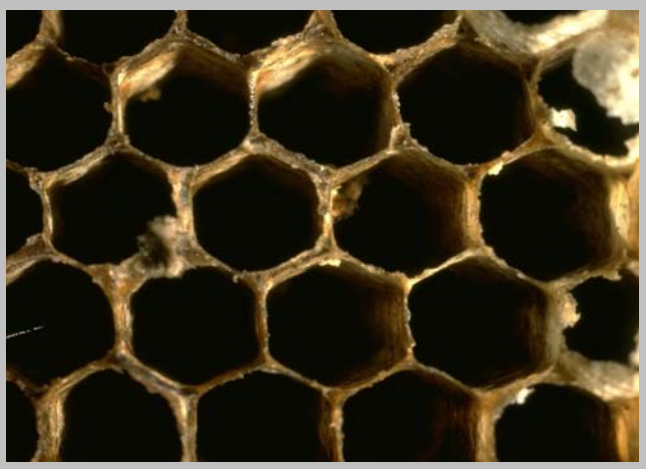
1. Represent and communicate mathematical ideas using oral, written, pictorial, and graphic representations, including line graphs.
2. Explain and justify mathematical ideas and solutions to others, using correct mathematical vocabulary.
3. Analyse and evaluate conclusions presented by others.

At the end of Level 3

1. Formulate questions about data, information, and problem situations.
2. Explain and justify mathematical ideas and solutions to others, using correct mathematical vocabulary.
3. Analyse and evaluate conclusions presented by others.

MATHEMATICS

MODULE 3



Keeping Tabs On Space Using Numbers

Table of Contents

MODULE 3: KEEPING TABS ON SPACE USING NUMBERS

Rationale 2–M3–1
Aim 2–M3–1
Goals 2–M3–1

Unit 1 — Recognizing, Determining, and Representing Patterns and Relationships

General Outcomes 2–M3–3
Specific Outcomes by Levels
- Level 1 2–M3–3
- Level 2 2–M3–4
- Level 3 2–M3–4

Content 2–M3–5
Instructional Strategies/Methods 2–M3–5
Suggestions for Assessment 2–M3–6
Resources 2–M3–6
Sample Lessons 2–M3–7

Unit 2 — When Objects Move

General Outcomes 2–M3–15
Specific Outcomes by Levels
- Level 1 2–M3–15
- Level 2 2–M3–16
- Level 3 2–M3–16

Content 2–M3–17
Instructional Strategies/Methods 2–M3–17
Suggestions for Assessment 2–M3–18
Resources 2–M3–18
Suggested Teaching and Learning Activities 2–M3–18
Sample Lessons 2–M3–20

Bibliography/Webliography 2–M3–26

Standards 2–M3–27

KEEPING TABS ON SPACE USING NUMBERS

Rationale

Keeping Tabs on Space offers the student opportunities to better understand real-world contexts and to analyse and solve authentic problems. The module is also aimed at developing students' appreciation of simple mathematical relationships in real life and the need to interpret them, bearing in mind the experiences and level of development of students.

Aim

This module is designed to develop aspects of problem solving, communication, personal development, and technological competence through the use of content in Set Theory, Algebra, and Geometry. The module is presented in two units, each of which will focus on different combinations of appropriate curriculum content.

It is designed to help the student to analyse and solve problems through the use of algebraic thinking, geometric models, and abstract and spatial reasoning. Furthermore, it seeks to develop advanced reasoning and problem-solving skills, as well as to provide opportunities to apply mathematics in cultural, environmental, and global contexts. In addition, it will provide opportunities for students to develop understanding and appreciation of the role of mathematics in past, present, and future social, political, and economic contexts.

Goals

This module is designed to support the attainment of outcomes in which students are expected to:

1. enhance their problem-solving skills through the use of algebraic thinking and the use of models (PS);
2. enhance problem-solving skills through the use of abstract reasoning and mental visualization (PS);
3. enhance critical thinking and enquiry skills (PS);
4. stimulate intellectual curiosity (PS);
5. enhance technological skills (TS);
6. develop an appreciation of the use of patterns to express mathematical ideas (AE, C);
7. develop skills in using charts, graphs, and symbols to communicate (C);
8. enhance spatial reasoning in analysing patterns and relationships of points in space (PS);
9. develop awareness of, and the skills to use, connections between operations with numbers and operations with symbols (C);
10. enhance problem-solving skills through the use of geometric models (PS);

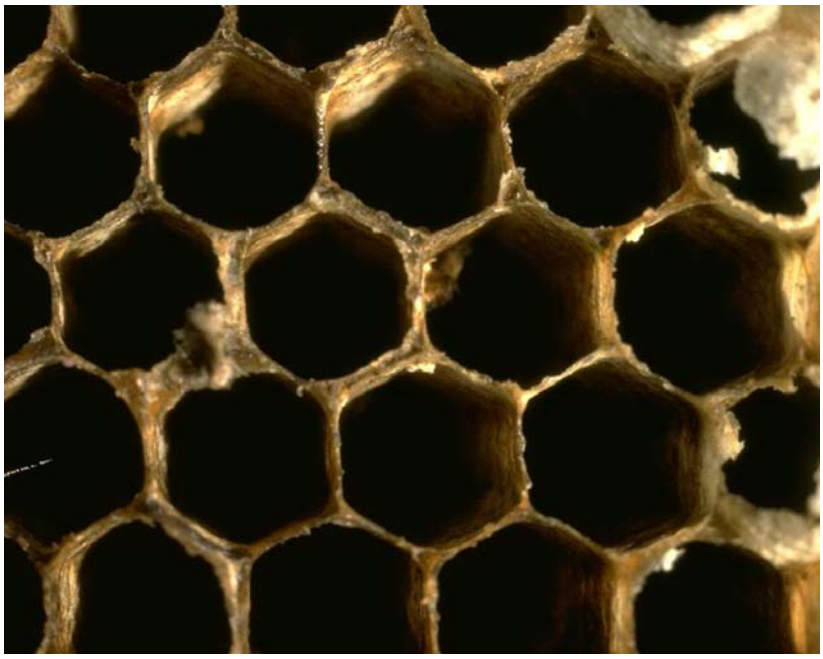
11. facilitate the construction of new knowledge through problem solving (PS);
12. enhance the skills required to analyse and evaluate mathematical thinking of others (PS, C);
13. enhance problem-solving skills through the use of mathematical representations (PS);
14. adapt and share mathematical information using a variety of sources as well as existing and developing technologies (TC);
15. enhance the skills required to understand, interpret, and analyse change in a variety of situations (PD, TC);
16. develop understanding and appreciation of the role of mathematics in past, present, and future social, political, and economic contexts (CIT);
17. develop skills in using symbols to model and interpret physical, social, and mathematical phenomena (C);
18. apply knowledge of mathematics concepts and procedures to address authentic issues in various cultural, environmental, and global contexts (CIT).

This module comprises two units:

Unit 1 — Recognizing, Determining, and Representing Patterns and Relationships

Unit 2 — When Objects Move

Unit 1



UNIT 1 — RECOGNIZING, DETERMINING, AND REPRESENTING PATTERNS AND RELATIONSHIPS

GENERAL OUTCOMES

Students are expected to:

1. demonstrate an understanding of patterns, relations, and functions (AE, C);
2. use a variety of representations, including symbols and simple line graphs, to express relationships between quantities (AE, C, TC);
3. make inferences about spatial relationships using coordinate geometry (PS);
4. use spatial reasoning to analyse mathematical situations and solve problems (PS).

SPECIFIC OUTCOMES BY LEVELS

Level 1

Students are expected to:

1. distinguish among groups of objects based on attributes or functions (objects should also be drawn from various local contexts, e.g., local birds, flowers, sportsmen, etc.);
2. determine different ways in which groups of objects could be classified;

3. apply the knowledge of classification to divide a set into a given number of subsets;
4. explain how to sort or classify a set of objects;
5. use drawings to represent different sets;
6. define sets by listing the elements or describing them in words;
7. use set builder notation to define sets;
8. distinguish among empty, equal, equivalent, finite, and infinite sets;
9. apply the concepts of universal sets, complement of a set, union of sets, intersecting sets, subsets, and disjoint sets to real-life situations;
10. use Venn diagrams to represent the relationships between two sets;
11. identify simple relationships linking two sets of numbers;
12. use arrow diagrams to illustrate relationships between sets;
13. represent a relation as a set of ordered pairs;
14. determine if a given set of ordered pairs represents a linear relationship;
15. identify missing terms, given part of a relation expressed as ordered pairs and involving two sets.

Level 2

Students are expected to:

1. locate points on the Cartesian plane;
2. represent a set of ordered pairs on the coordinate plane;
3. write a set of ordered pairs satisfying the equations $y = mx$ and $y = mx + c$, and use them to draw the graph of a linear relation for a given domain;
4. use a Venn diagram to solve problems (including word problems) involving two sets.

Level 3

Students are expected to:

1. represent the relationship between natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers in various forms, for example, on the number line, using a number tree, and Venn diagrams;
2. represent linear relations (written in any form) as graphs on the Cartesian plane;
3. represent a pair of linear relations on the same Cartesian plane;
4. use Venn diagrams to determine, in discrete cases, an ordered pair that satisfies two linear functions simultaneously;
5. use graphical methods to determine an ordered pair that satisfies two linear functions simultaneously;
6. apply knowledge of the point of intersection to solve a pair of linear equations.

Content

- ❖ Introduction to Set Theory
- ❖ Arrow Diagrams
- ❖ Ordered Pairs
- ❖ The Cartesian Plane
- ❖ Linear Equations
- ❖ Linear Graphs
- ❖ Venn Diagrams
- ❖ Subsets and Venn Diagrams (Real Numbers)

Instructional Strategies/Methods

- Abstracting
- Acting out a problem
- Activating prior knowledge
- Application teaching
- Audio-visuals
- Blogs
- Brochures
- Cheat notes
- Class publications
- Collective notebook
- Competitions
- Computer-assisted instruction (CAI)
- Concentration
- Error analysis
- Find the fib
- Find your partner
- Gaps
- Journals
- Manipulatives
- Mental arithmetic techniques
- Non-examples
- Pop quizzes

- Portfolios
- Problem-based learning (PBL)
- Quickdraw
- Quicktalk

Suggestions for Assessment

- ✓ Performance tasks
- ✓ Problem-solving tasks
- ✓ Examination of student journals
- ✓ Observation
- ✓ Worksheets
- ✓ Portfolios
- ✓ Presentations
- ✓ Socratic questioning

Resources

- Activity sheets
- Calculators (graphing)
- Cameras
- Charts
- Computers and software
- Flip chart sheets
- Manipulatives
- Markers
- Overhead projector
- Presentation software and hardware
- Scanners

SAMPLE LESSON # 1

Unit 1: Level 1

Bales of Straw

Overview

Students classify straws and make disjoint and overlapping Venn diagrams. In an extension, they make and record linear patterns.

Learning Outcomes

Students will be able to:

- classify straws according to given attributes;
- identify the attributes by which objects were sorted.

Materials

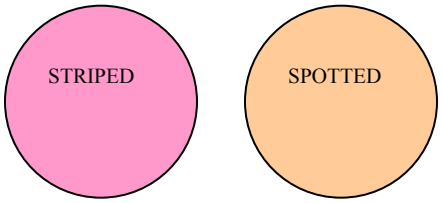
- Plastic bag containing 20 straws of varying lengths and colours for each student
- File cards or pieces of paper of equivalent size
- Wool or large rings

Instructional Plan

Each student is given a small plastic bag containing about 20 straws. Ask students to dump their straws onto their desk and tell them to find straws in their set of straws that fit each description you will give.

First, name an attribute such as "blue" and ask students to hold up a straw with that attribute. Other attributes, for example, long, red, and striped should be used.

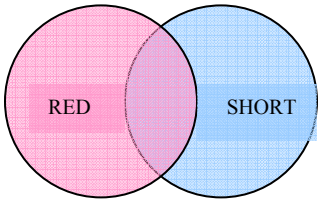
Now hold up one straw and ask the students to describe it in as many ways as they can.



Choose two descriptive words that are mutually exclusive (such as striped and spotted) and write them on separate file cards or pieces of paper. Next use two rings or make two rings out of wool and place a file card with an attribute written on it in each of the rings or circles. Tell the students that the figure is called a Venn diagram.

Now hold up a straw and ask where it should be put. [If the straw is neither striped nor spotted, place it outside the circles.] Repeat with several more straws, then remove all the straws and the rings/circles and repeat with other attributes.

Next, place two circles to overlap each other and write words that might describe a single straw (e.g., red and short) on file cards in each circle. Place a straw in the correct position and elicit reasons why it goes in that place. Now hold a straw and ask where it should be placed in this new Venn diagram. Then call on volunteers to place other straws on the diagram. [Straws that have both attributes should be placed in the overlapping section.]



Next, remove the straws and without revealing the sorting rule, place several straws in the circles, then ask for volunteers to try to place additional straws. If a student places a straw incorrectly, move it to the proper position without explaining why.

When several straws have been correctly placed, ask a volunteer to tell how he or she figured out what was the attribute used. Then verify the students' hypotheses. Repeat the activity with other attributes, using a different pair each time. To record the activity, ask each student to draw two overlapping circles, label them, and draw straws in the circles.

Sample Lesson # 2

Source: http://mathforum.org/cgraph/cplane/pexample.html

Unit 1: Level 2

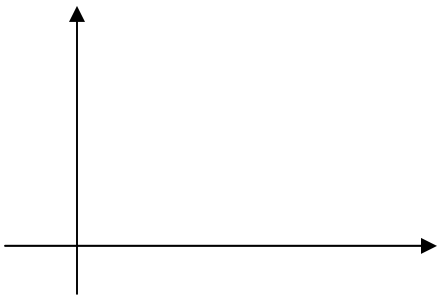
Navigating A 2-D Plane

This is Sam.

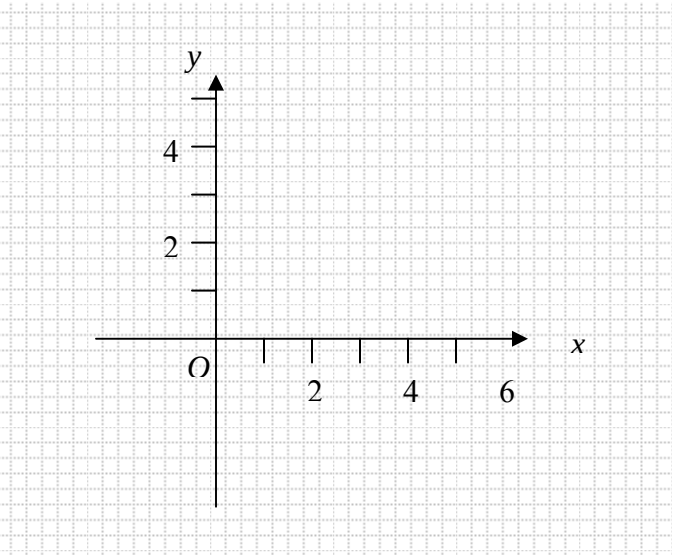
His wedding ring fell overboard while he was relaxing on his boat. Sam is not worried because he is very good at finding the right place.

The Plane

Luckily for Sam the sea bed he has to search is a plane, that is, it is a flat surface. To help Sam, two lines are drawn inside the plane. Each of these lines is called an *axis* (plural *axes*). The axes are like landmarks that are used to find different places in the plane.



The axes are labelled to make them easier for Sam to tell apart. The axis that goes from side to side is usually denoted as the *x-axis*, and the axis that goes straight up and down as the *y-axis*.



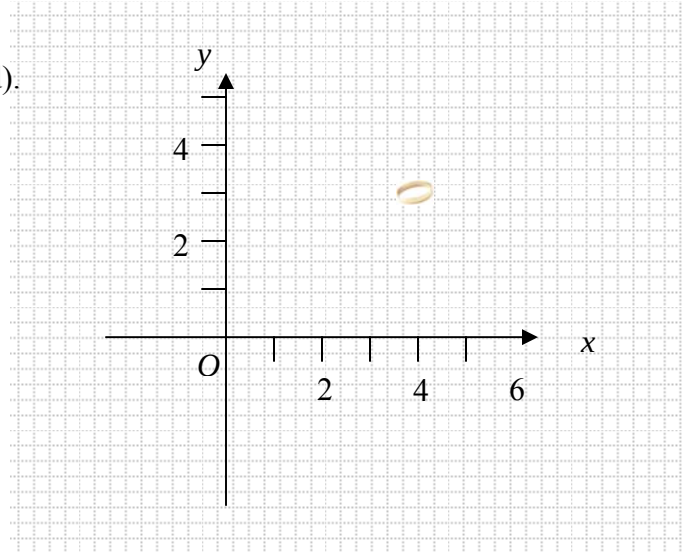
Some of the points on each axis have been marked to make them easier for Sam to find. The point where the two axes cross has a special name: it is called the *origin* and is denoted *O*.

Finding Points in the Plane

We can find every point in the plane using two numbers. These numbers are called *coordinates*. We write a point's coordinates inside parentheses, separated by a comma, like this: (5, 6). Sometimes coordinates written this way are called an *ordered pair*.

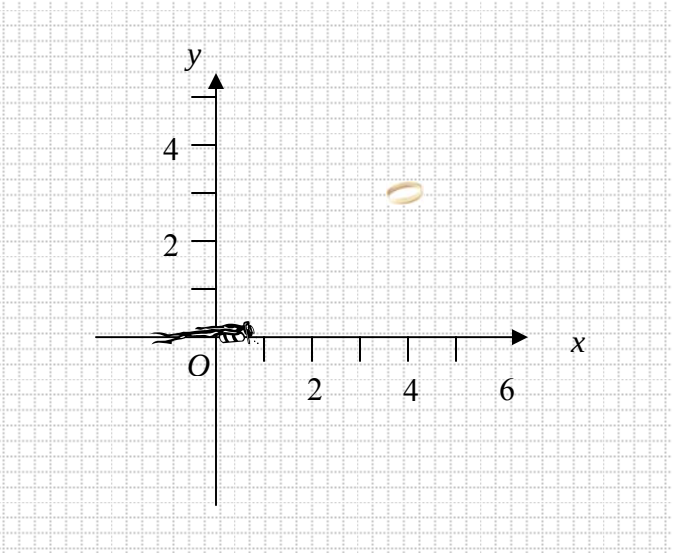
- The first number in an ordered pair is called the ***x*-coordinate**. The *x*-coordinate tells us how far the point is along the x-axis.
- The second number is called the ***y*-coordinate**. The *y*-coordinate tells us how far the point is along the *y*-axis.

Let's try an example.
The ring is on the sea bed (a plane field).

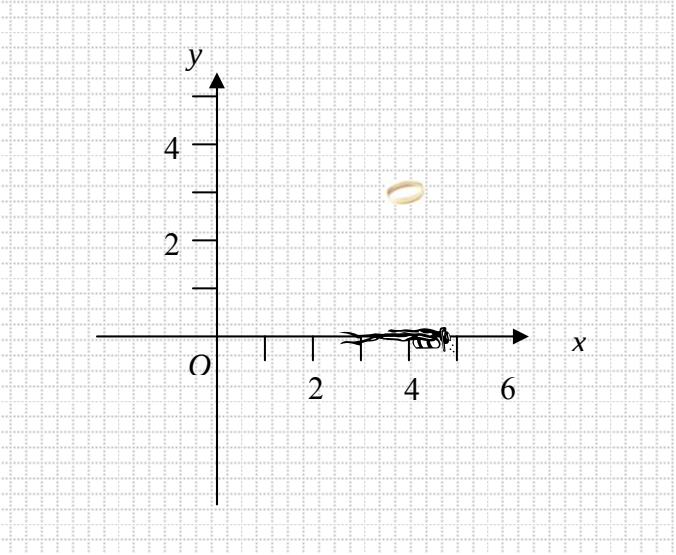


Sam knows that the ring is at point (4, 3). What should he do?

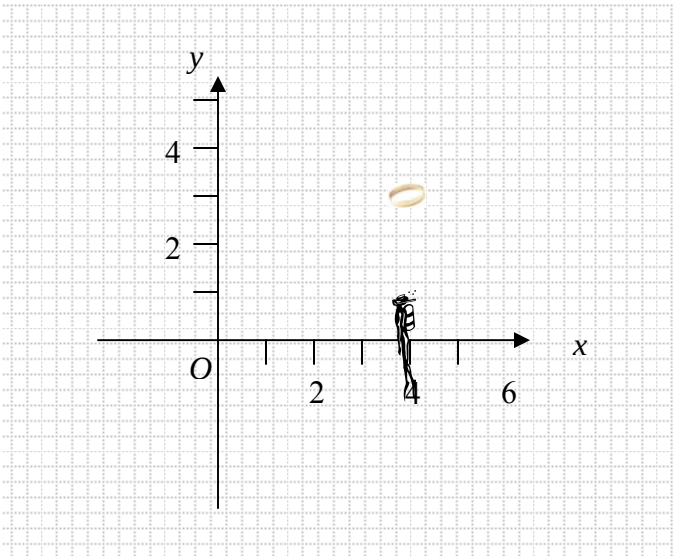
Sam starts at the origin. So far, he has not moved along the *x-axis* or the *y-axis*, so he is at point (0, 0).



Because he wants to find (4, 3), Sam swims four units along the x-axis.

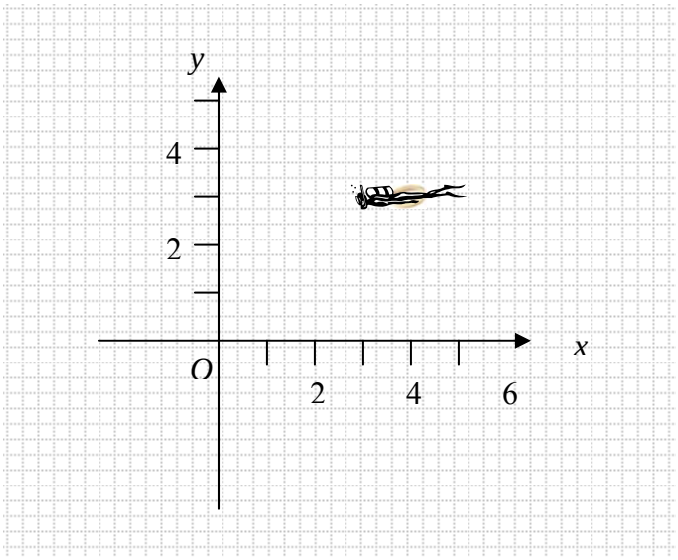


Next, Sam turns around and swims three units. He swims straight up, in the same direction that the y-axis travels.



Sam has found the point (4, 3).

He swims happily off with the ring

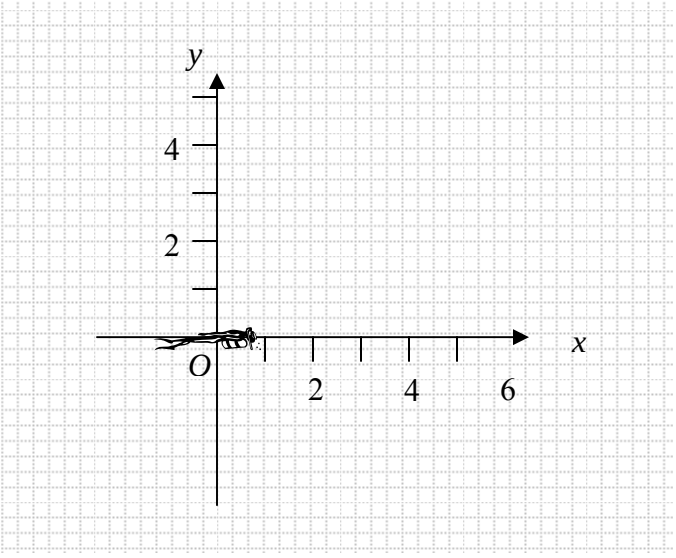


Graphing Points in the Plane

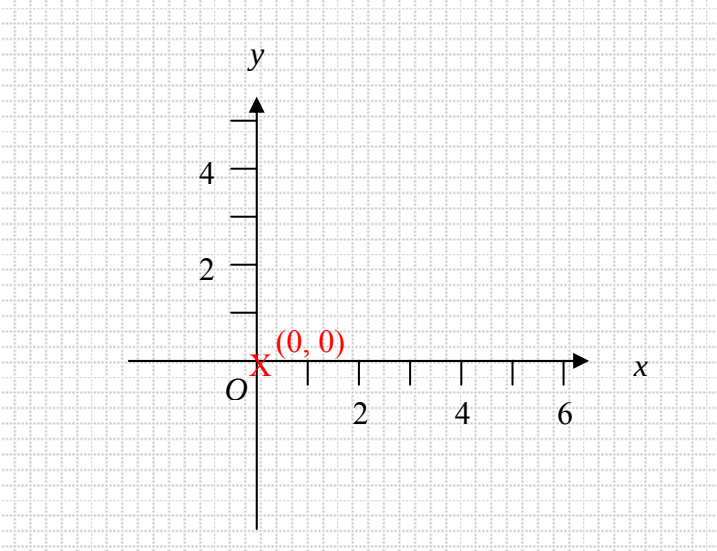
Points can be graphed the same way that Sam found the ring. Let's practise graphing different points in the plane.

We'll begin by graphing point (0, 0).

Sam starts at the origin and swims 0 units along the x-axis, then 0 units up. He has found (0, 0) without going anywhere!



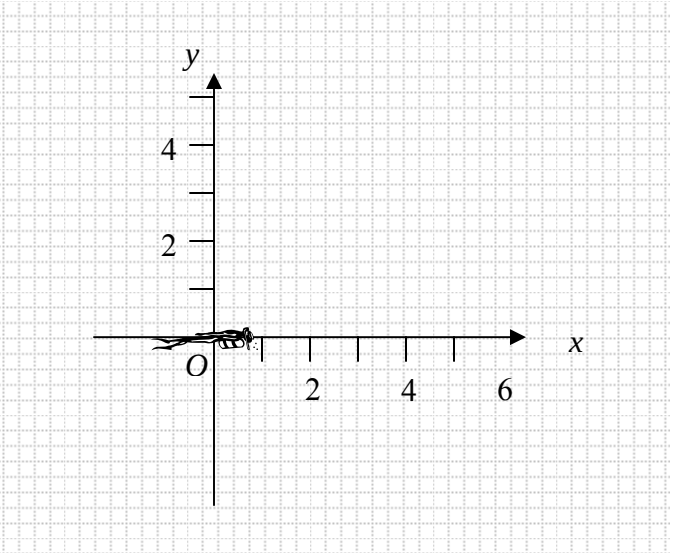
Sam marks the point with a Big ‘X’, and labels it with its coordinates.



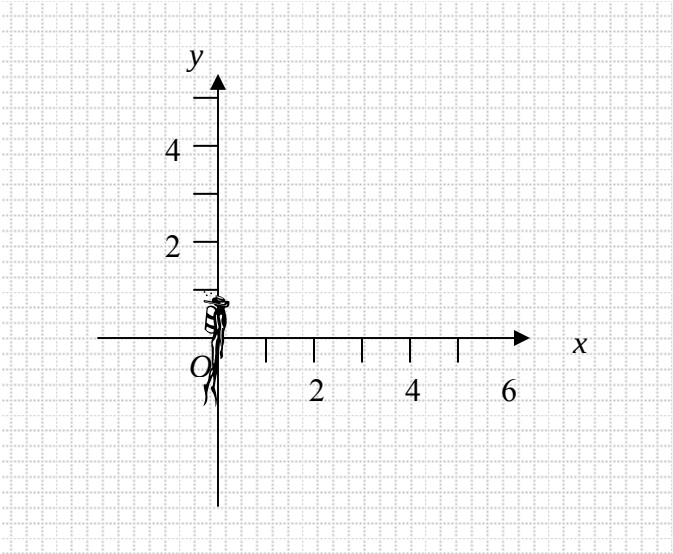
Sam has finished graphing point (0, 0).

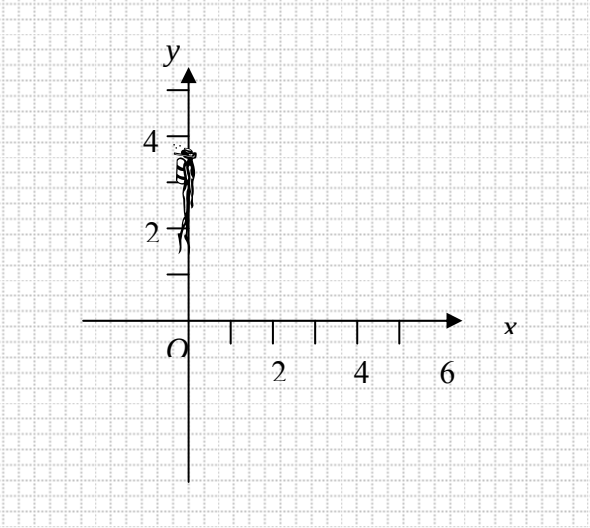
Next, let’s graph point (0, 3).

Sam starts at the origin, just like always. He swims 0 units along the *x-axis*, because the *x*-coordinate of the point he is trying to graph is 0.

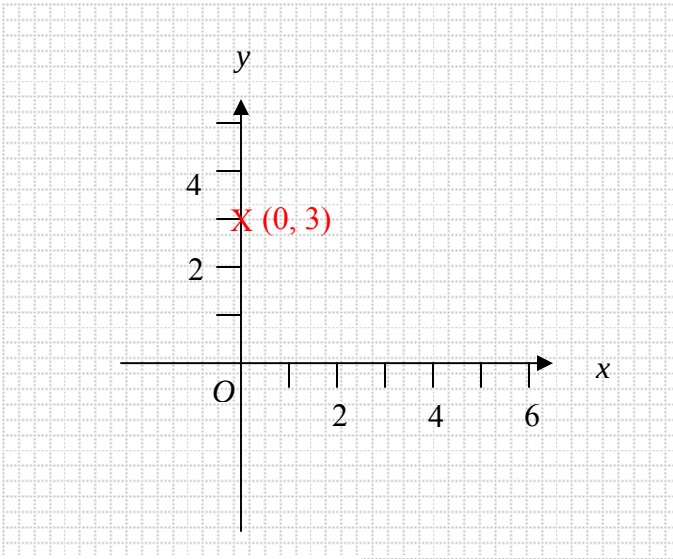


Sam swims to move 3 units straight up.

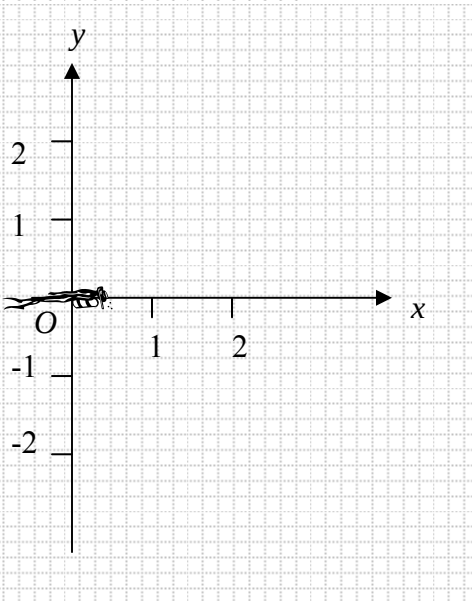




The final step is labelling the point



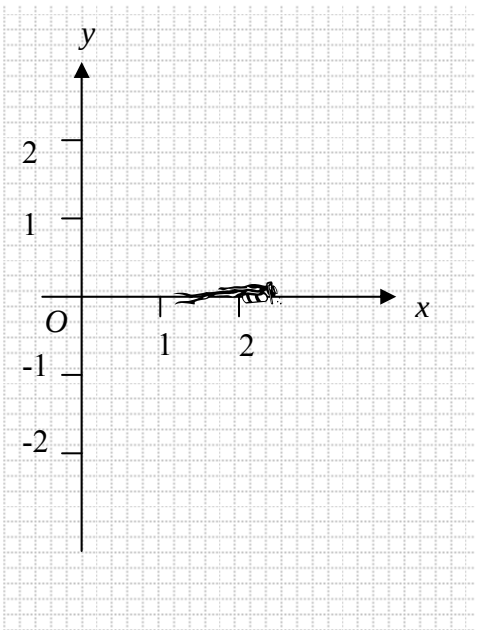
The point (0, 3) is on the *y-axis* and its *x*-coordinate is 0. Every point on the *y-axis* has an *x*-coordinate of 0, because you don't need to move sideways to reach these points. Similarly, every point on the *x*-axis has a *y*-coordinate of 0.



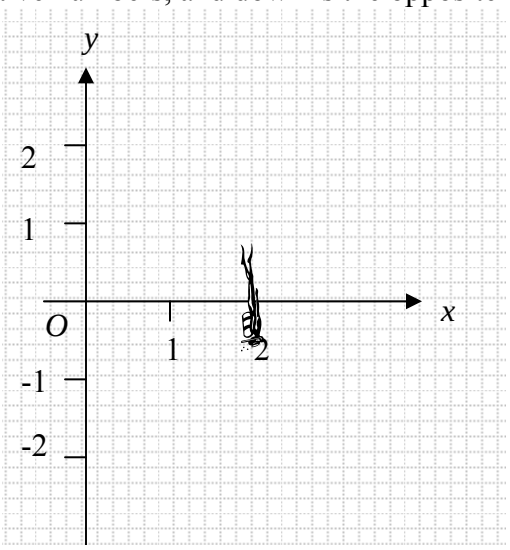
Let's end with a more complicated example: graphing point (2, -2).

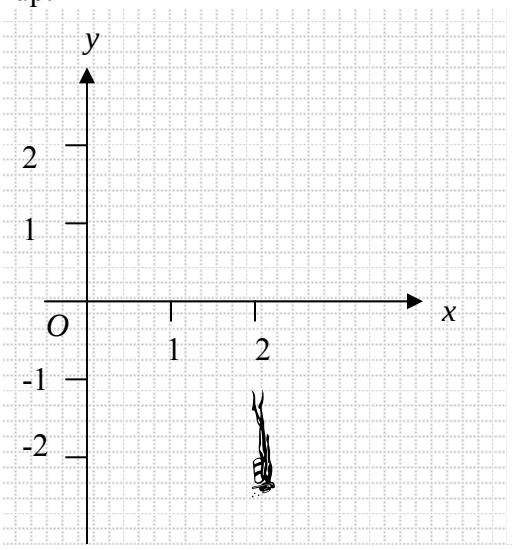
Sam begins at point (0, 0).

He swims 2 units along the *x-axis.*

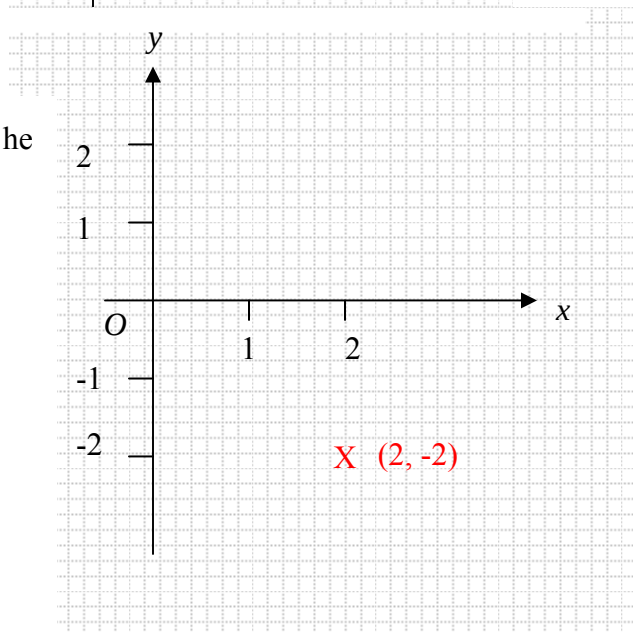


The *y*-coordinate of the point Sam wants to graph is -2. Because the number is negative, Sam swims down two units. Negative numbers are the opposite of positive numbers, and down is the opposite of up.

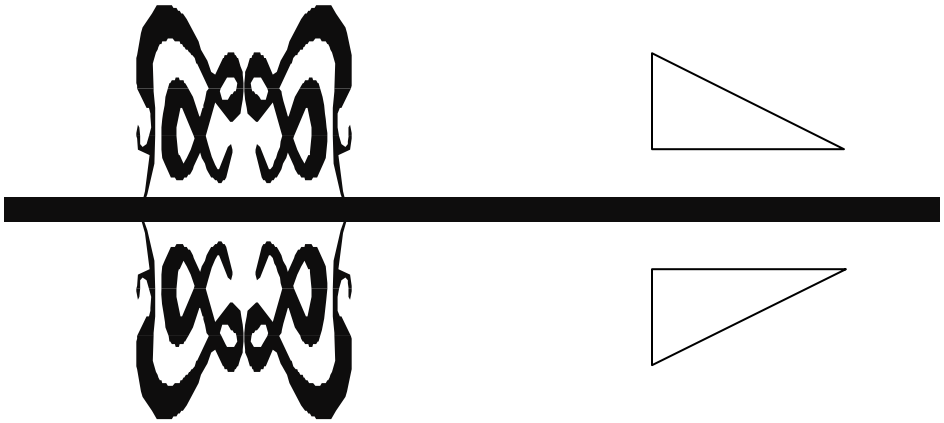




Before he leaves, Sam labels the point he graphed.



Unit 2



UNIT 2 — WHEN OBJECTS MOVE

GENERAL OUTCOMES

Students are expected to:

1. use knowledge of characteristics and properties of shapes and solids to express mathematical ideas, develop arguments, and form and analyse spatial and geometric relationships (AE, C);
2. analyse mathematical situations using symmetry and transformations of space (PS);
3. use geometrical modelling to solve problems (PS).

SPECIFIC OUTCOMES BY LEVELS

Level 1

Students are expected to:

1. classify patterns and pictures (of architecture, nature, sports, graphic design, and objects that surround them every day) in terms of translational or reflective/line symmetry;
2. describe the properties of a basic translation (including type of movement, location of a point or points relative to two axes, orientation of figures);
3. apply the knowledge of the properties of translation to generate patterns and translate an object to its image;

4. describe the properties of a single reflection in a line or surface (properties to include location, distance from axes of reference, symmetry, and geometrical properties of shapes under reflection);
5. apply the knowledge of the properties of reflection to generate patterns and reflect an object.

Level 2

Students are expected to:

1. translate figures on the Cartesian plane;
2. describe translations using horizontal and vertical directions, for example, 3 units to the right followed by 4 units down;
3. determine the coordinates of the object or the image given the "translation directions" and the coordinates of the image or the object respectively;
4. locate the line/axis of reflection given the object and its image;
5. determine and state the coordinates of the object or the image given the axis of reflection and the coordinates of the image or the object respectively;
6. reflect objects on the Cartesian plane using the x and y axes as lines of reflection.

Level 3

Students are expected to:

1. apply the concept of congruency of triangles to explain simple shapes and patterns;
2. prove congruency of triangles using the minimum conditions:
   - three sides in one triangle equal three sides in the other
   - two sides and the angle included in one triangle equal two sides and the angle included in the other triangle
   - two angles and one side in one triangle equal two angles and the corresponding side in the other triangle
   - in right-angled triangles the hypotenuse and one side are equal;
3. use the properties of congruency of triangles to solve problems;
4. use the properties of enlargement of plane geometrical figures (positive scale factors only) to solve problems;
5. apply the concept of "similarity" through concrete, visual, and abstract representations to explain shapes;
6. determine the conditions required for a set of triangles to be similar, that is:
   - the ratios of the corresponding sides in the triangles are equal
   - the ratios of two corresponding sides and the angle included are equal
   - two angles in one triangle equal two angles in the other triangle, that is, all three angles are equal;

7. use conditions to determine whether or not triangles are similar, given information about the triangles;
8. use the properties of similar triangles to solve problems.

Content

- ❖ Line Symmetry (Plane Shapes)
- ❖ Translation
- ❖ Reflection
- ❖ Cartesian Plane
- ❖ Construction
- ❖ Congruent Triangles
- ❖ Enlargement
- ❖ Similar Triangles

Instructional Strategies/Methods

- Acting out a problem
- Activating prior knowledge
- Analogies
- Audio-visuals
- Competitions
- Computer-assisted instruction (CAI)
- Craft
- Demonstrations
- Discovery teaching
- Field trips
- Guided discovery
- Idea spinner
- Inside-outside circle
- Investigation
- Journals
- Luck of the draw
- Manipulatives
- Mix/Freeze/Group

- Mental arithmetic techniques
- Non-examples
- Pattern forming
- Posters
- Quickdraw

Suggestions for Assessment

- ✓ Performance tasks
- ✓ Worksheets
- ✓ Presentations

Resources

- Manipulatives
- Computer and software
- Worksheets
- Graphing calculators

SUGGESTED TEACHING AND LEARNING ACTIVITIES

The following is an introduction to some strategies that may be used to teach this unit and to provide realistic contexts for understanding the content and achieving the outcomes:

---

1. *Classifying*

The teacher can give different groups of students the same set of approximately 12 pictures and ask them to classify the patterns in any manner they wish. One might suggest that they classify them by colour, size, shapes—whatever they want. Once the students have been introduced to the various types of planar symmetry, challenge them to reclassify their pictures according to this structure. Ask the students to make a chart or Venn diagram to classify their patterns. It is important that they understand that some patterns have more than one type of symmetry. After the students have classified their original group of pictures according to the patterns of planar symmetry, they can be encouraged to surf the Internet and look at some other patterns.

---

2. Fabric Design Printing

Art and craft can be integrated in this unit. Students can engage in fabric printing to get a hands-on experience of the concept and properties of translation. Fabric can be folded and tie-dyed in such a way as to bring out the concepts and properties of symmetry and reflection. Students should observe that both properties of translation and reflection can be observed.

---

3. Reflection

- Students can work individually to write a journal entry "reflecting" on the different uses of the word *reflection* and how reflections are used in different contexts (e.g., architecture).
- Students should be encouraged to look for a relationship between the segment (which joins a point to its image) and the mirror line. Students should notice that the mirror line is the perpendicular bisector of the segments.
- Extend practical activities to the Cartesian system by letting both axes (x and y) of the Cartesian plane act as mirrors. Students are encouraged to look for a relationship between the coordinates of the image and the original point. Upon reflection in the *y-axis* the coordinates (x, y) become $(-x, y)$. Upon reflection in the x-axis the coordinates (x, y) become $(x, -y)$.

---

4. Constructing Triangles

Teachers can start by focusing on the construction of triangles in order to lead pupils to the realization that a triangle consists of 6 parts (3 angles and 3 sides), but that one only needs to know 3 parts to be able to construct a triangle. Pupils then generate different combinations for the construction of triangles, for example, 3 sides (SSS); 3 angles (AAA); 2 sides and 1 angle (SSA, SAS); 2 angles and 1 side (AAS, ASA), and so on.

Allow pupils to use the different combinations that they have established to construct triangles and to lead them to making conjectures about which combinations would result in cases of congruency.

SAMPLE LESSON # 1

Unit 2: Level 1

Relationships Between Reflections and Symmetry

Overview

Many patterns and designs have bilateral symmetry. In this lesson, students will learn the properties of designs that have this kind of symmetry.

Learning Outcome

Students will demonstrate understanding of the properties of designs that have bilateral symmetry.

Materials

Worksheets

Instructional Plan

Bilateral symmetry: A shape has *bilateral symmetry* if the image of a shape after reflection is exactly the same as the shape itself. The mirror line that leaves the shape unchanged is called the *line of symmetry* of the shape. Many things in nature possess bilateral symmetry, for example, the human body and many kinds of leaves have bilateral symmetry. Artists often use bilateral symmetry to give balance and harmony to their work.

1. Locate the line of symmetry for each of the following pictures.


a


b


c


d

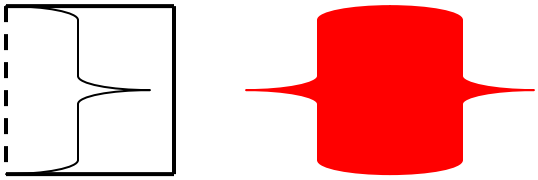

e

2. Which of the following designs have bilateral symmetry?

   For each design that has bilateral symmetry, draw in the line of symmetry on the diagram.

a b c d

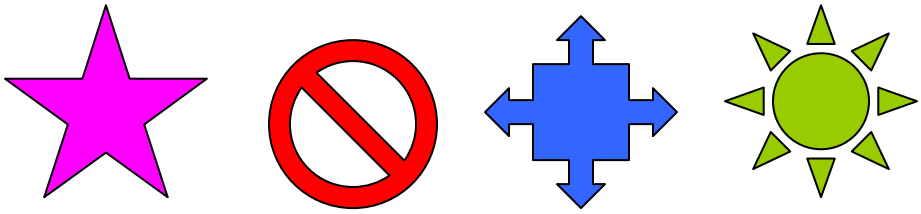
3. Make your own designs by folding a piece of paper in half and then cutting out a shape. When you unfold the shape, you will have a design with bilateral symmetry. Where is the line of symmetry?



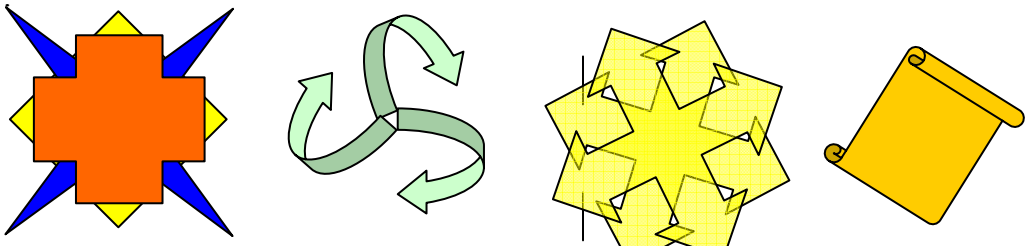
4. Find some examples of bilateral symmetry in nature and locate the line of symmetry for each object.
5. Find some examples of bilateral symmetry around you—advertising logos, familiar objects, or artistic designs—and locate the line of symmetry for each design.

If a figure or design has 3 mirror lines, we say the figure has symmetry *of order 3*, and so on.

6. What order symmetry does each of the following designs have? Find the lines of symmetry.



7. What do you notice about how all the mirror lines of a specific design are related to each other? Can you explain why this relationship holds in these designs? Will this relationship always hold in a design that has more than one line of symmetry?
8. Which of the following designs have lines of symmetry? Draw the line/lines of symmetry.



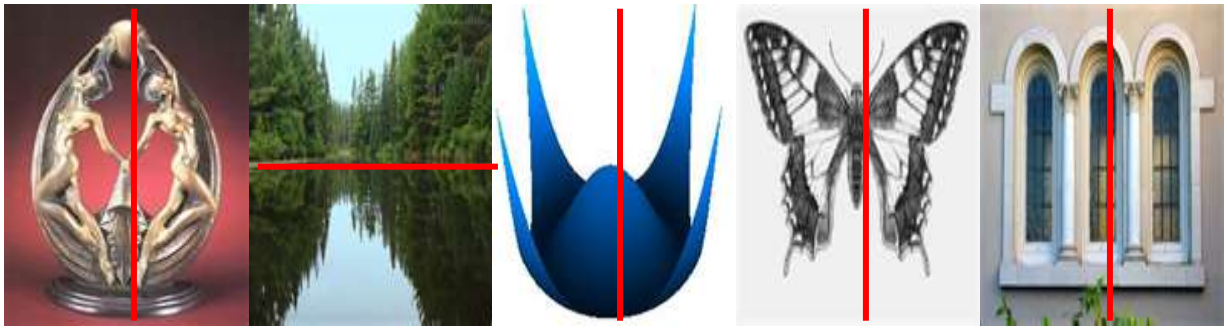
a b c d

9. Draw several designs with symmetry of order 3, 4, and 6.

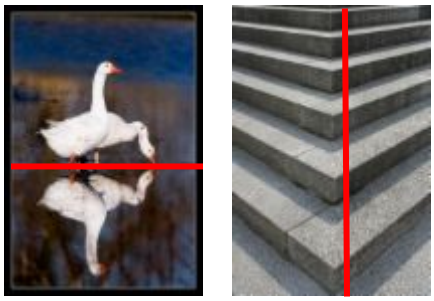
Answers

Relationships between Reflections and Symmetry

1. The red lines show the lines of symmetry for the different pictures.



2. Only a and c have bilateral symmetry. Their lines of symmetry are shown below.



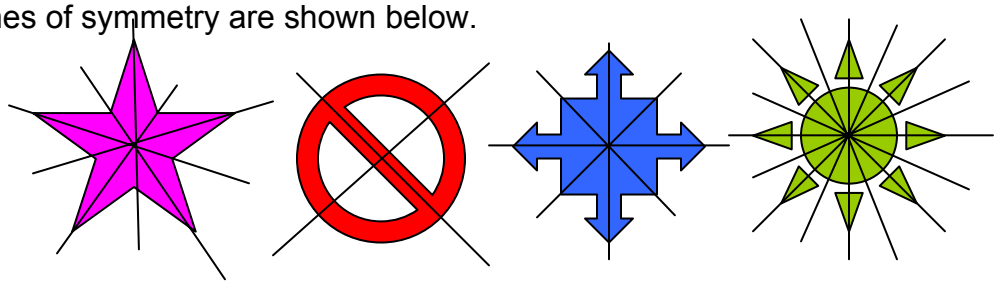
3. The line of symmetry for the cut-out shape will be along the fold line.

4. Designs will vary.

5. Designs will vary.

6. a - order 5 b - order 2 c - order 4 d - order 8

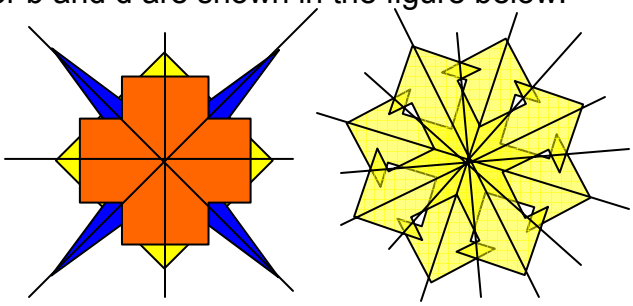
The lines of symmetry are shown below.



7. All the mirror lines of a specific design intersect at one point in the centre of the design. Each mirror line divides the design in half and passes through the centre of the design; therefore, the point of intersection of any two mirror lines will be the centre of the design. In any design with more than one line of symmetry, the mirror lines will always intersect at the centre point of the design.

8. a - order 4 c - order 8

Mirror lines for b and d are shown in the figure below.



9. Designs will vary.

SAMPLE LESSON # 2

Unit 2: Level 3

Similar Triangles

Overview

In this lesson, students will have an opportunity to engage in a hands-on approach to developing an understanding of similar triangles.

Learning Outcomes:

Students will understand similar triangles using hands-on experience of the side-side-side law. They will also demonstrate an understanding of the concept of congruency.

Prerequisite Skills

The concept of triangles.

Materials Needed

- a set of three different colour straws of three different lengths—long, medium, and short—for each student. The ratio of the long straw to the medium straw must equal

the ratio of the medium straw to the short straw for each colour; however, the corresponding lengths for each colour need not be the same

- 12 pipe cleaners for each student
- a yardstick
- an overhead projector

Procedure

The teacher will:

- decide in advance on a unique triangle that can be built with the straws, and write down the components on the board. For example, if the straws are blue, red, and yellow:
  - one long blue straw
  - one long red straw
  - one medium yellow straw
  - three pipe cleaners
- distribute a set of straws and pipe cleaners to each student;
- explain that they are to build as many different geometric shapes as they can using only the pieces in the "recipe." The shapes must be closed so that every straw end is connected to another straw by means of the pipe cleaners.

  How many different shapes can we make with these 3 straws and 3 pipe cleaners?
- As your students build, walk around the class and collect all the triangles on a yardstick. Make sure to always hang them from the same vertex, preferably the most acute angle. The triangles hanging on the yardstick provides a very strong visual cue that the shapes are the same.

  How many different shapes have the class made? Why did everyone build the same triangle?
- Repeat the exercise with a different "shape recipe," using a medium blue straw, a medium red straw, a short yellow straw, and three pipe cleaners. As the triangles are being picked up, it will become obvious to the class that the triangles are "all the same, but two different sizes."
- Place the two sizes on the overhead projector and lead the class in a discussion of what happened:
  - *Why are they all the same, but two different sizes?*
  - *Why couldn't anyone make a different triangle?*
  - *What is the relationship between the straws in the little triangle and the straws in the big one?*
  - *What if we had used 2 short yellow and 1 short blue straw?*
  - *How can we be sure that two shapes are exactly the same?*

- Explain that two shapes that exactly overlap with each other are **congruent**.

Students should make summaries and write ideas in their journals. They should also try to write rules for similar and congruent triangles.

Similar Triangles

More advanced students, can try the exercise in three dimensions. Instead of triangles, they can build a similar **tetrahedral** using 4 pipe cleaners and 6 straws each.

Does the rule hold in three dimensions as well?

Assessment

Take note of students' ideas during discussions, and review individual work recorded in math journals.

Bibliography/Webliography

Bibliography

Caribbean Examinations Council. (2006). *The new secondary level programme: Mathematics*. St. Michael, Barbados: Author.

State of Delaware. Department of Education. (1995). *Mathematics curriculum framework: Content standards*. Dover, DE: Author.

Trinidad and Tobago. Ministry of Education (2004). *Secondary Education Modernization Programme, national curriculum for Forms 4 & 5 mathematics*. Port of Spain, Trinidad: Author.

Webliography

The Math Forum @ Drexel
http://mathforum.org/

NCTM Illuminations
http://illuminations.nctm.org/LessonDetail.aspx?ID=L470

Associations/Organizations

National Council of Teachers of Mathematics
http://www.ascd.org/portal/site/ascd

New Jersey Department of Education
http://www.state.nj.us/education/

Pennsylvania Department of Education
http://www.pde.state.pa.us/

STANDARDS

MODULE 3

Mathematics Standards

MODULE 3: KEEPING TABS ON SPACE

1. Students will use algebraic methods to represent, analyse, and solve mathematical and practical situations involving patterns and functional relationships, with and without technology.

At the end of Level 1

1. Recognize and determine relationships or rule connecting two sets, drawing situations from real life.
2. Use concrete objects and combination of symbols and numbers to create expressions that model mathematical situations.
3. Create and solve linear equations.

At the end of Level 2

1. Find solutions to inequalities from a given replacement set.
2. Translate among and use tables of ordered pairs and graphs on coordinate planes, and linear equations as tools to represent and analyse patterns and relationships.
3. Create and solve linear inequalities.

At the end of Level 3

1. Solve systems of equations using graphing calculators, symbol manipulations, and other software.
2. Formulate expressions, equations, and systems of equations to model problem situations.

2. Students will apply the properties and relationships of geometric shapes and figures to represent, investigate, analyse, and solve problems, using tools and technology when efficient.

At the end of Level 1

1. Identify and create examples of line symmetry.
2. Identify reflections.

At the end of Level 2

1. Use comparative directional words (such as right, left, up, down).
2. Describe, identify, and model translations.
3. Describe and model reflections.

4. Draw and describe the results of translations and reflections.
5. Identify coordinates for a given point or locate points of given coordinates on a grid.
6. Use coordinate geometry to represent and interpret relationships defined by equations and formulas, translating among ordered pairs, graphs, and equations.

At the end of Level 3

1. Identify and model geometric figures that are congruent or similar.
2. Apply properties of equality and proportionality to solve problems involving congruent or similar shapes.
3. Describe and apply geometric properties and relationships, namely, congruence and similarity.
4. Use coordinate geometry to graph linear equations.
5. Construct geometric models, and transformation and scale drawings using a variety of methods and tools (such as paper folding, technology).
6. Identify congruent and similar figures using coordinate geometry; apply this information to solve problems.

3. Students will select and use a wide variety of tools and technology-supported methods to increase either the efficiency or quality of results.

At the end of Level 1

1. Use scientific and graphing calculators.
2. Identify and solve problems arising from mathematical situations and everyday experiences.
3. Represent and solve problem situations with concrete, symbolic, and graphic models.
4. Verify the correctness and reasonableness of results.

At the end of Level 3

1. Use a variety of technologies, including scientific calculators and computers, to evaluate and validate problem solutions.
2. Use graphing calculators, computers, and computer software effectively and efficiently to define and solve various types of problems.
3. Apply processes of mathematical modelling in mathematics, other disciplines, and practical situations.

4. Students will read, listen, and discuss to obtain mathematical information; analyse and use the information; and present and justify mathematical ideas in written, oral, and visual formats.

At the end of Level 1

1. Represent and communicate mathematical ideas using tools such as manipulatives, calculators, and computers.
2. Explain and justify mathematical ideas, strategies, and solutions,, using the correct mathematical vocabulary.
3. Identify and explain key mathematical concepts and model situations using geometric and algebraic methods.
4. Use mathematical language, notation, and symbols to represent problem situations and mathematical ideas.

At the end of Level 2

1. Identify, explain, and model key mathematical concepts and situations using oral, written, concrete, pictorial, and graphic methods, making certain that the situation is represented unambiguously and accurately.

At the end of Level 3

1. Use properties, models, known facts, and relationships to explain and defend thinking.

5. Students will relate and link mathematical ideas, concepts, and procedures within the discipline, among other areas, and to everyday situations.

At the end of Level 1

1. Use mathematical ideas from one area of mathematics to explain an idea from another area of mathematics.

At the end of Level 2

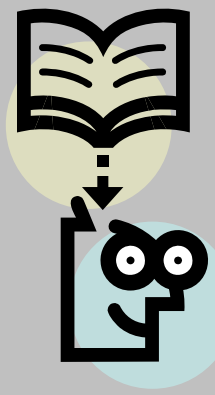
1. Identify and use connections among various mathematical topics.

At the end of Level 3

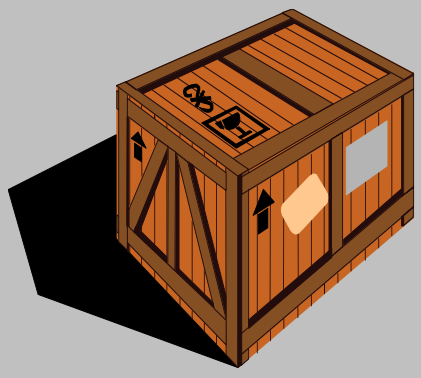
1. Use the connections among mathematical topics to develop multiple approaches to problems.

MATHEMATICS

MODULE 4



IT'S ALL ABOUT REASONING



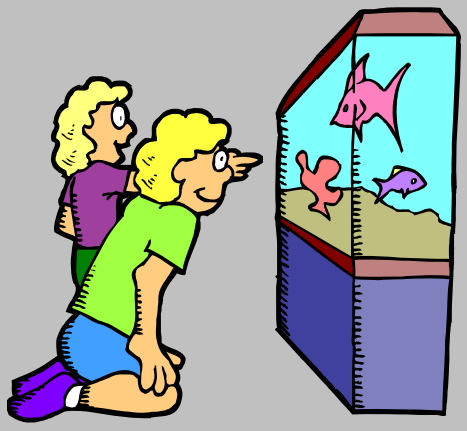


Table of Contents

MODULE 4: IT'S ALL ABOUT REASONING

Rationale 2–M4–1
Aim 2–M4–1
Goals 2–M4–1

Unit 1 — Understanding Relations

General Outcomes 2–M4–2
Specific Outcomes by Levels
- Level 1 2–M4–2
- Level 2 2–M4–3
- Level 3 2–M4–3

Content 2–M4–4
Instructional Strategies/Methods 2–M4–4
Suggestions for Assessment 2–M4–4
Resources 2–M4–4

Unit 2 — Using Geometry to Understand Space

General Outcomes 2–M4–5
Specific Outcomes by Levels
- Level 1 2–M4–5
- Level 2 2–M4–6
- Level 3 2–M4–6

Content 2–M4–7
Instructional Strategies/Methods 2–M4–7
Suggestions for Assessment 2–M4–7
Resources 2–M4–8
Suggested Teaching and Learning Activities 2–M4–8
Sample Lessons 2–M4–9

Bibliography/Webliography 2–M4–14

Standards 2–M4–15

IT'S ALL ABOUT REASONING

Rationale

Reasoning with space offers students the opportunity to explore mathematical situations using spatial reasoning. It enables students to develop geometric thinking, which is an essential skill needed if they are to become effective problem solvers.

Aim

Module 4 is designed to develop aspects of problem solving, communication, aesthetic expression, and technological competence through the use of content in Number, Geometry, Relations and Functions, and Algebra. The module comprises two units, each of which will focus on different combinations of appropriate curriculum content.

Goals

The module will support attainment of outcomes in which students:

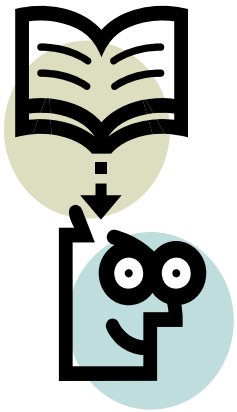
1. use spatial reasoning to analyse mathematical situations and solve problems (PS);
2. use models to represent, analyse, and interpret quantitative relationships (AE, PS);
3. understand how relationships can be expressed numerically and geometrically (C, AE);
4. improve their problem-solving skills through development of geometric thinking (PS);
5. enhance critical thinking, creative, and enquiry skills using mathematical relationships, geometric shapes, and objects (PS);
6. appreciate the importance of spatial reasoning to interpret and understand the world around us (AE);
7. communicate ideas using number, symbols, and graphs (C);
8. enhance presentation skills (C);
9. engage in activities that stimulate intellectual curiosity (PS);
10. enhance evaluation skills (PS);
11. enhance technological skills (TC);
12. prove hypotheses and theorems using different methods (PS);
13. understand how mathematical ideas relate to one another (PD, C);
14. compute fluently (PD, TC);
15. adapt and share mathematical information using a variety of sources and technologies (CIT, TC).

This module consists of two units:

Unit 1 – Understanding Relations

Unit 2 – Using Geometry to Understand Space

Unit 1



UNIT 1 — UNDERSTANDING RELATIONS

GENERAL OUTCOMES

Students are expected to:

1. communicate their understanding of various types of relations in various ways (PS, C);
2. use relations to model data (C, TC);
3. develop problem-solving skills (PS);
4. develop cooperative and inquiry skills (CIT).

SPECIFIC OUTCOMES BY LEVELS

Level 1

Students are expected to:

1. discuss and state relationships that exist in everyday situations, including family and social situations;
2. use arrow diagrams to show relationships that exist in everyday situations involving people, places, and objects;
3. use words to explain simple relationships between given sets of numbers;
4. perform basic arithmetic calculations to determine whether or not numbers belonging to specific sets satisfy a given relation;

5. draw arrow diagrams to show simple mathematical relations;
6. identify characteristics of the following relations:
    - one-to-one
    - many-to-one
    - one-to-many
    - many-to-many
7. draw arrow diagrams to show relations identified in specific outcome number 6;
8. identify different types of relations that exist among a given set of arrow diagrams.

Level 2

Students are expected to:

1. write the domain and range, given a set of ordered pairs representing a relation;
2. identify and explain the relationship that exists between elements in a given domain and its range;
3. determine whether or not an ordered pair satisfies a given relation;
4. write ordered pairs to satisfy a given relation;
5. represent relations given as a set of ordered pairs on the Cartesian plane. (values limited to integers and mixed numbers involving halves, e.g., 1½, 2½ ...)

Level 3

Students are expected to:

1. solve problems involving the concept of slope;
2. verify that slope, gradient, and steepness all represent the same concept;
3. identify straight lines that have positive slope;
4. identify straight lines that have negative slope;
5. make comparisons between lines that have similar slope;
6. explore the relationship between lines that have the same slope (parallel lines);
7. identify and state the solution to two linear equations whose graphs are given;
8. verify if a pair of x and y values satisfy two linear equations involving these variables.

Content

- ❖ Relationships
- ❖ Relations
- ❖ Arrow Diagrams
- ❖ Ordered Pairs
- ❖ The Cartesian Plane
- ❖ Graphs of Linear Equations
- ❖ Simultaneous Linear Equations
- ❖ Algebra: Substitution, Solving Equations
- ❖ Graphical Approach to Solving Simultaneous Equations

Instructional Strategies/Methods

- ▪ Cooperative learning
- ▪ Demonstrations
- ▪ Peer group teaching
- ▪ Problem-based learning
- ▪ Simulations
- ▪ Use of charts

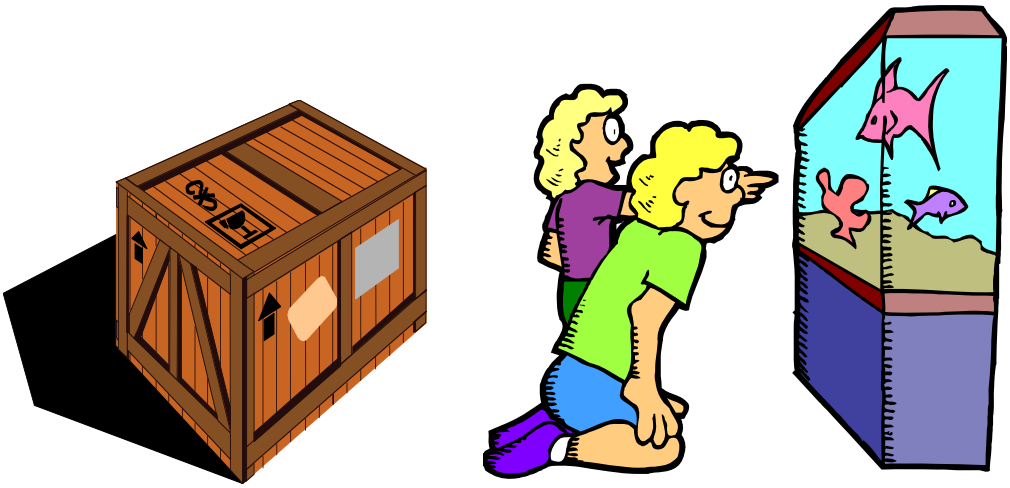
Suggestions for Assessment

- ✓ Interviews
- ✓ Observations
- ✓ Oral questioning
- ✓ Performance tasks
- ✓ Presentations
- ✓ Problem-solving tasks
- ✓ Worksheets

Resources

- Computer and software
- Calculators
- Charts
- Books

Unit 2



UNIT 2 — USING GEOMETRY TO UNDERSTAND SPACE

General Outcomes

Students are expected to:

1. solve geometric problems using spatial reasoning (PS);
2. develop problem-solving skills (PS);
3. communicate mathematical ideas in speech, in writing, and graphically (C, TC);
4. develop skills in inquiry to investigate space and objects in the environment (PS);
5. cooperate effectively with others (CIT).

Specific Outcomes by Levels

Level 1

Students are expected to:

1. explain what is meant by the terms *point*, *straight lines*, *segments*, *rays*, and *planes* and show how these may be represented on a Cartesian plane;
2. display how points, straight lines, segments, rays, and planes relate in terms of their properties;
3. use geometrical instruments to measure and draw angles in the range 0º – 360º (limit to protractor and ruler);
4. classify angles according to type (acute, right, obtuse, straight, and reflex);

5. investigate and state the properties of angles when two lines intersect;
6. use angle properties of intersecting lines to calculate related missing angles;
7. identify 3D solids and state their properties (including cubes, cuboids, cylinders, prisms, cones, pyramids, spheres);
8. identify and draw the nets of solids (including cubes, cuboids, cylinders, prisms, cones, pyramids);
9. classify triangles based on their properties as right angled, obtuse angled, isosceles, equilateral, and scalene;
10. draw triangles using ruler and protractor;
11. explain the properties of triangles such as the sum of the interior angles and the relationship between the longest side and the largest angle;
12. use the properties of triangles to solve problems;
13. outline and explain the features of parallel and perpendicular lines;
14. draw parallel and perpendicular lines using appropriate instruments;
15. recognize and name various quadrilaterals (parallelogram, rhombus, kite trapezium, rectangles, and squares);
16. identify the properties of various quadrilaterals (parallelogram, rhombus, kites, trapezium, rectangles, and squares);
17. use the properties of quadrilaterals to solve geometric problems involving missing angles.

Level 2

Students are expected to:

1. use the relationship between the exterior angle of a triangle and the opposite interior angles to solve problems;
2. explain and use the properties of parallel lines cut by a transversal to solve angle problems;
3. use the angle relationships involving parallel lines cut by a transversal and angles of triangles to solve problems (i.e., corresponding angles, alternate angles, co-interior angles, and vertically opposite angles).

Level 3

Students are expected to:

1. use geometrical reasoning to develop understanding of Pythagoras' theorem;
2. solve real-world problems using Pythagoras' theorem;
3. explore and state the reasoning underlying the three trigonometric ratios;

4. state the three trigonometric ratios;
5. use trigonometric ratios to calculate the lengths of sides in right-angled triangles;
6. develop the concept of a polygon through concrete, visual, and abstract representations;
7. solve simple geometric problems involving polygons.

Content

- ❖ Points and Lines
- ❖ Angles
- ❖ Using Geometrical Instruments
- ❖ 3D Shapes
- ❖ Triangles
- ❖ Quadrilaterals
- ❖ Pythagoras Theorem
- ❖ Trigonometric Ratios
- ❖ Polygons

Instructional Strategies/Methods

- ▪ Demonstrations
- ▪ Guided instruction
- ▪ Peer group teaching
- ▪ Multiple representations
- ▪ Real-life applications
- ▪ Simulations
- ▪ Use of charts

Suggestions for Assessment

- ✓ Problem-solving tasks
- ✓ Oral questioning
- ✓ Performance tasks
- ✓ Observations
- ✓ Interviews
- ✓ Worksheets
- ✓ Presentations

Resources

- Computer and software
- Charts
- Geoboards
- Solids
- Plane shapes
- Geometrical instruments
- Calculators
- Books

SUGGESTED TEACHING AND LEARNING STRATEGIES

The following activities can help students to better understand and apply mathematics in problem-solving and logical reasoning activities:

1. Emphasizing Real-Life Applications

Through guided discussion, the teacher helps students to appreciate the value and application of the mathematics they are studying by presenting a variety of real-life applications in context. Relating a mathematics topic to something relevant to a student's life can help to increase his/her interest in the topic, and help make mathematics more meaningful. This can be especially beneficial for struggling students who may not be able to see how the mathematics they are studying has any relevance to their daily lives. Many real-life applications of mathematics can make the content more interesting to struggling students. This can help to motivate students who may otherwise appear to dislike mathematics as a subject.

2. Incorporating Multiple Representations

The use of multiple representations can help to address the needs of students who have different learning styles. When introducing a new concept, it is beneficial to use as many representations of the concept as may be available. These may include the use of manipulatives and models, real-life examples, technology, and symbolic representations.

Classroom Application
Consider Level 1 Specific Outcome no. 13:
Outline and explain the features of parallel and perpendicular lines

1. Show examples of parallel and perpendicular lines in architecture and art.
2. Give students straws to model these lines.
3. Use dynamic geometry, such as appropriate computer software, to demonstrate parallel and perpendicular lines.
4. Have your students record several examples of lines that can be found in the world around them.

SAMPLE LESSON # 1

Unit 2: Level 3

Examining the Diagonals of Squares

Overview

A proper understanding of measurements, especially as they relate to the dimensions of plane shapes, is particularly useful in solving a variety of problems. With regard to working with squares, there are unique properties of their diagonals that can help in simplifying and solving a variety of problems.

Learning Outcomes

Students will be able to:

- determine and state the lengths of the sides and diagonals of squares;
- explore the relationship between lengths of side and diagonals of given squares;
- formulate a rule for finding the length of a diagonal of a square given the length of the side.

Materials

- Metric rulers
- Squares in the school environment
- Provided squares on worksheets

Instructional Plan

- Students are asked to estimate the length of a diagonal of a square in the environment, which is obviously too large to measure. Such a square could be drawn if necessary.
- Students should be encouraged to give reasons for their estimates.
- Students should be asked to repeat this process for measurable squares of various sizes. They should record the length of the side of each square given and also record the length of the diagonal. A table like the one shown below can be used.

  (Note that the side lengths of the squares provided should be in whole numbers measured in centimetres. It is therefore recommended that students measure the squares in centimetres.)

  As students record their data, a pattern should emerge. Students should notice that the ratio in the third column of the table is roughly 1.4. (The value will vary slightly because of measurement accuracy.) Students' tables should look like the one below:

Side Length	Diagonal Length	Ratio of Diagonal Length to Side Length
3	4.3	1.43
4	5.5	1.38
5	7.1	1.42

- Discuss the results with the class and lead students to generate the following rule for the length of the diagonal:

  Length of Diagonal = 1.4 × Length of Side

  If students are comfortable with algebraic notation, this could be written as:

  $d = 1.4s$

- Students should notice that this rule does not work exactly. In fact, the exact rule that relates the diagonal to the side length is $d = s\sqrt{2}$, and $\sqrt{2} \approx 1.4142135623\ldots$. Students may be guided to discover this rule on their own, and to reflect on the issue overnight. Some students may come up with the rule on their own, though most likely it will need to be shared and discussed during the next lesson in this unit.
- To conclude the lesson, students may be asked to use their rule to determine the length of the diagonal from the large square used at the beginning of the lesson.

Assessment

Students may be assessed in the following ways:

1. Provide appropriate worksheets on which are drawn squares for which they must determine the length of each diagonal.
2. Question or interview students on their understanding of the subject matter.
3. Identify a given square appearing in a real-world context on the school compound and have students determine the length of the diagonal.

SAMPLE LESSON # 2

Unit 2: Level 3

Exploring Diagonals in Rectangles and the Pythagorean Theorem

Overview

Further to working with squares, the approach is extended to involve wider applications. The use of the Pythagorean Theorem helps to simplify problems involving diagonal measurements and rectangular shapes.

Learning Outcomes

Students will be able to:

- determine the length, width, and length of diagonals of rectangles;
- understand the concept of the Pythagorean theorem.

Materials

- Centimetre rulers
- Calculators
- Rectangles

Instructional Plan

At the end of the previous lesson, students examined the pattern that relates the length of a diagonal to the side length of a square ($d = 1.4s$).

Use of a calculator to explore this relationship for a few sample squares would be an appropriate review activity at this point.

Students may now be asked: "What relationship do you predict between the length and width of a rectangle and its diagonal?" Students should individually record their predictions.

After students have recorded their predictions, display a Motivational Problem, which asks students to consider the following:

Starting from home, Johnny drives 5 kilometres south and 12 kilometres east. He knows of a diagonal shortcut that he can take to get back home. If Johnny takes the shortcut, how many kilometres will he travel on the return trip?

Discuss this problem with the class, and allow students to estimate and record their answers. Do not solve the problem now, as students will return to it at the end of the lesson.

Present students with many rectangles and give them time to explore the lengths, widths, and diagonals. These rectangles could also be floor tiles, chalkboards, windows, desktops, and so forth.

Encourage the students to also, on a blank piece of paper, sketch the rectangle that they are considering and record the measurements.

At the end of the activity, ask the students what methods they used to estimate the length, width, and diagonal. If any students used their shoe size, ask if those with small feet had different answers than students with larger feet. What difference does this make?

After students have made their predictions, they should pair up and compare. Students should not alter their predictions based on this discussion, because their predictions will be revisited at the conclusion of the lesson.

Regardless of the warm-up problem that students considered, they should recognize that the diagonal is always longer than either the width or the length of the rectangle. Ask students to make predictions about how much longer it will be. That is, will the diagonal double the shorter side? Will it be one-and-a-half times the longer side? Will it be longer or shorter than the sum of the length and the width?

Using the examples of rectangle now provided by the teacher, each pair of students should measure the side length of each rectangle using a ruler. Then, using a ruler, they should draw a diagonal and measure it. These three measurements should then be recorded. (Although this need not be a requirement, you may want to encourage students to use centimetres for their measurements. The rectangles provided should be dimensions that are whole numbers of centimetres.)

Length	**Width**	**Diagonal Length**	**Ratio of Diagonal Length to Side Length**
1	3	3.2	$\sqrt{10}$
2	3	3.6	$\sqrt{13}$
3	3	4.2	$\sqrt{18}$
4	3	5	$\sqrt{25}$
5	3	5.8	$\sqrt{34}$

Students should now be guided in order to complete the fourth column. [The relationship is that $n = \text{length}^2 + \text{width}^2$. Note that students can find the value of n by squaring the value in the third column and then rounding the answer to the nearest whole number. For instance, the diagonal length given in the third column of the second row is 3.6. When this value is squared and rounded to the nearest whole number, the result is $3.6^2 \approx 13$; and, the value in the fourth column of the second row is $\sqrt{13}$.]

Students' attention should be drawn to the rectangle with side lengths of 3 and 4 and a diagonal length of 5. Ask students, "What makes 3, 4, and 5 so special? What can you do mathematically to rewrite this in a different way, not using square roots?"

Students may not immediately see a relationship between the values of the first two columns and the value of the fourth column. If that happens, allow students to work in groups of 3–4 to discuss any patterns that they see. After several minutes of discussion, bring the class together to share their ideas. At this point, it is likely that at least one group will see a relationship between the length, width, and diagonal. Some possible relationships are:

$$l^2 + w^2 = d^2$$

$$d = \sqrt{l^2 + w^2}$$

If not, questions may be posed that lead students to this discovery.

When students have found the pattern, they should return to their predictions and compare them with the relationship that they discovered.

Before the end of the lesson, be certain to discuss the motivational problem about Johnny. Students may not realize that the original problem relates to the lesson, because it uses a triangle, not a rectangle. The teacher should point out that the shortcut is actually the diagonal of a rectangle with side lengths of 5 kilometres and 12 kilometres. Students should be able to then determine the length of the shortcut [13 kilometres].

Students may also be allowed to examine their measurements of the various objects that they measured at the beginning of class.

How did their measurements of the diagonal compare to the actual length?

Assessment Exercises

1. Ask students to measure the length and width of common rectangles (a sheet of paper, a book cover, etc.). Based on those measurements, challenge them to predict the length of the diagonal, and then measure the actual lengths to confirm their predictions.
2. Provide some written questions involving the use of Pythagoras' theorem.
3. In addition to the above problems, the teacher should also conduct observations and use questioning to monitor students' developing understandings:
   - as students complete the in-class activity;
   - as students discuss the problems with their partners.

Bibliography/Webliography

Bibliography

Bright, G. W., Waxman, H. C., & Williams, S. E. (Eds.). (1994). *Impact of calculators on mathematics instruction*. Lanham, MD: University Press of America.

Trinidad and Tobago. Ministry of Education. Secondary Education Modernization Programme. (2004). *National curriculum for Forms 4 & 5 mathematics*. Port of Spain, Trinidad: Author.

Van De Walle, J. A. (1994). *Elementary school mathematics: Teaching developmentally*. New York: Longman.

Webliography

Alaska Department of Education & Early Development
http://www.eed.state.ak.us/tls/Frameworks/content.htm
http://www.eed.state.ak.us/tls/Frameworks/mathsci/ms0_1tit.htm

Teaching Today
http://teachingtoday.glencoe.com/

Associations/Organizations

Association for Supervision and Curriculum Development
http://www.ascd.org/portal/site/ascd

National Council of Teachers of Mathematics
http://www.nctm.org/

STANDARDS

MODULE 4

Mathematics Standards

MODULE 4: IT'S ALL ABOUT REASONING

1. Students will use algebraic methods to represent, analyse, and solve mathematical and practical situations involving patterns and functional relationships, with and without technology.

At the end of Level 1

1. Recognize, reproduce, extend, create, and describe various types of relations.
2. Use verbal rules, open sentences, and arrow diagrams to represent relations.
3. Use arithmetic calculations to determine whether or not members satisfy a given relation.
4. Recognize and represent, using arrow diagrams, relations that are one to one, many to one, one to many, and many to many.

At the end of Level 2

1. Use rules and variables to represent relations as a set of ordered pairs.
2. Examine ordered pairs and determine the relationship that exists.
3. Use simple coordinate geometry to represent relations given as ordered pairs on the Cartesian plane (values limited to integers and mixed numbers involving halves).

At the end of Level 3

1. Recognize and compare the slopes of straight lines.
2. Examine the slopes of parallel lines.
3. Determine graphically the solution of two intersecting straight lines and verify the results.

2. Students will apply the properties and relationships of geometric shapes and figures to represent, investigate, analyse, and solve problems, using tools and technology.

At the end of Level 1

1. Use, describe, name, and represent points, straight lines, segments, rays, and planes.
2. Draw and measure angles using geometrical instruments.
3. Classify angles as acute, right, obtuse, straight line, reflex.

4. Solve problems involving missing angles when straight lines intersect.
5. Describe, name, and label related geometric two- and three-dimensional shapes such as triangles, parallelograms, rhombus, kites, trapezium, rectangles, squares, cubes, cuboids, and so on.
6. Draw two-dimensional shapes and three-dimensional geometric solids and construct triangles using rulers and protractors.
7. Draw the nets of geometric solids such as cubes, cuboids, cylinders, pyramids, prisms, and cones.
8. Solve simple geometric problems involving triangles and missing angles.
9. Identify and draw parallel and perpendicular lines.
10. Apply knowledge of the properties of quadrilaterals to solve geometric problems involving missing angles.

At the end of Level 2

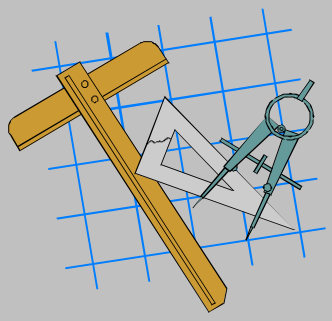
1. Describe and apply a variety of strategies for solving problems involving missing angles in situations involving triangles and parallel lines cut by transversals.

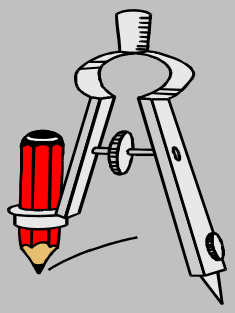
At the end of Level 3

1. Develop, explain, and apply the Pythagorean theorem.
2. Use basic trigonometric ratios (right triangle trigonometry) to solve problems.
3. Apply the properties of polygons to solve geometric problems involving polygons.

MATHEMATICS

MODULE 5





MEASURING WHEN IT MATTERS

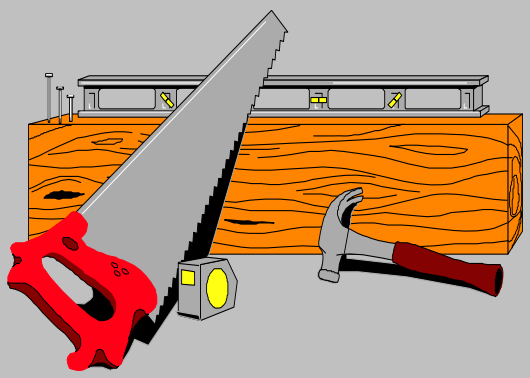


Table of Contents

MODULE 5: MEASURING WHEN IT MATTERS

Rationale 2–M5–1
Aim 2–M5–1
Goals 2–M5–1

Unit 1 — Making Accurate Measurements

General Outcomes 2–M5–3
Specific Outcomes by Level
- Level 1 2–M5–4
- Level 2 2–M5–4
- Level 3 2–M5–5

Content 2–M5–5
Instructional Strategies/Methods 2–M5–5
Suggestions for Assessment 2–M5–6
Resources 2–M5–6
Suggested Teaching and Learning Activities 2–M5–6

Unit 2 — Making Estimates of Area and Perimeter

General Outcomes 2–M5–10
Specific Outcomes by Levels
- Level 1 2–M5–10
- Level 2 2–M5–11
- Level 3 2–M5–11

Content 2–M5–11
Instructional Strategies/Methods 2–M5–11
Suggestions for Assessment 2–M5–12
Resources 2–M5–12
Suggested Teaching and Learning Activities 2–M5–12

Unit 3 — Construction of Shapes

General Outcomes 2–M5–17
Specific Outcomes by Levels
- Level 1 2–M5–18
- Level 2 2–M5–18
- Level 3 2–M5–18

Content 2–M5–19
Instructional Strategies/Methods 2–M5–19
Suggestions for Assessment 2–M5–19
Resources 2–M5–19
Sample Lesson 2–M5–20

Bibliography 2–M5–22

Standards 2–M5–23

MEASURING WHEN IT MATTERS

Rationale

Measuring When It Matters offers students the opportunity to develop basic reasoning and problem-solving skills as well as to use practical knowledge of measurements drawn from various contexts to better understand their environment. In addition, the module provides opportunities for students to understand and appreciate the social skills and values displayed by groups and individuals in a variety of situations.

The module lends itself to a student-centred and constructivist approach to instruction, employing practical activities, projects, and cooperative learning activities to support learning.

Aim

Module 5 is designed to use appropriate content in Measurement to develop aspects of aesthetic expression, citizenship, communication, personal development, problem solving, and technological competence.

Goals

The module will support attainment of outcomes in which students are expected to:

1. develop estimation skills (PS);
2. develop skills in doing geometrical constructions (PS, TC);
3. gain an understanding of the use of measurements (PD, TC);
4. gain an understanding of the use and importance of precision (PD, TC);
5. use direct and indirect measurements to compare, explain, and interpret scales and measures (C, PS);
6. communicate information on measures using appropriate units (C);
7. develop basic mathematical skills and operations with formulae (C, PS);
8. develop and apply a range of formulae and procedures for measurement (C);
9. demonstrate an understanding of the perceptions and feelings of others (PD);
10. improve personal communication with others (PD);
11. demonstrate improved self-awareness (PD);
12. express mathematical ideas in common everyday language (C);
13. express mathematical ideas to reflect logical thinking (PS, C);
14. develop collaborative and decision-making skills and attitudes (PD, C);
15. enhance critical thinking, creative, and problem-solving skills using mathematical connections, geometrical thinking, and spatial reasoning (PS).

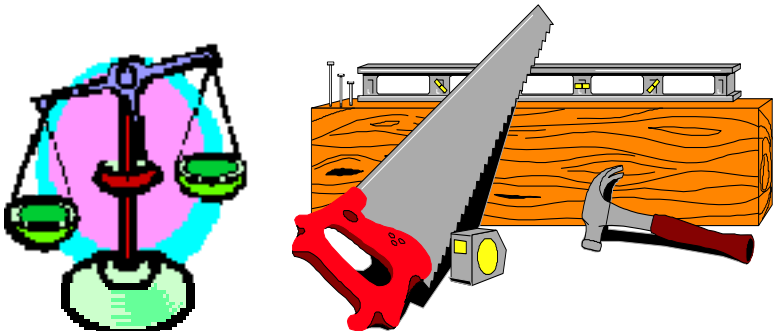
This module comprises three units:

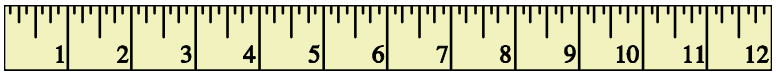
Unit 1 — Making Accurate Measurements

Unit 2 — Making Estimates of Area and Perimeter

Unit 3 — Construction of Shapes

Unit 1





UNIT 1 — MAKING ACCURATE MEASUREMENTS

GENERAL OUTCOMES

Students are expected to:

1. understand measurable attributes of objects, units of measure, and systems of units (PD);
2. appreciate the necessity to adopt standard units of measure (PD);
3. understand and use the relationships between simple units and sub-units in the metric and imperial systems (PD);
4. select and use a variety of appropriate processes, techniques, and tools, including technological tools, to make measurements (PD, PS, TC);
5. evaluate situations and processes that require varying degrees of precision (consistency) and accuracy (correctness) when making measurements, and choose appropriate units to reflect the required precision and accuracy (PD, PS);
6. use precise mathematical language and other representations to communicate measurements and mathematical results (C);
7. develop confidence and inquiry skills, and enhanced motivation to explore mathematics as a multi-faceted discipline (PD);
8. understand and appreciate the role of mathematics in past, present, and future social, political, and economic contexts (CIT);
9. enjoy "doing mathematics" and solving problems in a rapidly changing technological society (AE, PD, PS, TC).

SPECIFIC OUTCOMES BY LEVELS

Level 1

Students are expected to:

1. distinguish between standard and non-standard (informal) measures;
2. describe appropriate attributes of quantities for each unit of measure;
3. describe/interpret informal measures used in their local context (e.g., a "cwmh," zoot, handspan, real cheap, "maga," footprints, plastic straws, toothpicks, thimble, bottle caps, hefty, featherweight) and use them as a rationale for standard units;
4. use appropriate equivalent measures between metric and imperial systems to estimate measurements (e.g., 1 km is just over ½ mile; 1 m is about 1yard; 1 kg is a little more than 2 lbs; 1 ton (imperial) is a little more than a tonne (metric));
5. use the computer and other technologies to tell time in different countries and to compare with time in Trinidad and Tobago;
6. compare the metric system with the denary system and use the comparison to identify relationships between the sub-units and other units of the metric system.;
7. justify the use of a particular unit of measure for quoting a given quantity (e.g., use of yards instead of inches, or kilometres rather than metres in real-life situations);
8. locate and explain how the dual use of both the metric and the imperial systems of units apply in their local context.

Level 2

Students are expected to:

1. use both the metric and imperial systems of units to effectively compare large and small quantities in both systems of units (e.g., centimetre *vs* inch; kilometre *vs* mile; kilogram *vs* pound);
2. correctly convert quantities quoted in one set of units to other units (degrees to revolutions, millimetre to metre, and kilometre to mile);
3. read and interpret scales accurately;
4. measure quantities such as length and size of angle to a given degree of accuracy;
5. quote numerical results to an acceptable level of accuracy depending on established requirements;
6. communicate measurements made to a specified degree of accuracy;
7. recognize that all measurements are approximations and include error measurements to express varying degrees of accuracy;
8. select and use appropriate units and tools to measure quantities to a specified degree of accuracy in a particular situation;
9. solve problems involving simple conversions of square units such as m^2 to cm^2 and mm^2 to cm^2, and vice versa.

Level 3

Students are expected to:

1. determine distances on maps and models in accordance with a given scale;
2. create accurate 2D drawings of geometrical figures, charts, and graphs, given a particular scale requirement;
3. determine the change in area of a rectangle if the length or breadth changes by a given amount or by a given percentage;
4. create 3D models of prisms from nets;
5. determine the corresponding change in surface area of a prism when there are changes in length and/or breadth by a given amount or percentage;
6. select and use appropriate units and tools to measure quantities to a degree of precision needed in a particular problem-solving situation.

Content

- Metric System of Units – mm, cm, m, km, g, kg, tonne
- Imperial System of Units – inches, feet, yard, mile, ounces, pound, ton
- Degree of Accuracy – specifications
- Errors in Measurement – qualitative discussion
- Conversions – metric and imperial systems
- Operations With Units – metric and imperial systems
- Using Measuring Instruments to Measure Distance, Weight
- Scales – maps, scale drawings
- Standard Time
- Informal Measures – global and local contexts

Instructional Strategies/Methods

- Practical activities
- Brainstorming and discussion
- Games
- Direct instruction
- Computer-based activity
- Explorations – geometrical shapes and figures
- Demonstrations – of skills required in using and reading measuring instruments

Suggestions for Assessment

- ✓ Observation – of students' participation in group activities, contribution to group, use of manipulatives and measuring instruments
- ✓ Results – accuracy of reading and interpreting scales
- ✓ Reports of practical work and activities
- ✓ Calculations – reasonableness of estimates
- ✓ Peer evaluation – of students' performance in groups
- ✓ Posters – non-standard units
- ✓ Charts – metric and imperial systems, conversion flow diagrams
- ✓ Research – relevance, accuracy, presentation

Resources

- Charts – conversion arrow diagrams
- Computer and software – drawing shapes
- Geoboard – physical, interactive, dotted paper
- Internet facilities – research
- Measuring instruments including digital and laser (ruler, metre rule, measuring tape, thermometer, measuring cylinder, balance, stopclock)
- Maps & grid sheets
- Manipulatives – 2D geometric shapes, 3D models (prisms) and their nets
- Non-standard measures
- Elastic bands/rubber bands and scissors
- Bristol board

SUGGESTED TEACHING AND LEARNING ACTIVITIES

Principle: This module focuses on teaching children "what it means to measure" as opposed to using a procedural approach to teaching children "how to measure." The concept of measurement is embodied as a process. Some relevant activities to facilitate students' understanding of this are presented as follows:

1. Practical Work Stations

Activity: Non-Standard Measures

Non-standard measures are used to make rough estimates of length, mass, time, temperature, and so on. These include:

- a string with knots tied along its length for comparing dimensions of objects
- a rod of known length for measuring dimensions of classroom
- using the length of a shadow to tell time
- handspan for distance
- fingertips for estimating temperatures of liquids (caution – be sure to use moderate temperatures)
- an object of known mass or a heap of coins for weighing
- jars, cups, cubes, etc. for volume
- fingertips to measure pulse rate, etc. (N.B. timing for 15 seconds and multiplying by 4 magnifies the error)

Students may check the reasonableness of their estimates by using measuring instruments (ruler or metre rule, thermometer, measuring cylinder, balance, stopclock, and pulsemeter) to obtain more accurate results.

Activity: Body Measurements

Students use a dressmaker's measuring tape to measure and record body measurements such as:

- height, distance from shoulder to tip of index finger, distance from shoulder to elbow, distance from elbow to tip of index finger, distance from hip to knee, distance from knee to base of foot
- circumference of wrist, neck, waist, head, base of thumb
- length of nose, thumb, foot, handspan, strand of hair

Students will record measurements using centimetre and millimetre units and use actual measurements to compare handspans, heights, length of feet, etc.

Students can also calculate significant ratios such as distance from shoulder to tip of index finger, distance from shoulder to elbow, and so on.

Activity: Sketching Lines and Shapes Using the Computer

Students use the computer (Autoshapes or Draw feature) to sketch lines and regular geometrical shapes (triangles, squares, rectangles, rhombus, parallelograms, pentagons, and hexagons). They may make estimates of different dimensions and then use a ruler

to make more accurate measurements of the dimensions and compare the reasonableness of their estimates.

(Students may adapt this activity to use with a geoboard if computer is inaccessible.)

Activity: Making a Ruler

Students use suitable materials to make a 15 cm or 30 cm ruler showing clearly the cm and mm divisions. Students may add the inch scale to their rulers.

Activity: Standard Time

Students use the computer to tell time in different countries and to compare with time in Trinidad and Tobago.

Activity: Mass and Weight

Students will use a balance to determine the mass of various objects and record the measurements in kilograms.

The same objects will be placed on a Newtonmeter (an instrument for measuring weight) and their weight (i.e., the force that they exert on the instrument) recorded in Newtons.

Students will compare measurements made for mass and weight using a table (or graphical form) to establish quantitatively the relationship between mass and weight.

Activity: Recovery Time Activity

Students record their normal pulse rate and then engage in an activity such as running up and down a flight of stairs or running a measured distance for a short stipulated time (10-20 minutes). At the end of the activity, they measure and record pulse rate every minute until it returns to normal. They record, present, and use the results to determine the time taken to return to their normal pulse count.

---

2. Visual Demonstration

- Manipulatives such as regular 2D geometrical shapes, 3D solids (prisms), and their nets can be used to obtain actual measurements of the dimensions. The values obtained for each shape or solid can then be examined to see how these measurements relate.

---

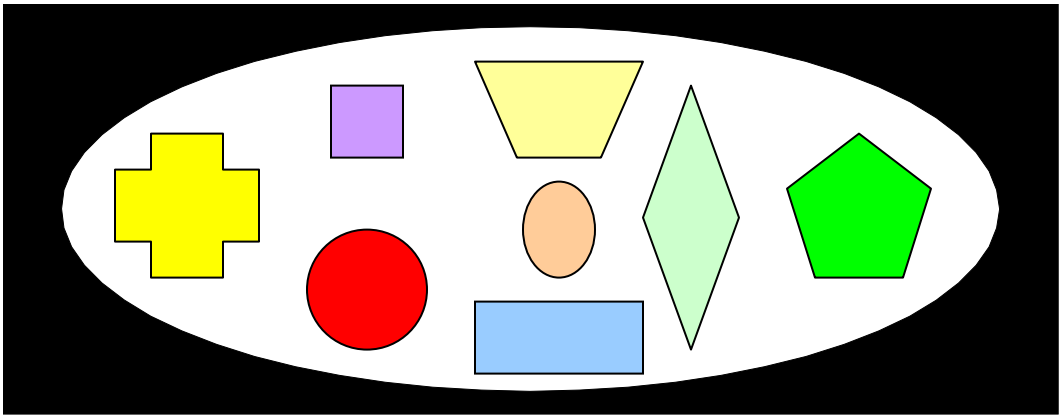
3. Use of the Internet

- research flight schedules for aircrafts, ships, and trains: standard time
- research use of shapes and solids in real world
- research laser measuring instruments
- locate and use maps
- draw and change dimensions of shapes to observe effect on area, using computer software (e.g., *Textease*, *Geometer's Sketch Pad*)
- tell time in different countries and compare to Trinidad and Tobago

Further work can extend to researching:

- very small and very large distances such as atomic and planetary dimensions
- how light and sound are measured

Unit 2



UNIT 2 — MAKING ESTIMATES OF AREA AND PERIMETER

GENERAL OUTCOMES

Students are expected to:

1. understand, apply, and critically assess the effectiveness of appropriate techniques for making estimations (TC, PD);
2. use estimations to gain insight into the reasonableness of results (C, PS);
3. use scaled models to represent authentic mathematical situations (C);
4. apply mathematical thinking and spatial reasoning to solve problems that arise in other disciplines such as art and landscaping, and in our rapidly changing technological environment (PS, PD, CIT);
5. describe how a change in one measurement affects the changes made in related measurable quantities, mathematical shapes, and models (C, PD);
6. appreciate the usefulness of making quick estimates in real-life situations (AE).

SPECIFIC OUTCOMES BY LEVELS

Level 1

Students are expected to:

1. estimate perimeter and area of regular and compound shapes comprising triangles and rectangles drawn on grids;
2. explore the corresponding effect on area of shapes caused by changing their dimensions;
3. estimate the circumference of a circle;
4. solve problems involving estimates of perimeter and area, including finding the dimensions of a shape, given its perimeter.

Level 2

Students are expected to:

1. estimate the area of a circle using grids;
2. develop and use the formula for the circumference and area of a circle;
3. calculate the perimeter and area of compound shapes involving parts of a circle (quadrants only);
4. use logic and reasoning to make and support conjectures about regular geometrical shapes (e.g., area of a rectangle is twice the area of a triangle).

Level 3

Students are expected to:

1. derive and use the formula for the length of an arc of a circle expressed as a fraction of its circumference;
2. calculate the perimeter of compound shapes involving parts of a circle;
3. derive and use the formula for the area of a sector in a circle by representing it as a fraction of the area of the circle;
4. calculate the area of compound shapes involving parts of a circle.

Content

- ❖ Triangles, Squares, and Rectangles
- ❖ Compound Shapes
- ❖ Circle
- ❖ Perimeter and Area Estimation
- ❖ Formulae – Perimeter and Area Computation
- ❖ Conjectures – Regular 2D Geometrical Shapes

Instructional Strategies/Methods

- Explorations – geoboard
- Project – small group activity
- Guided discovery – use of manipulatives and models
- Discussion

Suggestions for Assessment

- ✓ Observation – contribution to group
- ✓ Calculations – use of formulae
- ✓ Peer evaluation – performance in group
- ✓ Portfolio – including projectwork
- ✓ Evidence to support conjecture
- ✓ PowerPoint presentation of project – rubric

Resources

- Grid sheets
- Manipulatives and models
- Charts

Suggested Teaching and Learning Activities

In teaching this module, the following activities may be useful:

1. Spatial Ability

Clarifying concepts – height vs length of side

Students tend to confuse the length of a slant side with the height of a figure.
Most have had many early experiences with the "L x W formula" for rectangles, in which the height is exactly the same as the length of a side. Perhaps this is the source of this confusion.

It is important for students to understand that any side or surface of a figure can be called a base of a figure and that for every base of a figure, there is a corresponding height.

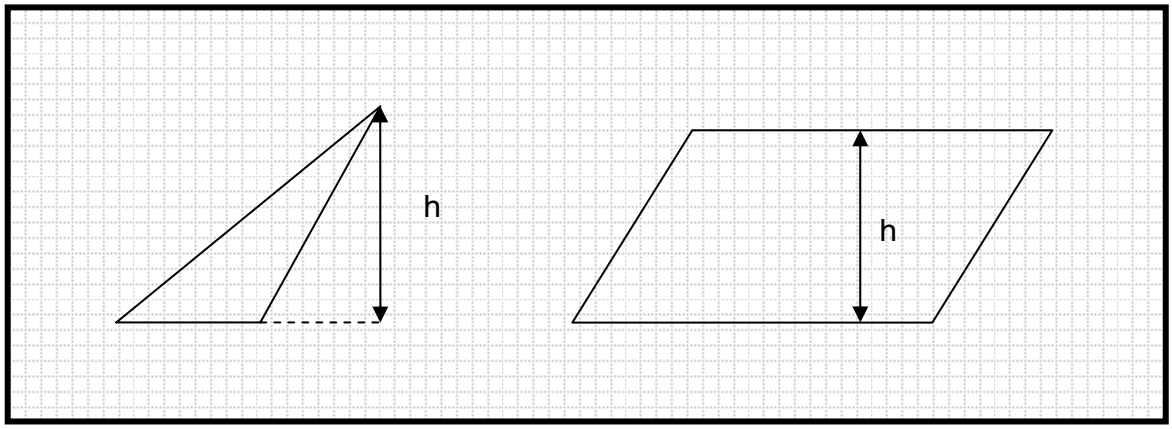


Figure 1. **The edge or surface of a figure may not always represent the height of the figure.**

2. Project: Exploring Geometrical Shapes and Figures — Perimeter and Area —

Guidelines for Project

Students will:

1. research basic geometrical properties of common 2D shapes, identify their uses in real-life situations, illustrate their usage, and present findings using suitable presentation methods;

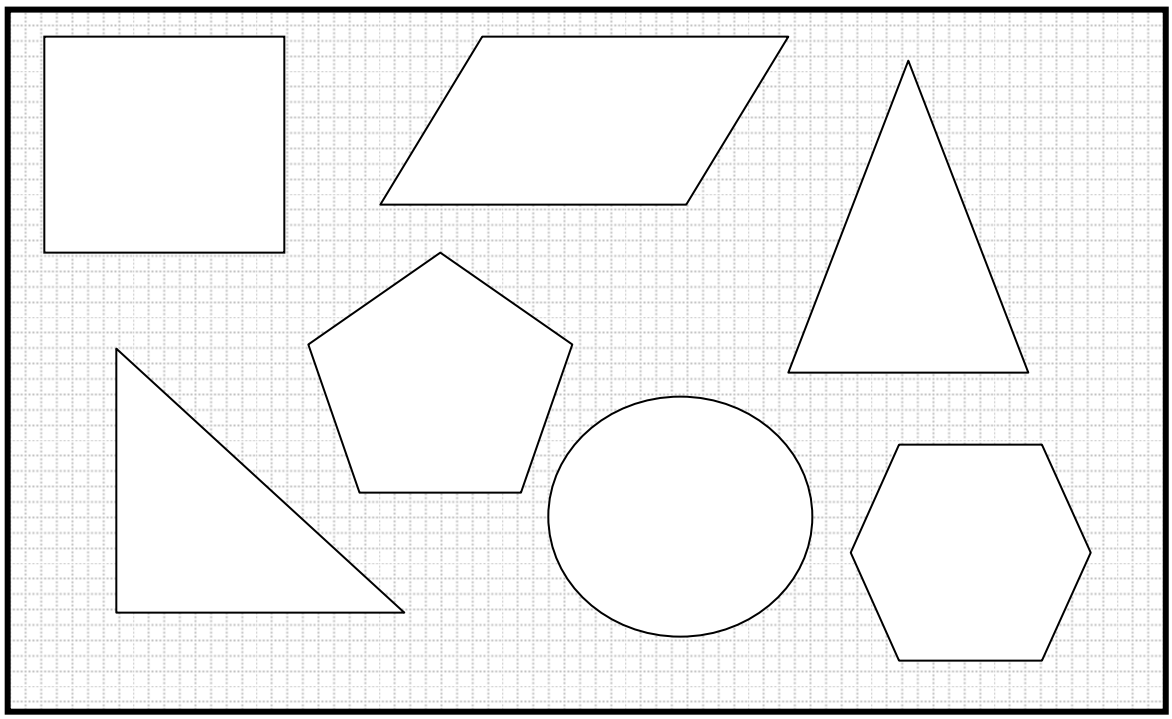


Figure 2. **Common 2D shapes.**

2. draw outlines of each shape on grid paper or stick cut-outs of each shape using suitable materials on grid paper (1 cm square) to match areas;
3. estimate the perimeter of each shape by counting along unit lengths of squares showing steps using a diagram;
   (Students may need to know that the length of the diagonal of a 1 cm square is approximately 1.5 cm)
4. estimate the area of each shape by counting squares showing steps using a diagram;
5. present findings using PowerPoint;
6. extend activity using 2D compound shapes including creative designs.

This project may be adapted to a computer-based project, with students using interactive computer software and grid paper to draw and create shapes, followed by PowerPoint presentations.

3. Developing Area Formulae
— Rectangle and Parallelogram —

The formula for the area of a rectangle is one of the first that was developed and is usually given as A = L x W or Length *multiplied by* Width. Looking at other area formulae, a more unifying idea might be A = b x h or base *multiplied by* height.

The base *multiplied by* height formula can be generalized to all parallelograms and is useful in developing area formulae for triangles and trapezoids. Furthermore, the same approach can be extended to three dimensions where volumes of prisms are given in terms of area of base *multiplied by* height. The concept of base *multiplied by* height can then help to connect a large family of formulae.

A. Guided Discovery Activity – using manipulatives

Developing area formulae for a rectangle and parallelogram

Students will:

1. trace the outline of a prepared shape of a rectangle on grid paper:
   - count the lengths of the base and height along the sides of squares and compute the area of rectangle using the product base x height
   - estimate the area by counting squares
   - present findings using a suitable method of presentation;

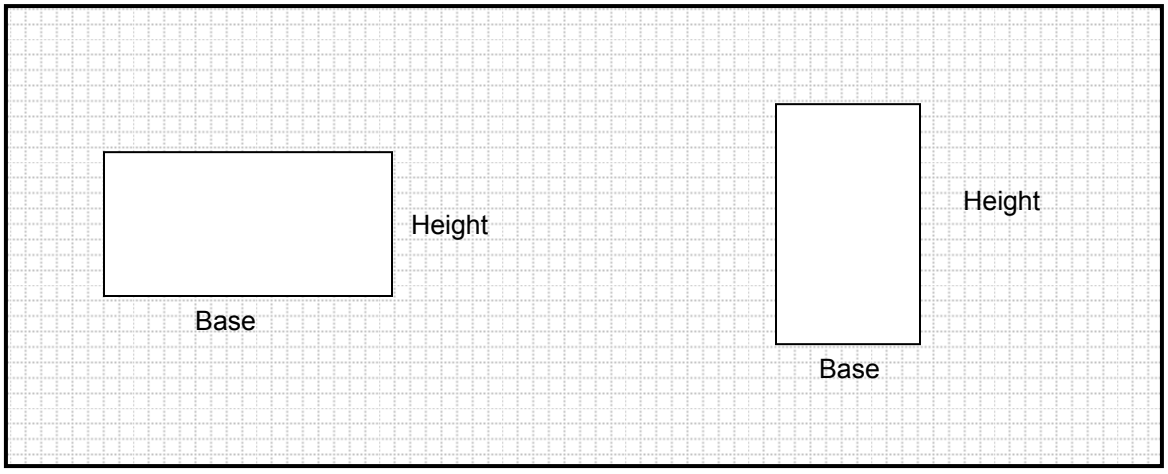


Figure 3. **The rectangle.**

2. use the same rectangle, turn it around to orient a "new" base and its corresponding height. Repeat Step 1 above for rectangles of varying sizes;
3. interpret findings by looking for patterns in order to deduce a formula:

   Area of rectangle = base x height;
4. repeat Steps 1 and 2 for parallelogram.

B. The Triangle Connection

Students will:

1. cut the shape of any triangle on paper and make two identical copies;
2. arrange both triangles on grid paper to form a parallelogram;

   (N.B. 3 different arrangements are possible)

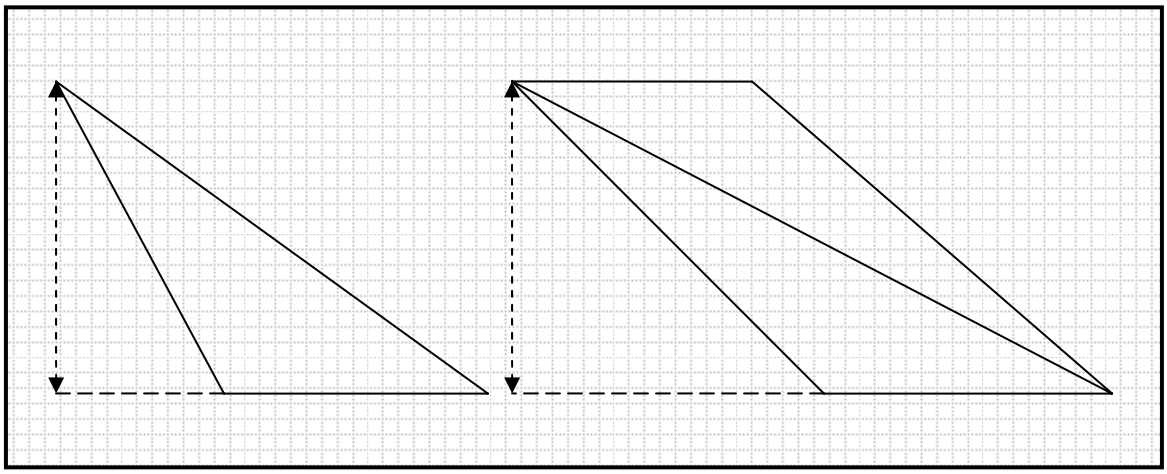


Figure 4. **Two copies of any triangle will always form a parallelogram with the same base and height.**

3. identify a base and a height for the parallelogram and the triangle and estimate their measurements;
4. compute areas of both shapes and use them to make a connection to establish formula for the area of a triangle:

   Area of parallelogram = base x height

   Area of triangle = ½ x base x height

C. Using a Model to Develop the Area of a Circle

Students will:

1. use a model of a circle that is cut into equal "wedges" and reassemble the wedges as shown in the diagram presented in *Figure 5*;
   (The "reassembled" circle is very close to being a rectangle)

   (Note that in this activity the circle should be cut into an even number of wedges so that upon reassembly the longer sides of the shape that is formed will be of equal length.)

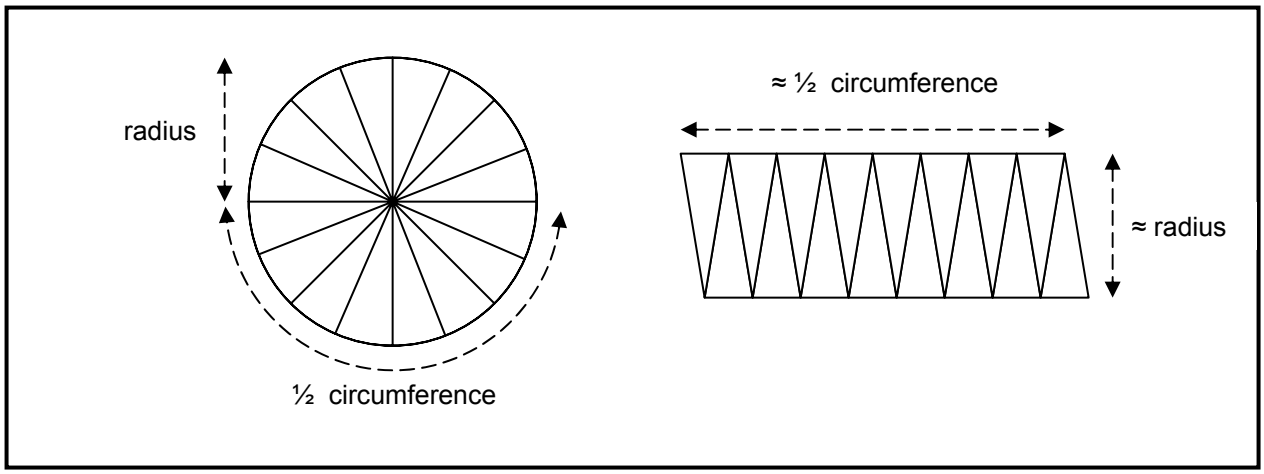


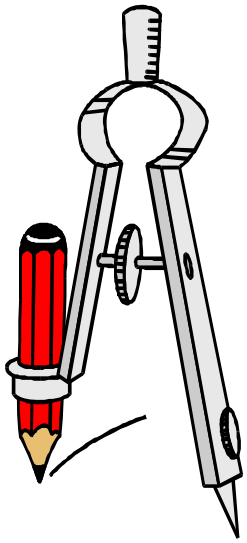
Figure 5. **Dis-assembling the circle.**

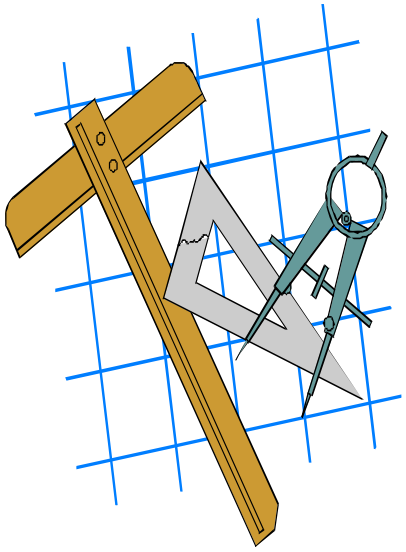
2. write an expression for area of the rectangle in terms of the radius, ***r,*** and circumference, **C,** of the circle;
3. replace the circumference, **C,** of the circle by the expression $2\pi r$ and derive the formula for the area of a circle:

   Area of a circle $= \pi r^2$

❖ This activity may be adapted by the teacher for presentation using an animated multimedia slide show to demonstrate how the cutting and reassembly can be done

Unit 3





UNIT 3 — CONSTRUCTION OF SHAPES

GENERAL OUTCOMES

Students are expected to:

1. understand and apply knowledge, concepts, and skills to perform geometrical constructions to a specified degree of accuracy (C, PD);
2. appreciate the value of precision and accuracy in making measurements (PD);
3. apply creative and critical thinking, and spatial and deductive reasoning to explore relations between shapes and solids in space (PS, AE, C);
4. use geometrical models and ideas to gain insights into and solve problems in other areas of mathematics such as art and architecture (PD, AE, PS, CIT);
5. analyse mathematical situations and solve problems (PS);
6. use precise mathematical statements to communicate logical deductions and evidence to support conjectures made about geometrical figures (C);
7. recognize and use connections among mathematical ideas and use appropriate formulae to determine measures (PD, C, TC);
8. use appropriate mensuration formulae to determine measures (PS, TC);
9. appreciate the role of technology in gaining a better understanding of shapes and space in general (PD, AE).

SPECIFIC OUTCOMES BY LEVELS

Level 1

Students are expected to:

1. construct a line segment using ruler and compasses;
2. bisect a line segment;
3. use a pair of compasses and a straight edge to bisect an acute or obtuse angle;
4. create designs of simple plane shapes and patterns in 2D related to architecture and landscaping, using bisection of line segments and angles.

Level 2

Students are expected to:

1. draw angles using a protractor;
2. use ruler and compasses only to construct standard angles (i.e., 30º, 60º, 90º, 120º);
3. draw angles of the same size as a given angle;
4. construct triangles, when given:

   (a) the lengths of 3 sides

   (b) the lengths of 2 sides and the included angle;
5. construct parallel and perpendicular lines;
6. use lines, angles, and the axes of reference to describe and represent directions (e.g., in using a navigational compass);
7. apply the properties of equality and parallelism of sides and equality of angles of triangles to construct compound shapes.

Level 3

Students are expected to use geometric knowledge, skills, and instruments to:

1. draw and construct a circle, given the radius;
2. draw and construct a circle, given two chords (i.e., using geometrical skill of bisection of line segments);
3. construct triangles, parallelograms, rectangles, and squares using given information;
4. construct regular polygons with up to 6 sides;
5. use logic and reasoning to make and support conjectures about geometrical shapes;
6. draw and construct representations of 2D and 3D geometric objects using a variety of tools such as geometrical instruments and computer software.

Content

- ❖ Geometric Constructions:
  – lines, angles, 2D geometrical shapes
- ❖ Properties of Polygons:
  – sides and angles
- ❖ Circles:
  – diameter, radius, chord, centre, arc length, area of sector

Instructional Strategies/Methods

- ▪ Cooperative learning groups, e.g., Jigsaw model
- ▪ Individual/group work
- ▪ Teacher-demonstrations

Suggestions for Assessment

- ✓ Constructed shapes and models
- ✓ Observation:
  – students' skills in using instruments and reading scales
- ✓ Portfolios showing various outputs
- ✓ Presentations of designs using a variety of media

Resources

- Geometrical instruments
- Writing utensils
- Appropriate computer software
  – for enrichment exercises
- Manipulatives and models: 2D and 3D shapes

SAMPLE LESSON

Unit 3: Level 3

Landscape Design

Overview

Students will apply spatial reasoning, visual thinking, and geometrical knowledge and skills to analyse, represent, and solve a real-world problem that requires a designated degree of accuracy.

Learning Outcomes

Students will be able to:

- analyse a specific measurement situation to determine the necessary degree of accuracy and/or the allowable error tolerance;
- design suitable models in collaboration with others to solve identified problems;
- demonstrate understanding for environmental concerns;
- select and use appropriate units and tools to measure quantities to a specified degree of precision and accuracy in problem-solving situations;
- communicate measurements made to a specified degree of accuracy using a scale drawing;
- communicate details of solution in written and oral form.

Instructional Plan

Your grandmother has been asked to babysit two of her grandchildren for an extended period. She needs your help to redesign her backyard. Her five-year-old granddaughter desires a playground equipment area. Her grandson would like to have a rectangular swimming pool.

In addition, your grandmother does not want to lose her garden area.

Your task is to design a backyard layout to provide for all three requests.

The dimensions of the backyard are 60 feet by 50 feet. The playground area cannot exceed 400 square feet. The pool must be at least 600 square feet. There are only 80 feet of fencing available to enclose the garden.

Draw your design on squared paper and label all dimensions. A final write-up must be presented to your grandmother to explain the perimeter and area of the playground, pool, and garden. You must also inform her of the amount of unused area available for landscaping.

Assessment

You will create a scale drawing of the backyard on squared paper and a written presentation in the form of a letter explaining details of your backyard design.

In addition, you will do an oral presentation to the class while displaying your design.

An assessment rubric can be used to score student performance. The following example may be appropriate in this context:

Rubric – Scoring Assessment Task

Criteria to be Assessed	4 Exemplary	3 Acceptable	2 Partial Success	1 Little Success
The process used to create the design of the backyard was within required specifications: Playground did not exceed 400 square feet, pool was at least 600 square feet, the perimeter of the garden was no more than 80 feet, and the total design was placed within the backyard measuring 60 feet by 50 feet	The student thoroughly demonstrated the needed skills, accuracy, and completeness required. The plan meets all of the specifications	The student sufficiently demonstrated the needed skills to solve the problem with only minor errors in computation and/or drawing. The result is close to the required specifications	The student partially demonstrated the needed skills to solve the problem, but has major computation errors and/or the drawing resulted in an unreasonable solution	The student had no understanding of the skills needed to solve the problem
Mathematical knowledge required in the use of formulae and calculations	The response demonstrates the knowledge of the proper use of the formulae for area and perimeter. The calculations needed to solve the problem are complete and accurate	The response demonstrates the knowledge of the use of formulae for area and perimeter for the most part, but may lack completeness and/or have minor calculation errors	The response is unreasonable and/or the use of formulae and calculations is fragmented or confused, leading to an invalid solution	The response is unreasonable and there is little or no understanding of how to use the formulae or how to go about solving the problem
The student's explanation of the process used is reasonable and clearly communicated	The student's explanation is clearly communicated, logical, accurate, and complete	The student's explanation is generally complete and for the most part, reasoning is logical, although it may be difficult to understand at one or two points	The student's explanation may be general and lacking in detail, or may be difficult to follow or somewhat illogical	The student's explanation may be completely illogical and/or very difficult to follow

Other lessons for this module may be sourced from appropriate books or online via the Internet.

Bibliography

Aronson, E. (2004). *Jigsaw classroom: History of the jigsaw classroom*. Available at: http://www.jigsaw.org/history.htm

Aronson, E. (2004). *Jigsaw classroom: Jigsaw in 10 easy steps*. Available at: http://www.jigsaw.org/steps.htm

Caribbean Examinations Council. (2006). *The new secondary level programme: Mathematics*. St. Michael, Barbados: Author.

Carr, J. F., & Harris, D. E. (2001). *Succeeding with standards*. Alexandria, VA: Association for Supervision and Curriculum Development.

Four leading models of cooperative learning. (2000, May/June). *Harvard Education Letter*. Available at: http://www.edletter.org/past/issues/2000-mj/models.shtml

SuccessLink. *Special quadrilateral PowerPoint presentation.* Available at: http://www.successlink.org/GTI/lesson_unit-viewer.asp?lid=2993.

Trinidad and Tobago. Ministry of Education. Secondary Education Modernization Programme. (2004). *National curriculum for Forms 4 & 5 mathematics*. Port of Spain, Trinidad: Author.

Van De Walle, J. A. (1994). *Elementary school mathematics: Teaching developmentally*. New York: Longman.

STANDARDS

MODULE 5

Mathematics Standards

MODULE 5: MEASURING WHEN IT MATTERS

1. Students will describe and apply relationships among numbers, their uses, and their representations.

At the end of Level 1

1. Make quick and rough estimates of quantities used in everyday life using non-standard (informal) units.
2. Apply multiple representations of numbers such as integers, decimals, and exponents in the use of and conversion of units of measurements, and appreciate that performing arithmetical operations on measurements affects precision.

At the end of Level 2

1. Make quick and rough estimates of quantities used in everyday life using appropriate equivalents between the metric and imperial systems of units.
2. Apply multiple representations of numbers such as integers, fractions, decimals, and exponents in quoting units of measurements from scales of measuring instruments, and appreciate that performing arithmetical operations on measurements affects precision.

At the end of Level 3

1. Make quick and rough estimates of quantities used in everyday life using appropriate equivalents between the metric and imperial systems of units, justifying the use of a particular unit.
2. Apply multiple representations of numbers such as integers, fractions, decimals, exponents, and scientific notation in the use of scales for map reading and accurate drawing of 2D and 3D geometrical figures, and appreciate that performing arithmetical operations on measurements affects precision.

2. Students will estimate and compute using mental math, estimation strategies, paper and pencil techniques, and technology-supported methods.

At the end of Level 1

1. Select and use the most efficient estimation and computational methods such as mental math, estimation strategies, paper and pencil techniques, and technology-supported methods:

   (i) in the use of and conversion of units of measurement;

   (ii) in making estimates of perimeter and area.
2. Appreciate that performing arithmetical operations on measurements affects precision.

At the end of Level 2

1. Select and use the most efficient estimation and computational methods such as mental math, estimation strategies, paper and pencil techniques, and technology-supported methods:
   (i) in quoting units of measurements from scales of measuring instruments;
   (ii) in making estimates of perimeter and area of 2D geometrical shapes and figures.
2. Appreciate that performing arithmetical operations on measurements affects precision.

At the end of Level 3

1. Select and use the most efficient estimation and computational methods such as mental math, estimation strategies, paper and pencil techniques, and technology-supported methods:
   (i) in the use of scales for map reading and accurate drawing of 2D and 3D geometrical figures;
   (ii) in computing perimeter and area of 2D geometrical shapes and figures using mensuration formulae.
2. Appreciate that performing arithmetical operations on measurements affects precision.

3. Students will estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools, and technologies.

At the end of Level 1

1. Students will use appropriate measuring instruments and units of length, time, temperature, and mass, in both the metric and imperial systems, to make estimates and accurate measurements of quantities in simple situations;
2. Students will use appropriate measurements and geometrical instruments to measure length and construct and bisect line segments.

At the end of Level 2

1. Students will use appropriate measuring instruments, technological tools, and units such as length, time, temperature, mass, and degrees, in both the imperial and metric systems, to make estimates and accurate measurements in simple situations, to a required degree of accuracy and precision;
2. Students will use appropriate measuring instruments and geometrical instruments to measure length and construct specific lines, angles, and triangles.

At the end of Level 3

1. Students will use appropriate measuring instruments, technological tools, and units such as length, time, temperature, mass, and degrees, in both the imperial and metric systems, to make estimates and accurate measurements in complex situations, to a required degree of accuracy and precision;
2. Students will use appropriate measuring instruments and geometrical instruments to measure length and construct compound 2D shapes involving triangles, squares, and rectangles.

4. Students will use algebraic methods to represent, analyse, and solve mathematical and practical situations involving patterns and functional relationships, with and without the support of technology.

At the end of Level 1

1. Students will use appropriate units of measurements when applying algebraic methods to represent and analyse results obtained from estimations and direct measurement situations to establish simple patterns.

At the end of Level 2

1. Students will use appropriate units of measurements when using algebraic methods to represent and analyse results obtained from estimations and direct measurements to solve mathematical and practical situations involving simple qualitative trends, with and without technology.

At the end of Level 3

1. Students will use appropriate units of measurements when using algebraic methods to represent and analyse results obtained from estimations and direct measurements to solve mathematical and practical situations involving simple quantitative linear functional relationships, with and without technology

5. Using tools and technology when these would be most efficient, students will apply the properties and relationships of geometric shapes and figures to represent, investigate, analyse, and solve problems.

At the end of Level 1

1. Use manipulatives such as the geoboard and the computer to explore quantitatively the properties and relationships of 2D geometrical shapes and figures by using and changing measurements of shapes and figures.

At the end of Level 2

1. Use geometrical instruments and the computer to explore quantitatively the properties and relationships of 2D geometrical shapes and figures by using and changing measurements of shapes and figures to represent, investigate, and solve simple problems in the real world.

At the end of Level 3

1. Use geometrical instruments and the computer to explore quantitatively the properties and relationships of 2D geometrical shapes and figures and prisms by using and changing measurements of shapes and figures to represent, investigate, and solve simple problems in the real world.

6. Students will select and use a wide variety of tools and technology-supported methods to increase either the efficiency or quality of results.

At the end of Level 1

1. Use the scientific calculator and other technology-supported methods to perform operations on, and conversions of, units of measurements both within and across the metric and imperial systems of units, and to check the reasonableness of results for a given situation.

At the end of Level 2

1. Conduct investigations using interactive geoboards and computer software to represent and analyse problems involving measurement of 2D geometric shapes and figures, in order to observe trends and describe qualitative relationships.

At the end of Level 3

1. Conduct investigations using interactive geoboards and computer software to represent and analyse problems involving measurements of 2D geometric shapes, figures, and prisms, in order to observe trends and describe both qualitative and quantitative relationships.

7. Students will apply a wide variety of mathematical concepts, processes, and skills to solve a variety of problems.

At the end of Level 1

1. Select and apply a wide variety of estimation techniques to solve authentic problems involving non-standard units of measurements.
2. Select and apply a wide range of estimation and measurement techniques to solve real-world problems involving perimeters and areas of regular geometrical shapes.
3. Apply basic skills of geometrical construction of line segments and bisection of angles to solve authentic problems involving perimeters and areas of regular geometrical shapes.

At the end of Level 2

1. Solve authentic problems by selecting and applying a wide variety of estimation and measurement techniques involving standard units in both the metric and imperial systems.

2. Select and apply a wide range of estimation and measurement techniques to solve real-world problems involving perimeters and areas of 2D geometrical shapes and figures.
3. Apply basic skills of geometrical construction of specific angles and triangles, and parallel and perpendicular lines to solve real-world problems involving perimeters and areas of 2D geometrical shapes.

At the end of Level 3

1. Select and apply a wide variety of techniques involving accurate measurements in both the metric and imperial systems to solve real-world problems.
2. Select and apply a wide range of estimation and measurement techniques to solve real-world problems involving perimeters and areas of circles, and 2D compound geometrical shapes and figures.
3. Apply basic skills of geometrical construction of polygons and circles to solve real-world problems involving perimeters and areas of 2D compound geometrical shapes.

8. Students will apply reasoning skills to make sense of, investigate, evaluate, and justify steps in devising approaches and solutions to mathematical situations.

At the end of Level 1

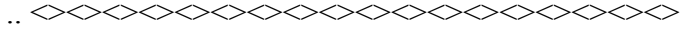
1. Make simple "agree with" and "conflict" statements about mathematical ideas involving dimensions, perimeter, and area of regular 2D objects.
2. Formulate logical and valid arguments to support or refute claims.

At the end of Level 2

1. Use simple "if…then" statements to construct conjectures about mathematical ideas involving dimensions, perimeter, and area of regular 2D objects.
2. Construct simple logical proofs to test conjectures.

At the end of Level 3

1. Make simple conjectures about mathematical ideas involving dimensions, perimeter, and area of regular 2D objects based on logical reasoning
2. Test conjectures by using counter examples.

.. 

MATHEMATICS 3

INSTRUCTIONAL STRATEGIES/METHODS

Table of Contents

INSTRUCTIONAL STRATEGIES/METHODS

Introduction 3–1

Concept Development 3–2

Algorithmic Procedures 3–3

Problem Solving 3–4

Suggested Activities 3–5

Suggested Resources 3–7

Details on Some Common Instructional Strategies/Methods 3–10

- Cooperative Learning 3–10
- Discussion 3–10
- Project-Based Learning 3–11
- Simulations 3–11
- The Internet 3–11

Glossary of Instructional Strategies/Methods 3–12

Bibliography 3–17

INTRODUCTION

The effective implementation of the mathematics curriculum has challenged teachers greatly over the years. Mathematics has traditionally been seen as a discipline with broad-based content organized along various strands: number operations and number theory, geometry, measurement, algebra, sets, relations and functions, and statistics and probability. However, developing mathematical literacy is much more than acquiring a body of knowledge. The mathematically literate person must be able to use his/her knowledge of mathematics to think mathematically, strategically negotiate the environment, and develop logical understandings to produce desired results. Mathematical thinking requires formulating questions, developing and using algorithms appropriately, communication, making connections, reasoning, and problem solving. Teachers need to balance the acquisition of procedural skills with the attainment of conceptual understanding and to apply mathematical principles in coherently addressing real-world issues.

The mathematics classroom thus needs to be one in which students are actively engaged in doing mathematics. It is no longer acceptable to view mathematics as a set of isolated rules and procedures that the teacher transmits to the student. Instead, the student must begin to see mathematics as a useful tool for understanding the world. Classroom activities and student assignments must be structured to provide opportunities for students to employ spatial reasoning, algebraic reasoning, and logical reasoning; to think strategically; to communicate mathematically; and to make real-world connections and solve problems that are relevant to students' needs and interests.

The learning environment should be one in which students and teachers regularly use mathematical resources and technology, actively discuss problems, make conjectures, share solutions, verify reasoning, and value multiple approaches and representations. The learning environment needs to be a safe one where students feel comfortable to share alternative approaches and to make suggestions.

Teaching strategies must be selected with the understanding that the learner actively constructs meaning and acquires knowledge in different ways. The curriculum must respond by suggesting a variety of instructional strategies that reflect constructivist approaches to satisfy the needs of different learners. These include:

- Discourse, including higher-level questioning
- Investigation and exploration of real-life issues
- Multiple approaches to problem solving, and justification of strategies employed
- Collaborative learning – working in small groups or pairs
- Providing opportunities for explanations, questions, justification, observation, and the use of resources
- Devising, examining, and refining strategies for solving problems
- Journal writing

- Portfolios
- Projects/presentations
- Integration with other core areas of the curriculum
- Learning organizers, for example, concept maps
- Infusing technology and values education into the curriculum
- Storytelling
- Brainstorming
- Role-playing
- Activity sheets
- Checklists
- Modelling processes for carrying out tasks involving the psychomotor, cognitive, and affective domains

Three aspects of mathematics must be emphasized as part of the instructional process. These are **concept development**, **algorithmic procedures** with emphasis on computational proficiency, and **problem solving**. Each aspect requires a different instructional approach, and each is discussed in more detail below.

CONCEPT DEVELOPMENT

Concepts are ideas that require understanding in order to provide a foundation on which other knowledge and skills can be developed. Concept development requires active teaching and learning using a student-centred approach. Through student-centred efforts, students are given the opportunity to actually experience learning through active engagement, and this allows them to investigate ideas and clarify their thoughts as they build concepts. This approach embraces a constructivist perspective, which should utilize sense perception, reflection, collaboration, consensus, and sharing. Students must therefore be allowed, and even encouraged, to communicate effectively, reason, make connections with other curriculum areas and real-life situations, apply problem-solving skills, and use various representational forms.

Value of Concept Learning for Students

- Decreases rote memorization and promotes relational understanding
- Enables students to examine an idea, problem, or question using the first principle approach
- Develops students' discriminatory and critical thinking skills
- Increases students' self-confidence in their ability to reason mathematically

Guidelines for Teaching Concepts

Provide opportunities for students to explain, reason, debate, question, and come to their own understanding of relevant concepts:

- Examine examples and non-examples
- Make connections with real-life situations
- Build on students' knowledge and experiences
- Engage students in exploring issues, and facilitate discussion
- Provide students with time and opportunities to move from the concrete representation to the abstract/symbolic
- Encourage the use of appropriate language and terminology. This may include the use of the vernacular and layman's concepts, where necessary.

ALGORITHMIC PROCEDURES

Algorithms are procedures for calculating or carrying out various processes in solving problems. Ideally, algorithmic procedures should be introduced after concepts are developed. The instructional approach should vary to meet the needs of students and should incorporate aspects of Howard Gardner's theory (Campbell & Campbell, 1999, p. 5) of multiple intelligences, which presently includes nine major areas of intelligences: musical, bodily-kinesthetic, logical-mathematical, linguistic, spatial, interpersonal, intrapersonal, naturalistic, and creative. The goal should be to achieve mastery learning. However, in order to attain excellence in mathematics, students should be taught through their strengths and nurtured to learn even in areas that are not their natural strengths. Mastery learning requires direct teaching, guided practice, and independent practice.

Value of Learning Algorithmic Procedures for Students

- Enables students to acquire procedural fluency
- Improves problem-solving skills
- Increases efficiency
- Improves mathematical achievement

Guidelines for Teaching Algorithms

- Develop rather than present an algorithm, where possible
- Encourage discussion of a variety of student-developed algorithmic procedures and point out that efficiency is desirable in using an algorithm
- Provide opportunities for guided and individual practice
- Apply algorithm in as many different situations as experienced by students
- Use creative ways to assist students in remembering an algorithm, acknowledging that students are intelligent in different ways

PROBLEM SOLVING

Students will only become prolific problem solvers by determining solutions to challenging problems. Routine problems such as application problems, taught in relation to each strand in mathematics, are insufficient to develop creative thinking skills, although they contribute significantly to the development of such skills. Non-routine problems are derived from real-life or recreational situations. The cooperative learning strategy is the recommended instructional approach. This requires that the classroom be organized for group work and that students be provided with the time and resources necessary to foster collaboration, effective communication, reflection, and sharing.

Value of Problem Solving for Students

- Develops students' creative and critical thinking skills
- Encourages teamwork and collaboration
- Promotes respect for self and others
- Fosters a positive attitude towards mathematics
- Enhances students' potential for learning

Guidelines for Teaching Problem Solving

Polya's (1957) model is recommended as a guide to be used in teaching problem solving. Using this model, he identifies four steps in the problem-solving process:

Step 1: Understanding the problem:

- Formulate questions
- Elicit information given
- Determine what is required

Step 2: Planning to solve the problem:

- Suggest a strategy
- Conduct an experiment
- Make a drawing, diagram, or model
- Use patterns
- Compute
- Organize a list or table
- Simplify or simulate
- Use deduction
- Work backwards
- Use an equation

Step 3: Solving the problem

- Carry out the selected strategy
- Obtain a solution
- If no solution is obtained, repeat Steps 2 and 3 until a solution is obtained

Step 4: Review the problem

- Discuss various strategies used to obtain a solution
- Create variations or entirely new problems by extending the given problem

SUGGESTED ACTIVITIES

Modern educational research points to the tremendous success achieved by adopting a more student-centred approach to mathematics instruction. In such an approach, students engage in the construction of mathematical knowledge and the development of appropriate mathematical skills. Students use their individual experiences to construct understanding in context while teachers facilitate the process. Teachers identify and engage their charges in challenging yet meaningful tasks that will support the development of *critical* and *creative thinking* skills.

A challenging task is one that:

- depicts a real-life issue or problem;
- uses manipulatives, drawings, or generalizations;
- is interesting;
- has a sound mathematical base;
- is accessible on some level to all students;
- allows for extensions and connections;
- can be solved in a variety of ways.

Each activity should involve the processes of communication, representation, reasoning, connection, and problem solving.

Critical thinking skills can also be developed using routine problems that provide opportunities for students to demonstrate convergent reasoning. Accordingly, by using this strategy, the learner operates as a convergent thinker and seeks the correct answer in one way only.

Creative thinking skills can be developed using non-routine problems through the development of divergent reasoning. In this way, the learner operates as a divergent thinker and produces many solutions to a problem, using familiar methods in new and original ways.

Communication/Representation

- Use of mathematical language and symbols
- Use of visuals, expressions, equations, and inequations
- Logical, oral presentations involving the use of appropriate technology
- Interpretation, analysis, synthesis, and evaluation of the mathematical thinking and strategies of others
- Use of a variety of representations to model and interpret physical, social, and mathematical phenomena
- Selection, application, and translation of mathematical representations to solve problems

Reasoning

- Use of deductive and inductive reasoning
- Development of mathematical proofs
- Evaluation of mathematical arguments
- Recognition of reasoning and proof as fundamental aspects of mathematics

Connection

- Recognition and use of connections across strands in mathematics
- Recognition and application of mathematics in contexts across the core curriculum subjects and in real-life situations
- Making connections with mathematics through projects and investigations

Problem solving

- Use of the process of mathematical problem solving in investigations and projects
- Providing opportunities for solving a variety of problems that occur in mathematics and other contexts
- Use of a problem-solving approach to introduce and teach concepts and skills
- Developing a range of problem-solving strategies

SUGGESTED RESOURCES

Implementation of the mathematics curriculum requires adequate resources to enable teachers to use a student-centred approach based on the constructivist perspective. For too long, teachers have used only chalk, chalk-board, and the lecture method to deliver mathematical content. The predominant use of the lecture method, which relies on print material and the skills of listening, speaking, reading, and writing, does not cater for the ways in which most students learn best. It ignores the variety of ways in which students learn, and the developmental stages of learners.

According to Jerome Bruner (Flores, 2001), mathematical concepts are best learnt when represented in three basic modes: enactive, iconic, and symbolic. The enactive mode presents materials in a concrete or physical form, thus supporting the use of manipulatives in the instructional process. The iconic mode, however, involves representing materials in a visual or pictorial form. It also includes the stage when an act is recreated mentally.

It is recommended that, whenever possible, mathematical instruction should proceed from the enactive through the iconic to the abstract modes of representation Accordingly, students should be encouraged to draw and use diagrams wherever possible to clarify concepts and to solve problems. In the symbolic mode, there is the movement towards greater abstraction. Symbolic representation is facilitated by language competence and presents materials using words or symbols.

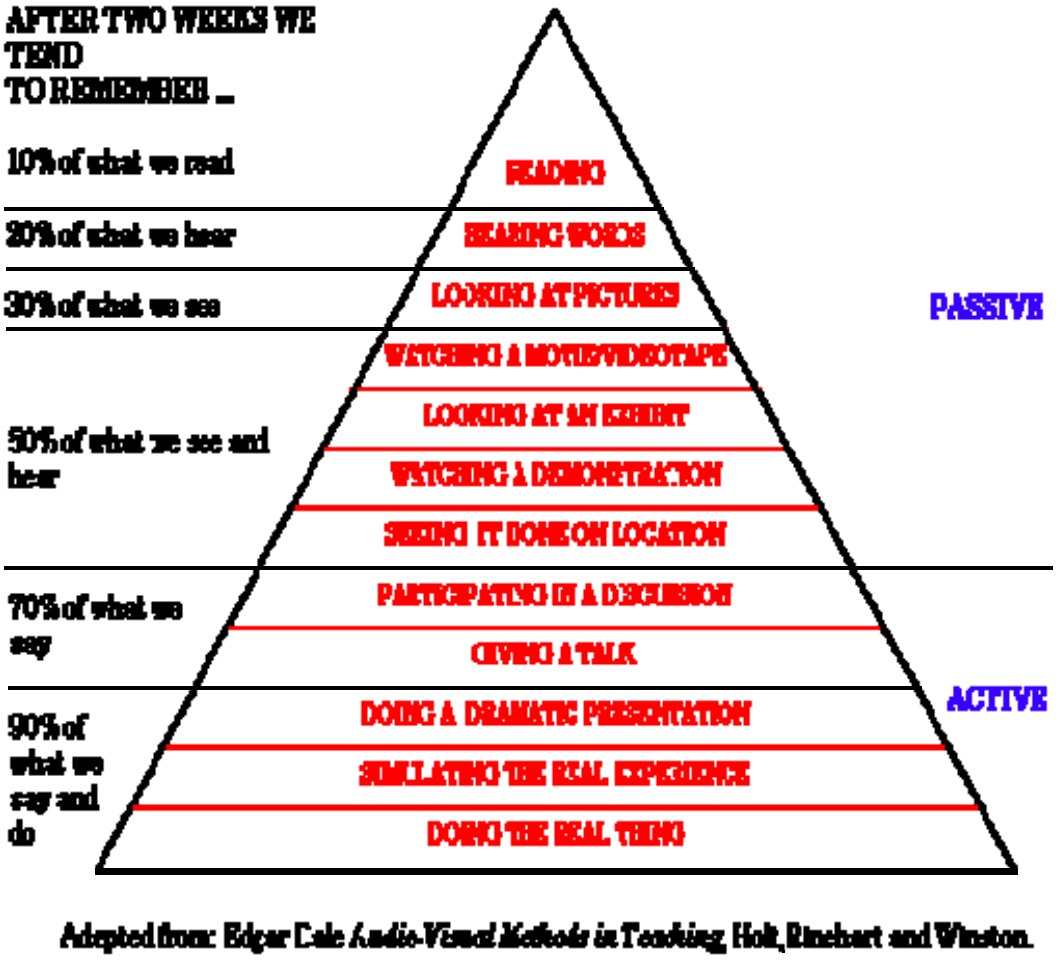
At the same time, teachers need to be aware of the individual differences of the learners in terms of their varying intelligences, as outlined by Howard Gardner, and the way that individuals tend to remember things as outlined in the "Cone of Learning," as developed by Bruce Hyland (McKeachie, 1998).

The Cone of Learning is a model that provides some insight into learners' capacity for retention of material. According to this model, students' retention of knowledge is better when the strategy for learning involves active learning experiences. Research shows that passive student involvement generally leads to limited retention of knowledge, as indicated in the Cone of Learning (See Figure 1; McKeachie, 1998).

Howard Gardner's research in multiple intelligences has identified the following teaching materials with the respective intelligence:

Intelligence	**Materials**
Linguistic	books, tape recorders, computers, tapes, stamp sets
Logical-mathematical	calculators, manipulatives, science equipment, games, rulers
Spatial	graphs, maps, video, Lego, art materials, optical illusions, cameras, picture library, other visuals

Intelligence	Materials
Bodily-kinesthetic	building tools, clay, sports equipment, tactiles
Musical	musical instruments, CDs, tapes
Interpersonal	board games, party supplies, props for role play
Intrapersonal	self-checking materials, journals, newspapers, materials for projects, activity sheets



Source: (McKeachie, 1998)

Figure 1. **The Cone of Learning.**

The Use of Technology

The use of technology to assist instruction in mathematics is essential not only to make instruction more effective, but also to prepare students to meet the technology needs of the 21st century. The debate on whether calculators should be introduced in the classroom is obsolete. Students must experience mathematical concepts in real-life contexts and should not be deterred by difficulties in computation. The appropriate use of the calculator in real-life situations certainly provides adequate justification for its use in the classroom, but it can also be used within the classroom context to assist students in finding patterns and relations among numbers. However, in order for this experience to be meaningful to the student, a proper understanding of the concepts must have been previously established. This, indeed, is the challenge for our teachers, and innovative ways must be employed to facilitate this aspect of learning.

The Computer

Today, the computer has practically replaced the typewriter as a word processing tool. It has the added advantage of being more versatile and can be extremely useful in the presentation of project reports. It also provides a volume of information for research via the Internet. The Internet can provide teachers with resource material for professional development in teaching practice, student learning, and curriculum development. The computer can be used to enhance instruction or to provide reinforcement, in terms of attending to the special needs of gifted students, as well as for remedial exercises for individual students who may be challenged in some way. It can also assist the teacher in a differentiated classroom by enabling him/her to provide activities for all students at the level required and to keep track of students' performance.

The Multimedia Library

The multimedia library in each school is a resource facility through which the technological and print resources, including mathematical journals, are made available for instructional use.

The Curriculum Guide

In the context of curriculum change, this curriculum guide is one of the key print resources. It provides an overview of the relevant curriculum. It identifies learning outcomes, instructional focus, suggested teaching and learning strategies, suggested integrated activities, and suggested assessment strategies as are applicable for the effective implementation of the curriculum.

Textbooks

Textbooks are resources that can be used to facilitate curriculum implementation. It is very important, though, for teachers to recognize this and use them to supplement and/or support the curriculum and not be misguided to treat them as if they were curriculum guides.

DETAILS ON SOME COMMON INSTRUCTIONAL STRATEGIES/METHODS

A. Cooperative Learning

Use small groups of students and have them work together to maximize their own and each other's learning. Cooperative learning requires that students work together to achieve goals that they could not achieve individually. It also helps them to attain higher levels of achievement, to increase time on task, to build cross-ethnic friendships, to experience enhanced self-esteem, to build lifelong interaction and communication skills, and to master the habits of mind (critical, creative, and self-regulated) needed to function as productive members of society.

Student Team Learning (STL)

Student Team Learning emphasizes team goals and team success for performances above the designated standard. Students are rewarded for improving on their own performances, and team scores are important motivators.

Learning Together

Students work in four- or five-member heterogeneous groups on a group assignment sheet. A single product is turned in, and the group receives rewards collectively.

Group Investigation

This is a general classroom plan where students form their own two- to six-member groups, choose subtopics from a class-wide unit, and produce group reports. Each group then makes a presentation or display to share its findings with the entire class.

Jigsaw Model

Students are assigned to six-member teams to work on segmented academic material. Each team member reads an assigned section, and then members from different teams who have studied the same sections meet in "expert groups" to discuss their sections. Then students return to their own teams and take turns teaching their teammates about their section.

The Jigsaw model reduces conflict among children, promotes better learning, improves student motivation, and increases enjoyment of the learning experience.

B. Discussion

This strategy can be meaningfully adapted to many classrooms and requires careful planning, so that the teacher can guide the discussion. It should conclude with consensus, a solution, clarification of insights gained, or a summary.

Strengths

- Whole-class discussion can help to build a positive classroom climate and lead to increased interest in the subject
- The teacher can model active listening and build on student responses
- It can lead students to conduct further research
- It is effective after a presentation or experience that needs to be analysed

Challenges

- It is time consuming
- A few students can dominate the activity if it is not well managed

C. Project-Based Learning

Students are assigned a problem/task in the form of a project to plan, research, study, analyse, implement, monitor, and report on. This is similar to Problem-Based Learning in that students may be challenged with a real-world problem, which may be best addressed by treating it as a project.

D. Simulations

The teacher presents an artificial problem, situation, or event that represents some aspect of reality. In simulations, levels of abstraction/complexity are purposefully reduced so as to focus on the underlying concept.

Strengths

- Serious risks or complications may be controlled or removed completely
- Simulations allow for experimentation that may not be possible in the real environment
- Students are active participants in the learning process
- They help to develop interpersonal and social skills, attitudes, and values

Challenges

- It is time consuming
- Not all situations can be adequately simplified.

E. The Internet

There are many sites on the Internet that can be accessed for relevant information. However, teachers are advised to access current information and locate sites before requiring students to do so. Furthermore, teachers may set up WebQuests to engage students in inquiry-oriented activities using information from resources on the Internet. Further information on WebQuests may be obtained from the Internet at: http://en.wikipedia.org/wiki/WebQuest.

GLOSSARY OF INSTRUCTIONAL STRATEGIES/METHODS

Abstracting

This is a thinking skill that involves summarizing and converting real-world events or ideas into models.

Acting Out a Problem

Students can act out mathematical, scientific, or social problems to improve their comprehension.

Activating Prior Knowledge

Helping learners connect to concepts about to be taught by using activities that relate to or determine the level of their existing knowledge.

Analogies

Examples of things that are similar to, but not identical with, a target example. For example, the Internet is analogous to the post office (because in both, multimedia information is delivered to specific addresses).

Application Teaching

This is a constructivist approach centred on activities that involve learning which proceeds from more basic ideas to more complex. The expected products generated by the students are determined by the teacher.

Audio-Visuals

These include many categories of educational materials, including posters, paintings, slides, videos, films, audio tapes, and videotapes.

Blogs

Blogs, also known as weblogs, are online journals that can be used by the teacher as a means of sharing thoughts, assignments, or resources; or blogs can be created by students for the purpose of reflection, intergroup communication, or to fulfil particular assignments.

Brochures

Brochures are pamphlets or short journals that are used to present information on a particular subject. Students research a topic then create a brochure to explain the topic to others.

Cheat Notes

This is a summarization technique. Students prepare a single note card of information they believe will be on a test. Students are allowed to bring these notes to the test. As students gain confidence, withdraw use of cards during tests.

Class Publications

Students collaborate to create a written work to be published. Formats might include: magazine, newspaper, brochure, map, newsletter, or yearbook.

Classroom Bulletin Boards

Classroom bulletin boards are effective ways of allowing students who may have difficulty writing to see themselves published, and to communicate their ideas. Students are encouraged to write their pieces and post them on the classroom bulletin board.

Collective Notebooks

A notebook maintained by a group in which each member of the group is expected to add an idea or observation during a specified time period (typically each day or each week). The contents of the notebook are regularly shared or published and discussed.

Competitions

Competitions can be useful in motivating some students to learn. Team competitions are especially effective in the classroom if they are tied to a collaborative practice or review activity before the competition.

Computer-Assisted Instruction (CAI)

Students learn at their own pace with interactive computer programs**.** The programs provide all relevant instructions and prompts to facilitate student learning.

Concentration

Pairs of cards are created (name of concept on one, description on other, for instance). Students take turns. On each turn, student chooses two cards from face-down arrangement. Students keep pairs that they correctly identify as matching.

Cooperative Learning

Small groups of students work together to maximize their own and each other's learning. Students draw on the knowledge of each member of the learning group to come up with more informed conclusions.

Craft

Students can illustrate concepts they have learnt while displaying their creativity by building relevant handwork.

Demonstrations

This is an activity that can be used to show students how things work or how they happen. Students may also be given the opportunity to give demonstrations that reflect the outcomes of their critical and creative thinking processes.

Discovery Teaching

Students begin learning with an activity designed to lead them to particular concepts or conclusions. Students acquire basic and advanced knowledge in random order.

Error Analysis

Teachers analyse the errors students make in mathematical computation, and use that analysis to guide further instruction.

Field Trips

A field trip is any activity that occurs outside the classroom for the purpose of providing hands-on experience with objects or people that only occur in certain places. Target locations for field trips can include museums, zoos, places of business, farms, nearby colleges, theatres, historical monuments or buildings, forests, wetlands, nature parks, or the grounds of the school itself.

Find the Fib

A team activity where groups of students write two true statements and one false statement, then challenge other teams (or the teacher) to "Find the Fib."

Find Your Partner

This is a method for assigning students to groups and at the same time reviewing previous concepts. Equations, sentences, or questions and answers are written on a single piece of paper, and then the parts of the sets are cut apart. The parts are distributed to students who compare papers with other students until they find their match.

Gaps

Students are given sentences or sequences with gaps (missing words, numbers, or symbols) and are asked to fill in the gaps.

Guided Discovery

This is a teaching model where students learn through explorations, but with directions from teacher.

Idea Spinner

Teacher creates a spinner marked into four quadrants and labelled "Predict, Explain, Summarize, Evaluate." After new material is presented, the teacher spins the spinner and asks students to answer a question based on the location of the spinner. For example, if the spinner lands in the "Summarize" quadrant, the teacher might say, "List the key concepts just presented."

Inside-Outside Circle

Review technique. Inside and outside circles of students face each other. Within each pair of facing students, students quiz each other with questions they have written. Outside circle moves to create new pairs. Repeat.

Investigation

Identifying what is known about a topic and inquiring to discover further information. There are three basic types of investigations: Definitional (What are...?); Historical (How...? or Why...?); and Projective (What if...?).

Journal

This is a form of writing typically done for a few minutes each day. The writing is done in a notebook and is often used to encourage reflection or exploration of ideas of interest to the students. Journal writing is typically not graded and, in some instances, is not read by anyone but the student. In other instances, the journal can be used to establish an ongoing written dialogue between the student and the teacher.

Luck of the Draw

All students' names are put into a container. At the end of class, a student's name is drawn at random from the container. At the beginning of the next class, the student whose name was drawn is required to present a 3-5 minute review of the previous day's lesson.

Manipulatives

Manipulatives are objects used in the classroom to allow students to make connections to concepts through touch. Examples might include a bag of beans for counting, or a microscope for scientific inquiry.

Mental Arithmetic Techniques

These are techniques to allow students to approximate answers to math problems. Mental math or mental arithmetic is important to allow students to be able to recognize when the answers they obtain using calculators are accurate.

Mix/Freeze/Group

In this activity, the teacher poses questions to which the answer is a whole number and the students (as a group) answer the question by moving through the classroom to form groups of that size. For example, if the question was, "How much is 24 divided by 8?" the students would cluster to form groups of 3.

Non-examples

A technique used in direct instruction to help students distinguish between similar concepts. Students are asked not only to explain concepts but to provide examples that do not conform to the prescribed definitions.

Pattern Forming

The ability to recognize and create patterns is central to many different fields. The use of lessons with "pattern forming" activities typically starts in pre-schools and continues into higher education.

Pop Quiz

This is a form of assessment given without notice. Usually written, and used to motivate students to study each day.

Portfolio

Portfolios are carefully selected samples of student work accompanied by formal criteria to allow the reader to judge the materials in the portfolio. Portfolios typically include work selected by the students to show their best work, some pieces to show

progress, and other work that reflects on what was learned and what remains to be learned.

Posters

Student-created posters can be used at any stage of instruction. During early exposure to concepts, students can create "mini-posters" providing a visual overview of what they already know of a topic. Later in instruction, assigning a poster format encourages students to organize and prioritize materials. Group posters encourage negotiation and develop team skills as students debate how best to illustrate concepts. Posters can also be combined with other forms such as oral presentations and visually enhanced essays. Commercially made posters are useful as quick reference and to augment lectures or discussions.

Problem-Based Learning (PBL)

This is an inductive teaching method with no direct instruction. Teacher poses an authentic (real-world) problem. Students learn particular content and skills as they work cooperatively to solve the problem.

Project-Based Learning

Students are assigned a problem/task in the form of a project to plan, research, study, analyse, implement, monitor, and report on. This is similar to Problem-Based Learning in that students may be challenged with a real-world problem that may be best addressed by treating it as a project.

Quickdraw

Pair activity in which students have a short period (typically 30 seconds) to share all they know by writing with symbols or drawings.

Quicktalk

Pair activity in which students have exactly 30 seconds to share all they know.

Simulations

Real-world activities are reproduced or modelled under controlled conditions so that key features may be closely studied.

Bibliography

Aronson, E. (2004). *Jigsaw classroom: History of the jigsaw classroom*. Available at: http://www.jigsaw.org/history.htm

Aronson, E. (2004). *Jigsaw classroom: Jigsaw in 10 easy steps*. Available at: http://www.jigsaw.org/steps.htm

Campbell, L. & Campbell, B. (1999). *Multiple intelligences and student achievement: Success stories from six schools*. Alexandria, VA: Association for Supervision and Curriculum Development.

Flores, N. (2001). *The educational theory of Jerome Bruner.* Available at: http://www.newfoundations.com/GALLERY/Bruner.html

Four leading models of cooperative learning. (2000, May/June). *Harvard Education Letter*. Available at: http://www.edletter.org/past/issues/2000-mj/models.shtmlG.

McKeachie, W. J. (1998). *Active learning*. Available at: http://courses.science.fau.edu/~rjordan/active_learning.htm

Polya, G. (1957). *How to solve it* (2nd ed.). Princeton, NJ: Princeton University Press.

Trinidad and Tobago. Ministry of Education. Secondary Education Modernization Programme. (2004). *National curriculum for Forms 4 & 5 mathematics*. Port of Spain, Trinidad: Author.

Trinidad and Tobago. Ministry of Education. Secondary Education Modernization Programme. (2001). *Draft national curriculum for Forms 1–3 mathematics*. Port of Spain, Trinidad: Author.

MATHEMATICS 4

ASSESSMENT AND EVALUATION

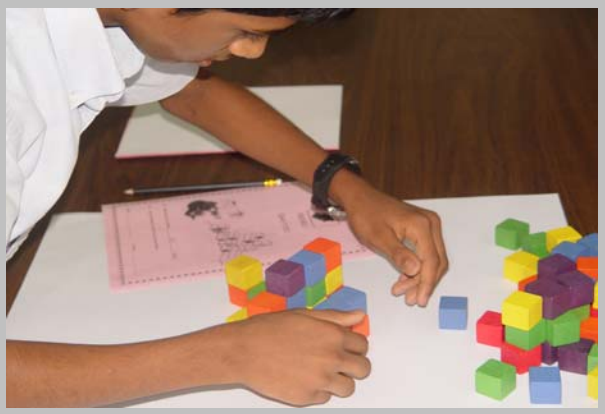


Table of Contents

ASSESSMENT AND EVALUATION

Nature and Functions of Assessment and Evaluation 4–1

Evaluation Standards 4–2

Evaluation Tools and Strategies 4–4

Examples of Different Recommended Assessment Strategies 4–7

Bibliography 4–8

NATURE AND FUNCTIONS OF ASSESSMENT AND EVALUATION

Assessment is a method for collecting data about what students know, understand, and are able to do. When the quantitative or qualitative descriptions obtained from assessments are judged in relation to some goal, objective, or outcome, the process is called evaluation. The purpose of evaluation is to provide feedback to:

- **students and parents** on the students' progress towards achieving the learning outcomes of the curriculum, and to assist the students in identifying their strengths and weaknesses and to guide them in decision making when choosing career options;
- **teachers** on the instructional process, to guide them in programme planning to meet the needs and interests of their students and the national curriculum, and to improve their delivery of the curriculum;
- **administrators, educators, stakeholders, and policy makers** on the effectiveness of the curriculum in achieving the goals of education, and in the placement of students in appropriate groupings to continue their education.

The purpose of assessment is to collect sufficient data about each student in order to make informed judgements based on the use of valid, reliable, and relevant measuring instruments.

An instrument is reliable when it consistently measures an individual's performance, giving the same results. An instrument is valid when it measures what it is designed to do. According to the National Council of Teachers of Mathematics (NCTM) in the United States, "A standard is a statement that can be used to judge the quality of a mathematics curriculum or methods of evaluation. Thus standards are statements about what is valued." The mathematics curriculum is designed so that the learning outcomes for each content strand are derived from the goals of mathematics, which are linked to the goals of education and the essential learning outcomes. In summary, the intent of these goals is that students will become mathematically empowered and technologically competent to make an effective contribution to their society, thus realizing the vision of the mathematics curriculum.

Attainment targets or benchmarks are established to ensure that standards set are being achieved. Attainment targets cover the range of knowledge, skills, and understanding that students are expected to master as they progress towards the standard.

Assessment should reflect the instructional process, but it is distinctly different to it. The curriculum is designed to be delivered using student-centred approaches and assessment is considered an integral part of the instructional process. Thus, it is necessary to examine student performance in order to measure understanding and competence in the content covered. Performance assessment gained popularity through an effort to deal with the limitations of traditional paper and pencil testing (Wraga, 1994) and the application of this mode of assessment has grown significantly within recent times.

Evaluation and assessment should be used to determine the readiness of the student to receive a new concept or skill. The process should also be used to check the progress of students at various points of the instructional process. This is referred to as formative or continuous assessment and evaluation. When the process is conducted at the end of instruction this is referred to as summative assessment and evaluation.

Standard setting is also necessary to make clear what will be considered an acceptable level of performance for each activity. Consequently, the establishment of evaluation standards that are relevant and appropriate is essential. For each curriculum area, however, these standards will focus on features of performance that are unique to that subject and level of operation.

EVALUATION STANDARDS

The evaluation standards for student assessment are derived from the vision and goals of the mathematics curriculum and are summarized as follows:

1. **Mathematical Power** – the ability to explore, make conjectures, reason logically, compute efficiently, and solve problems using a variety of mathematical methods.
2. **Mathematical Thinking** – the ability to be a creative and critical thinker who can solve both routine and non-routine problems.
3. **Mathematical Inquiry** – the ability to investigate one's environment using scientific and statistical processes.
4. **Mathematical Communication** – the ability to use mathematical terminology, symbols, and representations efficiently and accurately and to present mathematical responses orally, graphically, or in writing, clearly, coherently, and concisely.
5. **Mathematical Reasoning** – the ability to recognize the logic in a mathematical argument through its structure and principles, and to present proofs, verifications, and justifications as evidence.
6. **Mathematical Problem Solving** – the ability to apply the four steps in solving problems (understanding, planning, solving, and reviewing) to issues or tasks related to real-life situations across all strands in the mathematics curriculum.
7. **Mathematical Knowledge, Concepts, and Procedures** – the ability to demonstrate number and spatial sense, master computation, make estimations, and apply mathematical skills to their everyday lives.
8. **Mathematical Disposition** – attitudes to others and to mathematics, motivational level, willingness to take risks in mathematical thinking, and to work as part of a team.
9. **Mathematical Connection** – students' ability to relate mathematical concepts and ideas within the discipline, across other subject areas, and to real-life situations.

Thus, it must be reiterated that in meeting the standards articulated for each module of the curriculum, there are different levels of performance that need to be qualified and these must be explained in terms of the pre-defined performance indicators.

Levels of performance standards in mathematics are defined as follows:

- **Level 1 –** performance is below expectation
- **Level 2 –** performance is satisfactory but some assistance is required
- **Level 3 –** performance is satisfactory and independent
- **Level 4 -** performance is beyond expectation

Performance indicators are descriptions of exactly what students would be able to do at each level for each standard and learning outcome. It is these descriptive statements that are used in alternative assessment and authentic assessment procedures to qualify different levels of performance in designing rubrics.

Rubrics are "printed sets of guidelines that distinguish performances or products of different quality" (Blum & Arter, 1996). Standards, levels, and indicators of performance allow educators to measure a wider range of learning and to do so using a variety of strategies other than paper-pencil tests.

EVALUATION TOOLS AND STRATEGIES

Evaluation tools are the measuring instruments that are selected for assessing students' learning. They are designed to examine and address a cross-section of issues and concerns that relate to learning.

Considerations

When selecting a measuring instrument, teachers should:

- align the instrument with the learning outcome that is being measured. For example, students constructing triangles should be measured using a checklist and observation while students are engaged in an activity. Measurement could be done over the entire unit of instruction. The teacher could decide to collect a series of marks using a variety of strategies and use these marks to evaluate the students at the end of the unit. This may contribute to the students' course work evaluation;
- ensure that the items to be used for measuring are reflective of the three levels of thinking in mathematics—knowledge/recall/computation, algorithmic thinking, and problem solving;
- choose content that is meaningful, relevant, and of interest to students;
- ensure that the duration of the activity is sufficient and appropriate to the age and level of development of the students;
- prepare and pilot the measuring instrument before the assessment date;
- decide on the rubric, grading process, or marking scheme before using the instrument;
- decide on the performance indicator before using the instrument.

Tools and Strategies

There are many strategies, other than the paper-pencil test, that can be used for assessment. Among those suggested in the course outline are journals, presentations, projects, interviews, observation checklists, portfolios, activities, performance tasks, record keeping, worksheets, mental quizzes, homework assignments, and peer and self-evaluation. Some details associated with these strategies and the basic purposes for which they may be used are listed below.

1. *Pencil and paper tests*

These provide opportunities for students to:

- show understanding of a concept in various ways. For example, represent $3x + 2y$ concretely and pictorially;
- explain, compare, describe, and show what they know;
- demonstrate mastery of skills (solve, calculate);
- decide which mathematics to apply and which strategies to use to solve problems.

2. *Journal writing*

This provides opportunities for students to:

- convey understanding;
- explain their approach to solving a problem;
- respond to prompts, for example, "What was the main idea of today's lesson?" or "Explain to a friend in a letter how the formula for area of a circle was developed".

3. *Presentations*

These provide opportunities for students to:

- explain to the class their solution to a problem;
- report on findings;
- demonstrate creativity and the use of aesthetic expression;
- learn to become active listeners;
- become critical thinkers and learn questioning techniques;
- value others' ways of thinking as they listen to each other.

4. *Projects*

These provide opportunities for students to:

- consolidate different areas of mathematics;
- use research skills and technology;
- use strengths and interests, and enhance multiple intelligences;
- acquire self-confidence and respect for self and others through collaboration and discussion;
- acquire skills in self- and peer evaluation by their awareness of the criteria for assessment, as communicated in rubrics;
- view examples of excellence.

5. *Interviews*

These provide opportunities for students to:

- be assessed meaningfully through engagement in a social context;
- interact with teachers either individually or in groups to assess understanding of concepts—a useful strategy for remediation;
- respond to higher-order questioning and also to ask higher-order questions.

6. *Observation checklists*

The teacher observes pre-determined behaviours and uses a checklist to record observations. The technique provides opportunities for students to:

- be observed by the teacher as they work individually, in pairs, or in groups;
- work in their natural way without being interrupted by the assessor.

7. *Portfolios*

These provide opportunities for students to:

- collect samples of their work so that they themselves can see evidence of progress in mathematics;
- reflect on what they already know, what they have recently learnt, and what they still need to know;
- observe their self-development;
- express themselves aesthetically.

8. *Performance tasks and activities*

These provide opportunities for students to:

- relate mathematics to real-life issues through authentic activities;
- acquire positive attitudes to mathematics;
- acquire and integrate knowledge;
- extend and refine knowledge;
- model productive habits of the mind;
- use manipulatives to demonstrate understanding of concepts, skills, and problems.

9. *Peer and self-evaluation*

These provide opportunities for students to:

- provide feedback in a constructive manner;
- show respect for each other;
- become aware of their own thinking (metacognition).

EXAMPLES OF DIFFERENT RECOMMENDED ASSESSMENT STRATEGIES

1. Learning Outcome: Represent large and small numbers in scientific notation

Suggested Assessment Strategy: Journal entry

Methodology: Students are asked to write, in their journals, two examples of numbers represented in scientific notation using a negative and a positive power of 10, respectively. Through discussion they will indicate clearly what each number represents and rewrite each number in decimal form. The teacher will examine each student's journal for conceptual understanding of representing numbers in scientific notation.

2. Learning Outcome: Calculate the volume of prisms

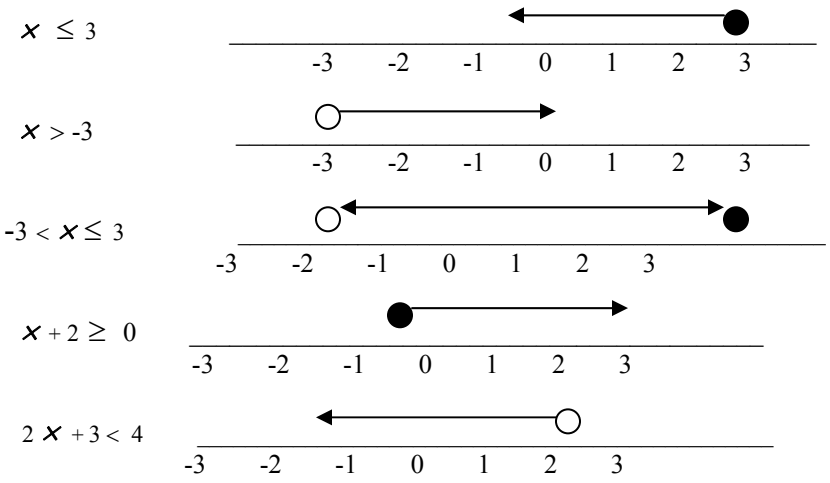
Suggested Assessment Strategy: Make a portfolio

Methodology: Students present the solutions to three problems in a portfolio, being as creative, efficient, and accurate as possible. The teacher will review each student's solutions, and students will have the opportunity to improve on their responses based on guidelines given by the teacher. This interchange will continue until the final submission date for evaluation.

3. Learning Outcome: Solve linear inequalities using a number line

Suggested Assessment Strategy: Observation of the representation of solutions on a number line

Methodology: Students will display the solution of linear inequalities on a number line using the appropriate symbolism. Example:



The teacher marks each display as it is completed by the student.

Bibliography

Blum, R. E., & Arter, J. A. (1996). *A handbook for student performance assessment in an era of restructuring*. Alexandria, VA: Association for Supervision and Curriculum Development.

Gardner, H. (1997). Our many intelligences: Kinds of minds. *The Mini Page* (a children's supplement to The Gazette, edited by Betty Debnam).

Herman, J. L., Ashbacher, P. R., & Winters, L. (1992). *A practical guide to alternative assessment.* Alexandria, VA: Association for Supervision and Curriculum Development.

Marzano J. R., Pickering, D., & McTighe, J. (1993). *Assessing student outcomes: Performance assessment using the dimensions of learning model.* Alexandria, VA: Association for Supervision and Curriculum Development.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Wraga, W. G. (1994). Performance assessment: A golden opportunity to improve the future. *NASSP Bulletin, 78*(563), 71–79.

MATHEMATICS

5

GLOSSARY OF MATHEMATICAL TERMS

A

Acute Angle
An angle the size of which is between 0 and 90 degrees.

Algebraic Equations
Mathematical statements involving an equal sign and at least one variable.

Algebraic Expressions
A set of numbers, symbols, and/or letters connected by '+' or '-' signs,
e.g., $3x + 2y – 5$.

Algebraic Term
A part of a polynomial, e.g., in the expression $5x^2 + 3x + 1$, $5x^2$, 3x, and 1 are all terms.

Algorithm
A systematic step-by-step process that can be followed in arriving at conclusions.

Amount
The sum of money received or repaid on an investment or loan at the end of a specific period.

Angle
A measure of how much something has turned.

Appreciation
The increase in value of some asset.

Approximation
Any amount that is close to, but is not the exact amount.

Arc
A line that connects two points along the *circumference* of a circle.

Area
A measure of the amount of surface of a plane shape.

Array
An ordered set of data.

Axes
Reference lines on a graph.

B

Bar Chart
A graph that uses separate bars of different lengths to show and compare data.

Bar Graph
Same as **Bar Chart.**

Base
The size of the groups used when counting things.

Binary
Base two.

Bisect
Cut into two equal parts.

C

Capacity
The internal volume of a container, i.e., the amount a container will hold when filled.

Cartesian Coordinate
The position of a point on a grid can be given by a pair of numbers, which are its distances from two lines called the *x-axis* and the *y-axis*. The first point on the x-axis is called the *x coordinate* and the second on the y-axis is called the *y-coordinate*.

Centre of Enlargement
The point from which lines are drawn to construct the enlargement of a diagram.

Centre of Rotation
The fixed point from which rotated points move so that they stay the same distance away.

Circle
The path traced out by the set of all points in a plane at a fixed distance from a fixed point called the centre.

Circumference
The perimeter or distance right around the edge of a circle.

Closure
A set is closed under an operation when any two elements of the set combine, using the operation, to give another element belonging to the set.

Commission
Percentage of a business transaction paid to the agent for his/her work.

Complementary Angle
Two angles that add up to 90 degrees.

Composite Number
A whole number that is composed by multiplying two other whole numbers. The dots of composite numbers can be lined up to form rectangles.

Compound Shape
A shape made up of more than one simple geometrical shape.

Congruent Triangles
Triangles that are identical in every dimension. They have same shape and size.

Constant
A number or variable that maintains a fixed value.

Continuous
When a variable can take any number in an interval it is said to be continuous.

Convert
Change to another form.

Cube
A solid that has *squares* for all of its faces.

Cuboid
A solid that has *rectangles* for all of its faces.

D

Data
Name given to a collection of numerical facts or information.

Decimal
A number written in base 10 notation.

Denary
Base 10.

Depreciation
The reduction in value of some asset.

Diameter
Any straight line that joins two points on the circumference of the circle and passes through the centre of the circle.

Dimension
The dimension of a space is the number of coordinates needed to fix the position of a point in it.

Discount
The amount of money deducted from the marked price of an item, so that a lesser price is paid for it.

Discrete
A variable is said to be discrete if it can take just whole number values.

Disjoint Sets
Sets that have no members in common. The intersection of the sets is the empty set.

Domain
The domain of a function is the set of elements on which the function operates.

E

Edge
The line where two faces of a solid meet.

Element
An individual member of a set.

Elimination
A method used to remove one variable from two equations in order to find a value for one of them.

Empty Set
A set that has no elements in it. It is symbolized by ∅ or { }.

Enlargement
A transformation that changes the size of an object without changing its shape.

Equal Sets
Sets that contain exactly the same elements, no matter the order.

Equivalent Sets
Sets with the same number of elements, no matter the elements.

Estimation
An answer that is close to the exact answer and may be found by "rounding off" or applying approximations.

Exterior Angle
Angle formed when the side of a polygon is extended beyond the vertex. It is the angle that is formed between the extended side and the adjacent side.

F

Face
A distinct surface of a solid shape.

Factor
A whole number that divides into other whole numbers exactly.

Factorization
The method used to express a polynomial expression as a product of its factors.

Finite Set
A set that contains a countable number of elements.

Formula
A general rule used to show the connection between related quantities.

Fraction
A part of a whole. It is commonly represented by a number less than 1.

Frequency
The number of times a particular item or event occurs when collecting data.

Frequency Polygon
A line graph that is formed when the mid-points of the tops of the bars in a *histogram* are joined.

Frequency Table
A tabulation of data with the matching frequency with which each value occurs.

Function
A rule connecting two sets such that for each item in the first set there is just one item that it is related to in the second set.

G

Gradient
The steepness or slope of a line.

Graph
A diagrammatic representation of data or information.

H

Highest Common Factor (HCF)
The largest number that can divide all of a set of numbers exactly.

Hire Purchase
A system of paying part of the cost of an item and then paying the rest in equal monthly or weekly amounts called installments.

Histogram
A frequency graph similar to the bar graph but with the bars touching along their sides, and in which the area of each bar, rather than the height of each bar, gives the frequency.

Hypotenuse
The longest side of a right-angled triangle. It is the side that is opposite to the right angle.

I

Identity Element
A number that when combined with another number under an operation remains unchanged.

Image
The result when an object is transformed.

Image Point
A point after it has undergone a transformation.

Imperial System
Traditional units of measurement, e.g., inches, feet, pounds, tons.

Index
A number or variable placed as a superscript to the right of another number or quantity indicating the number of times that the number or quantity is to be multiplied by itself (e.g., for 2^4, 4 is the index). [See also *power*]

Inequality
A statement showing the comparison between two quantities. Such a comparison usually involves the use of the inequality signs: $<$, $\leq$, $>$, $\geq$, e.g., $2x \geq 8$.

Infinite Set
An infinite set is one that contains a countless number of elements.

Interior Angles
The angles formed at the vertices inside a plane figure.

Intersection
The intersection of two sets A and B is the set that contains elements that are common to both sets. The intersection of sets is shown by using the symbol $\cap$ (e.g., $A \cap B$).

Inverse Element
Under a particular operation, it is the number that combines with a selected number to give the *identity element* for the operation.

Investment
The use of money to make a profit.

Irrational Number

A number that cannot be written in the rational form $\frac{a}{b}$, where *a* and *b* are integers and *b* is not zero, e.g., $\sqrt{7}$.

K

Kite

A quadrilateral with two pairs of equal sides and one *line of symmetry*.

L

Line Graph

A graph formed by plotting points and connecting these points to form a line.

Line of Symmetry

A line about which a shape can be reflected onto itself.

Line Segment

Part of a straight line between two given points.

Linear Equation

An equation in which the highest degree or power of any variable is 1.

Loss

The difference in value when an item is sold for less than what was paid for it.

Lowest Common Multiple (LCM)

The smallest number that any number from a specified set of more than one number can divide exactly.

M

Manipulatives

Tactile resources that are used to facilitate teaching and learning.

Mapping
A *relation* in which every element from the first set or domain relates to an element in the range.

Mass
A measure of the amount of matter obtained in an object.

Mean
This is obtained by dividing the sum of all the data by the number of items of data (also called *arithmetic mean*).

Median
The middle value (or mean of the two middle values) of a set of data once the data have been arranged in order of size.

Mode
The most frequently occurring item.

Multiple
A multiple of a number *n* is any number that n divides exactly, i.e., *k* x *n* where *k* is a counting number.

N

Natural Numbers
These are the counting numbers: 1, 2, 3, 4, 5,…..

Net
A plane shape that, when folded, can be made into a three-dimensional or solid shape.

Nonagon
A plane shape with nine sides.

O

Object Point
A point before it has undergone a transformation.

Obtuse Angle
An angle of more than 90 degrees but less than 180 degrees.

Octagon
A plane shape with eight sides.

Operation
A way of combining numbers or elements in a set. The most common kind of operation is a binary operation that combines two numbers to give a single number, e.g., for the binary operation '+' , 3 + 2 = 5.

Ordered Pairs
A pair of numbers in which there is a first number and a second number and the order in which they are presented is important. Ordered pairs are usually presented in curly brackets, e.g., {a, b}.

P

Parallel Lines
Lines that never meet, no matter how far they are extended.

Parallelogram
A *quadrilateral* formed by two pairs of parallel lines.

Percentage
A percentage is a way of expressing a fixed fraction of a hundred, e.g., 10% = $\frac{10}{100}$.

Perimeter
The distance around the outside of a shape.

Perpendicular Lines
Lines that cross at right angles.

Pictograph
A pictorial representation of information, similar to a bar graph, but using appropriate symbols to correspond to the frequencies of different kinds of data.

Pie Chart
A representation of information in statistics in a visual form, using a circle that is divided into sectors, so that the areas of the sectors represent the data.

Place Value
The value of a digit because of its position in the numeral representation of number.

Plan
A drawing or diagram that gives a layout of how something is arranged. A design plan is a view from above.

Plane
A flat surface. A *plane shape* is a shape that can be drawn with all its points in one plane.

Plane Shape
A shape that can be drawn with all its points in one plane.

Point
A position in space. It has no size and can be located by giving its coordinates relative to some fixed point of origin.

Polygon
A *plane shape* bounded by only straight lines.

Polynomial
An algebraic expression containing two or more terms.

Population
A target group from which *samples* are drawn for statistical analysis.

Power
Same as *index*.

Prime Number
A number that has two and only two factors, i.e., itself and 1. Examples of prime numbers are 2, 3, 5, 7, 11, 13, …

Principal
The sum of money borrowed or invested.

Prism
A solid shape with flat faces that are all polygons and has the same cross-sectional shape along its length.

Probability
The measure of how likely an event is. The probability of an event is represented by a number between 0 and 1.

Product
The result of multiplying two or more numbers.

Profit
The difference in value when an item is sold for more than what was paid for it.

Proportion
Two sets are proportional to one another when one set can be written as a *constant* times the other.

Protractor
An instrument used for measuring angles.

Pythagoras' Theorem
This states that in a right-angled triangle, the area of the square on the longest side is equal to the sum of the areas of the squares on the two smaller sides.

Q

Quadrant
Each of the four regions into which the x-axis and the y-axis divide the Cartesian plane.

Quadratic Expression
A polynomial in which the highest power of the variable(s) is two.

Quadrilateral
A plane shape with four sides.

Quotient
The answer you get when you do a division.

R

Radius
The distance from the centre of the circle to a point on the circumference.

Random
Numbers or objects that are chosen without bias.

Range
The set of numbers onto which the elements of the first set or domain in a *relation* is mapped.

Rate
A percentage of the sum of money borrowed or invested, paid on a periodic basis.

Ratio
A comparison of two quantities expressed in like units.

Rational Number
A number that can be written in the form $\frac{a}{b}$, where a and b are integers,
e.g., $\frac{5}{8}, \frac{12}{3}, \frac{7}{1}$.

Real Number
Any number that corresponds to a point on a number line. The set of real numbers is the union of the set of rational numbers and the set of irrational numbers.

Rectangle
A quadrilateral formed by two pairs of parallel lines meeting at right angles.

Rectangular Number
A number with two factors that are not equal or equal to 1—composite numbers. The dots of composite numbers can be lined up to form rectangles.

Recurring Decimal
A decimal where the pattern of digits after the decimal point keeps repeating itself, e.g., 0.3333…. or 0.23823823823….

Reflection
A transformation that transforms a shape in the same way as a mirror.

Reflex Angle
An angle of size between 180 and 360 degrees.

Relation
A rule that connects the elements of two corresponding sets, such as sets of things, numbers, or people.

Rhombus
A quadrilateral with four sides of equal length.

Right Angle
An angle that measures 90 degrees. A quarter turn.

Root of an Equation
Any number or value for the variable that satisfies the equation. It is the solution to the equation.

Rotation
A transformation that moves points so that they remain the same distance from a fixed point called the centre of rotation.

S

Sample
A group of people or objects chosen from a larger group to provide data to make predictions about the larger group.

Sector
The shape enclosed by two radii and an arc of the circle.

Set
A collection of well-defined things.

Similar Triangles
Triangles with the same shape but different in size.

Simple Interest
The amount of money paid or earned on a sum of money over a given period of time.

Simultaneous Equations
Two or more independent equations involving more than one variable for which there is a common set of roots.

Speed
The distance travelled by a body per unit of time, e.g., 50 km/h.

Statistics
The branch of mathematics concerned with the collection, organization, representation, interpretation, and analysis of data and the making of predictions based on the information gathered.

Straight Angle
An angle of 180 degrees.

Subsets
If every element of a set B is also an element of the set A, then B is a subset of A.

Substitution
The process of replacing letters in a formula or expression with numbers.

Supplementary Angles
Two or more angles that add up to 180 degrees.

Surd
An *irrational* number, e.g., $\sqrt{2}$.

Symmetry
The way that a shape can be reflected or rotated to fit onto itself.

T

Tangent
A straight line that touches a circle or curve at a single point.

Temperature
Degree of heat as shown by a thermometer. It is usually measured in degrees Celsius (^{o}C) or in degrees Farenheit (^{o}F).

Terminating Decimal
A fraction that can be worked out exactly to a fixed number of decimal places (i.e., without recurring).

Tessellation
A way of covering a plane shape using a pattern and without leaving gaps.

Transformation
A change from one position to another position by a defined movement.

Translation
A transformation in which a shape slides without turning. Every point moves the same distance in the same direction.

Transversal
A line drawn across two or more parallel lines.

Trapezium
A quadrilateral with one pair of opposite sides parallel.

Triangle
A plane shape with three straight sides.

Triangular Number
A number that can be shown as a triangle of dots, e.g., 3, 6, 10…

U

Union
The union of two sets A and B is the set that contains all elements that are in A, or in B, or in both A and B. The union of sets is shown by using the symbol $\cup$ (e.g., $A \cup B$).

Unitary Method
A method of carrying out a calculation to find the value of a number of items by first finding the cost of one of them.

Universal Set
The set of all elements being considered.

V

Variables
Letters or symbols that are used to represent any of a variety of different numbers.

VAT
Value Added Tax. A charge of 15% on specified goods and services.

Venn Diagram
A visual display that is used to represent sets and their relationship with each other. A typical diagram will show each set represented by a circle inside the *Universal set*, which is represented by a rectangle.

Vertex
The point where the sides of a plane shape or solid meet.

Vertically Opposite Angles
A pair of angles that are opposite to each other, where two lines cross.

Volume
A measure of the amount of space that a solid occupies. Also called capacity.

W

Whole Numbers
The set of natural or counting numbers in addition to the number 0, e.g., 0, 1, 2, 3, 4,...

Z

Zero
The identity element under addition of numbers. The first whole number.